Contents

List of Figures xi
Acknowledgments xiii

1. Infinity, Paradox, and Mathematics 1
   1. Paradox and Causal Finitism 1
   2. Some Mathematical and Logical Notes 4
   3. Modality 7
      3.1 Metaphysical possibility and necessity 7
      3.2 Rearrangement principles 7
         3.2.1 Defeasibility 7
         3.2.2 Causal powers 9
   4. Finitism: An Alternate Hypothesis 10
      4.1 Time and finitism 10
      4.2 Non-causal paradoxes: An advantage? 11
      4.3 Mathematics: A disadvantage 13
         4.3.1 Infinitely many primes 13
         4.3.2 Potential infinity 15
         4.3.3 *If-thenism 15
      4.4 Future infinities 17
   5. *Defining the Finite and the Countable 18
      5.1 The finite 18
      5.2 Acceptable models for the axioms of arithmetic 20
   6. Evaluation 23
   Appendix: *Counting Future Things 23

2. Infinite Regresses 25
   1. How to Violate Causal Finitism 25
   2. Infinite Causal Regresses 26
   3. Type (i): Uncaused Regresses 27
      3.1 Viciousness 27
      3.2 Vicious regresses and the Hume–Edwards Principle 29
      3.3 Regresses and explanatory loops 30
   4. Type (ii): Causation Passing through Infinitely Many Steps 32
   5. Type (iii): Outside Cause Directly Causing Each Item 33
      5.1 Options 33
      5.2 Regresses with outside overdetermination 35
   6. *Analogy with Axiom of Regularity 36
   7. Evaluation 37
   Appendix: *Two Kinds of Violations of Causal Finitism 37

OUP UNCORRECTED PROOF – REVISES, 30/5/2018, SPi

3. Supertasks and Deterministic Paradoxes 40
1. Introduction 40
2. Thomson's Lamp Revisited 40
2.1 Introduction 40
2.2 Causal finitism 40
2.3 Non-standard analysis 41
2.4 Special Relativity 42
2.5 Benacerraf's solution and the Principle of Sufficient Reason 43
2.6 Two counterfactuals 44
2.7 Evaluation 46
3. Grim Reapers 46
3.1 Introduction 46
3.2 Causal finitism 47
3.3 The absurd conclusion objection 48
3.4 A rearrangement objection 49
3.5 The mereological objection 50
3.5.1 Fusion 50
3.5.2 Necessary emergence of organic wholes 52
3.6 Uncaused lighting 52
3.6.1 Objection 52
3.6.2 The Causal Principle is true 53
3.6.3 Is the lamp lighting really uncaused? 54
3.6.4 A mysterious correlation 55
3.7 Discrete time 55
3.8 Evaluation 56
4. Infinite Newtonian Universes 56
4.1 An argument against causal finitism and a riposte 56
4.2 Smullyan's rod 58
4.3 The conditional 60
5. Another Eternal Life 60
6. Time Travel and Causal Loops 61
6.1 Grandfathers and togglers 61
6.2 Time travel and backwards causation without causal loops 63
7. Evaluation 63

4. Paradoxical Lotteries 64
1. Introduction 64
2. Countably Infinite Fair Lotteries 64
2.1 Background 64
2.2 Expected surprise 65
2.3 A guessing game 66
2.4 Symmetry 66
2.4.1 Symmetry and lotteries 66
2.4.2 *Symmetry and expected utility 68
2.5 Bayesian manipulation 71
2.5.1 The paradox 71
2.5.2 *A switchover point? 74

OUP UNCORRECTED PROOF – REVISES, 30/5/2018, SPi

2.5.3 *Countable additivity and conglomerability 75
2.6 Improving everyone's chances 77
3. Constructing Paradoxical Lotteries 79
3.1 Fairness and paradoxicality 79
3.2 Lucky coin-flip sequences 79
3.3 What it is to construct a countably infinite fair lottery 81
3.4 *Coin-flips and the Axiom of Choice 83
3.5 Random walks 85
4. Objections 86
4.1 Infinite lotteries and uniform distributions 86
4.1.1 The problem 86
4.1.2 Response I: No continuous distributions 87
4.1.3 Response II: Measurement of infinite precision data 88
4.1.4 Response III: The use of the Axiom of Choice 88
4.2 *A non-normalizable quantum state 89
4.3 Limitations on our reasoning 90
5. Evaluation 91

5. Probability and Decision Theory 93
1. Introduction 93
2. Guessing with Finitely Many Errors 93
2.1 Doing a little better than one can 93
2.2 A contradiction 95
2.3 Doing much better than one can 96
2.4 *Construction of strategy guaranteeing at most finitely many errors 98
2.5 A multipersonal synchronic version 98
2.5.1 An angelic announcement 98
2.5.2 An objection and a tweak 100
2.5.3 *Making the paradox robust 101
2.6 A parody? 102
2.6.1 The story 102
2.6.2 Evaluating the parody 103
3. Satan's Apple 106
3.1 The story 106
3.2 Synchronic version 107
3.3 Diachronic version 108
3.4 Objection: Scores, desires, and promises 108
3.5 Evaluation 110
4. Beam's Paradox 111
4.1 *The mathematical formulation 111
4.2 *Synchronic version 112
4.3 *Diachronic infinite future version 113
4.4 *Diachronic supertask version 114
4.5 Evaluation of Beam's paradox 114
5. Evaluation of Decision-Theoretic Paradoxes 114
Appendix: *Proof of the Theorem from Section 2.1 115

OUP UNCORRECTED PROOF – REVISES, 30/5/2018, SPi

6. The Axiom of Choice Machine 117
1. Less Technical Introduction 117
2. *The Axiom of Choice for Countable Collections of Reals 119
3. *Paradoxes of ACCR 120
3.1 Die-guessing games 120
3.2 Non-measurable sets 120
3.3 Banach–Tarski paradox 121
4. *An Argument for ACCR 122
5. *A Choice Machine 125
5.1 Strange mathematics and paradox 125
5.2 Coin-flips and Dutch Books 129
5.3 How to construct a Choice Machine 130
5.3.1 Angels 130
5.3.2 A four-dimensional machine 131
5.3.2.1 Making the machine 131
5.3.2.2 Using the machine 133
5.3.2.3 Causal infinitism and verifying the machine's match 134
5.3.3 A three-dimensional machine 136
5.3.4 **Is AC needed? 136
5.3.5 Luck 137
6. Evaluation 137
Appendix: **Details of Coin-Toss Rearrangement 138
7. Refinement, Alternatives, and Extensions 140
1. Introduction 140
2. Refinement 140
2.1 Event and trope individuation 140
2.2 Histories generated by partial causal relations 142
2.3 A closer look at Grim Reapers 143
2.4 Objections to causal finitism involving partial causation 147
2.5 Absences and omissions 148
3. Some Competitors to Causal Finitism 150
3.1 Finitism 150
3.2 No infinite regresses 151
3.3 No past infinities 151
3.4 No infinite intensive magnitudes 153
3.4.1 The basic theory 153
3.4.2 Some infinite intensive magnitudes 154
3.4.2.1 Center of mass and moments of inertia 154
3.4.2.2 Mental life 154
3.4.2.3 Black holes 155
3.4.2.4 Particles 155
3.4.3 Huemer's intensive magnitudes 156
3.4.3.1 Speed, Thomson's Lamp, and Hilbert's Hotel 156
3.4.3.2 *Smullyan's rod 157
3.4.3.3 Immaterial minds 158
3.4.4 Evaluation 159
3.5 No room 159

OUP UNCORRECTED PROOF – REVISES, 30/5/2018, SPi

4. Why is Causal Finitism True? 161
4.1 The question 161
4.2 Some explanatory suggestions 161
5. Further Extensions 162
5.1 Causal loops 162
5.2 Explanatory relations 164
6. Overall Evaluation 165

8. Discrete Time and Space 167
1. Introduction 167
2. Causal Finitism and Discreteness 167
2.1 The basic argument 167
2.2 From discrete time to discrete space? 167
3. Two Kinds of Discreteness 168
3.1 Subdivisibility and fixeity 168
3.2 Refining the Aristotelian picture 169
3.2.1 An objection to Aristotelian discreteness 169
3.2.2 Internal and external discreteness 170
4. Physics 172
4.1 An objection to causal finitism 172
4.2 Causation and physics 172
4.3 Quantum collapse 174
4.3.1 Some background 174
4.3.2 Causation 176
4.3.3 Back to discrete time 177
5. Fields and Discrete Space 178
6. Evaluation 180

9. A First Cause 181
1. Introduction 181
2. An Uncaused Cause 181
2.1 The quick argument 181
2.2 Towards a necessary being 182
2.3 Support for the Causal Principle 183
2.4 The Kalām argument 184
3. Compatibility with Theism? 184
3.1 Theism 184
3.2 Divine motivation 184
3.3 Divine knowledge 186
3.4 Divine action 188
3.5 Limits on metaphysical possibility 191
4. Evaluation 192

10. Conclusions 193

References 195
Index 201