Spectral Analysis of Pendulum Clocks (Tides, part 6)

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Introduction

In the early 1990's Prof. Hall decided to build the ultimate pendulum clock in his retirement. His story has been presented at conferences and in horological literature. With abundant resources and a career of scientific expertise he did everything right: ultra high vacuum, agate suspension, invar rod, invar bob, optical detection, electromagnetic impulse, and digital computer amplitude control. In addition the clock was given its own precise temperature controlled room built upon a massive insulated concrete foundation. The amazing creation is called the *Littlemore Clock*.

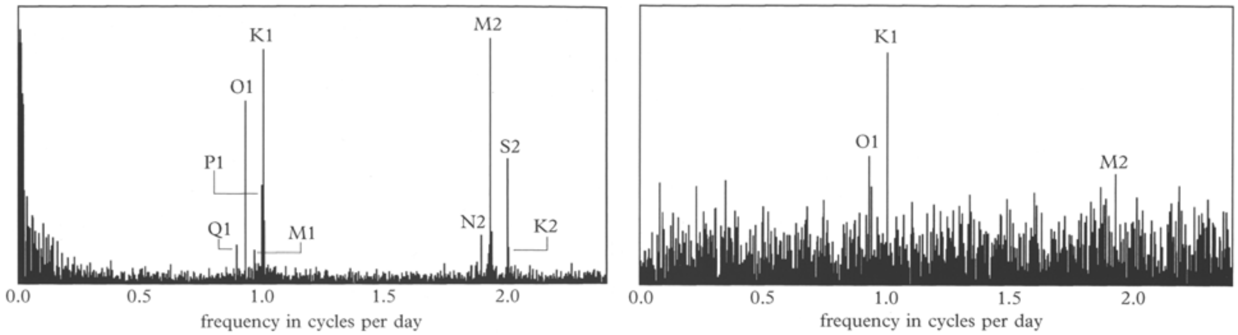
After years of work he gave his data to Philip Woodward for analysis. The expectation was that the Littlemore clock would perform significantly better than a Shortt-Synchronome clock. Alfred Loomis in the 1930's and Pierre Boucheron in the 1980's had shown Shortt was so good that the microscopic effects of lunar-solar solid earth tides were visible in the pendulum data.

The story ends badly. After all that work Littlemore did not see tides as clear as Shortt had. Prof. Hall gave up on the project never knowing for sure what the problem was.

Woodward's spectral plots of Shortt and Littlemore

From nearly a year's worth of hourly data Woodward created these two spectral plots.

Fig 1ab: Woodward's spectral plots from raw data for Shortt (left) and Littlemore (right):

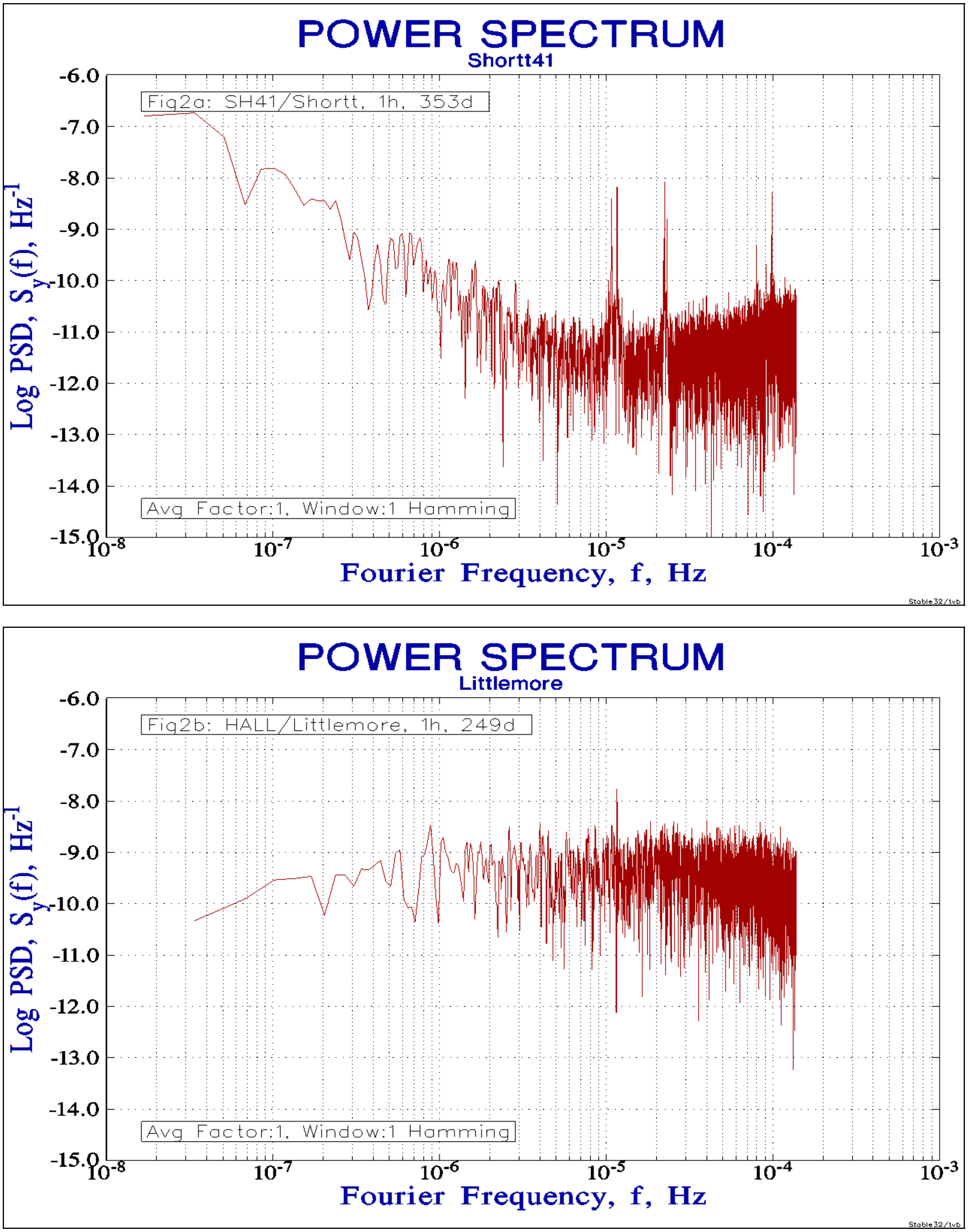


Many spectral peaks corresponding to known tidal effects are clearly visible in the Shortt data. But few of them also appear in the Littlemore data. In addition, there is dramatically more "background noise" in the Littlemore data. Most of the tidal signals, even if present, are lost in that noise. In previous articles I focused on the tidal *signal*. In this article the focus is on clock *noise*. The ratio of signal to noise (S/N, or SNR) is important in clocks.

Using Stable32 to generate spectral plots

Among the tools I use to analyze clock data is *Stable32*. Although the software was designed for laboratory quartz and atomic clocks it works equally well for data from tuning forks, watches, pendulums, and planets. Using the same raw data as Woodward the following are my two plots.

Fig 2ab: Spectral plots from raw data for Shortt (39º N, 1984) and Littlemore (52º N, 1995):



About spectral plots

Although the style of these plots is somewhat different than Woodward, they agree on the spectral lines. One difference is that these PSD (Power Spectral Density) plots are *calibrated*, that is, they have a y-scale (power) as well as the x-scale (frequency). This allows one to fairly compare PSD of different clocks and also to measure the actual power of signal(s) and noise.

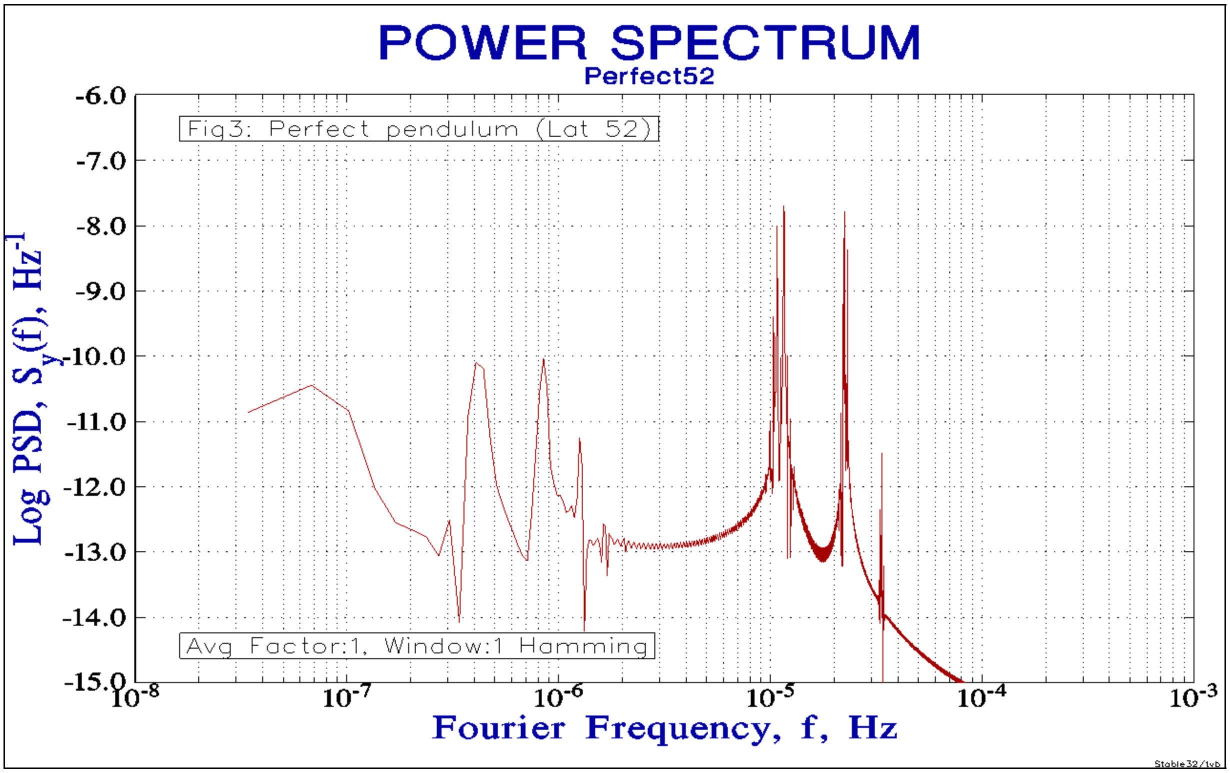
Note that logarithmic scales are used on each axis. Just as faint sounds are measured with *decibels*; dim stars are measured with *magnitude*; distant earthquakes are measured with the *Richter* scale, the power of perturbing periodic forces on a pendulum also lend themselves to a logarithmic scale. The effect of tides is incredibly small but never zero.

Years ago I met Philip and asked about his plots. I learned he wrote his own software; he didn't remember the exact parameters he used to make those two plots; and unfortunately he no longer had access to the files on his old Acorn computer. He may have used DFT. The PSD function in Stable32 is FFT based, but these are all Fourier methods so minor differences are not critical.

Simulating a perfect pendulum

To better understand what the spectral plot of Littlemore could have looked like we start with a simulation of gravity in Oxford, latitude 52° N. Astronomers have long since perfected the ability to predict the positions of the earth-sun-moon system so it is possible to generate a time series of a "perfect" pendulum clock. Slow periodic changes in distance cause changes in earth's acceleration of gravity which cause slow microscopic changes in pendulum period and rate.

Fig 3: Spectral plot from simulated data of "perfect" pendulum clock at 52° N latitude:

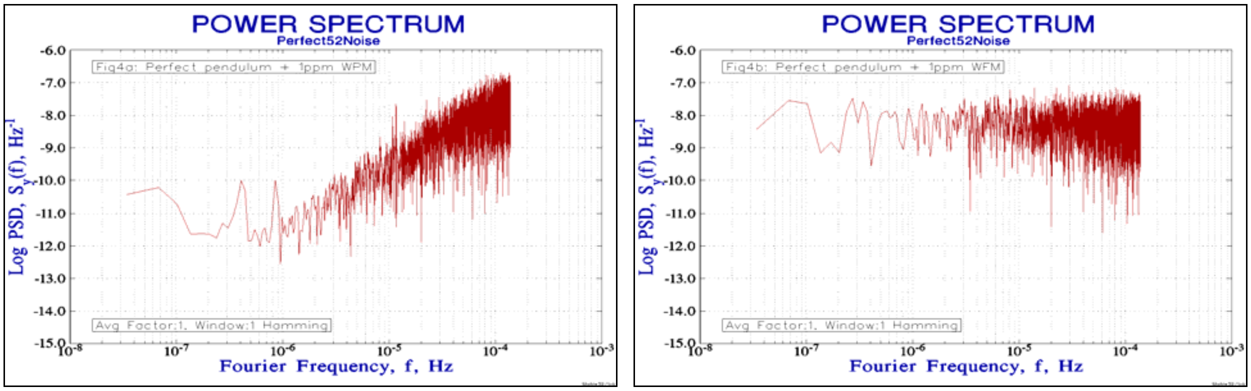


If Littlemore were flawless then Fig 3 is what Hall and Woodward would have seen. Obviously there is no perfect pendulum clock, but this gives us an idea of what the full spectrum of tides would look like if there were no noise in the clock. Notice this perfect pendulum has the same ~24 hour (~1e-5 Hz) and ~12 hour (~2e-5 Hz) tidal peaks at the same magnitude and frequency as seen in the actual data from Shortt and Littlemore. Computer models of g are extremely good.

Simulating a perfectly noisy pendulum

Since a simulation is simply a matter of generating computer data from an astronomical model, we can also simulate random noise, add it to the data, and see what the PSD looks like. There are many types of noise. Two likely candidates are *timing noise* (W**P**M, White **P**hase Modulation) and *rate noise* (W**F**M, White **F**requency Modulation). We can try both and see which one looks more like the actual Littlemore noise. For demonstration, we will add 1e-6 (1 ppm) of noise.

Fig 4ab: On the left we add a tiny bit of *timing* noise vs. on the right a tiny bit of *rate* noise:



The plots are very different. Fig 4a rises up at higher frequencies; Fig 4b is flat and independent of frequency. The latter is much closer to the shape we see in Shortt and Littlemore. Thus we conclude that Littlemore has a case of rate noise, not timing noise.

Timing noise would occur if you had a very good clock but your timing measurements were imprecise. This noise gets in the way of precision but at least it doesn't accumulate. Rate noise would occur if your measurement system was good but the clock had continuous small random variations in rate. It is additive over time because clock error is integrated rate error.

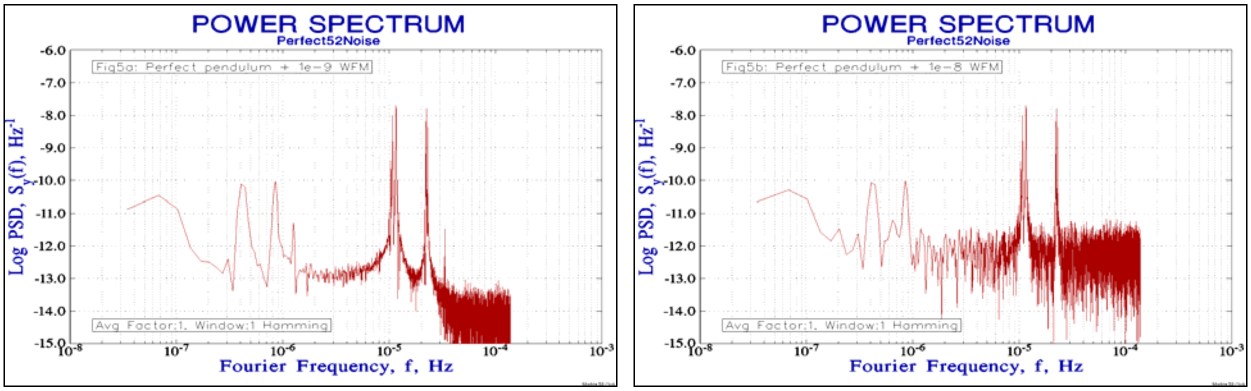
Magnitude of noise

For easy comparison, all PSD plots in this article have been created using the identical x and y scale. Observe that the 1 ppm of noise added to create Fig 4b has totally obscured the tidal peaks in the simulated plot. This alone tells us that any pendulum with more than 1 ppm of noise will never see tides. Comparing the Littlemore clock in Fig 2b with a simulated clock with noise in Fig 4b tells us Littlemore has somewhat less than 1e-6 noise. How much less?

There are several methods to determine this but for the sake of illustration we'll just use the trial and error method. This is one of the advantages of simulation. You can choose some noise level, generate simulated data, and plot it in a few seconds. Try that with a real pendulum clock!

So it's easy to test 1e-9, 1e-8, 1e-7, and 1e-6 levels of noise to see what they each look like.

Fig 5ab: We try 1e-9 WFM noise (left), and 1e-8 WFM noise (right):

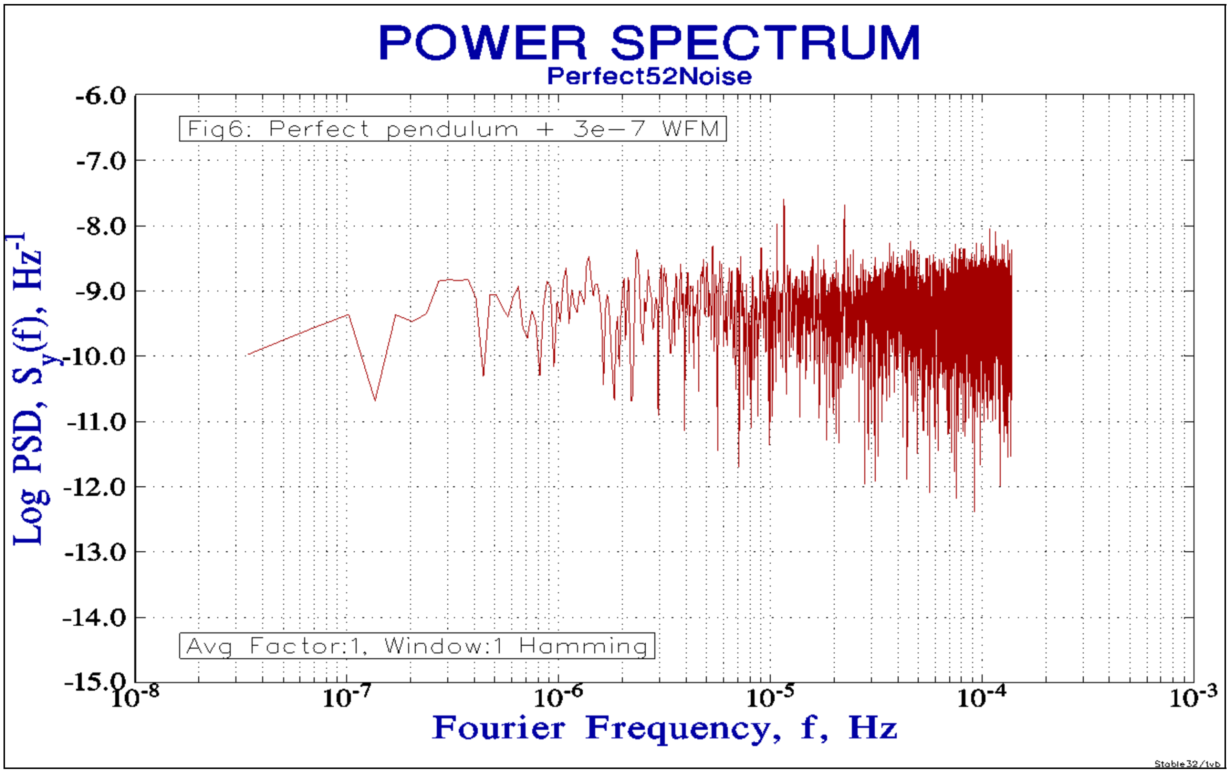


The left plot (1 ppb noise) still looks more like a perfect pendulum clock and not at all like Littlemore. The right plot (10 ppb noise) is closer. So now we know the noise in Littlemore is between 1e-6 (Fig 4b) and 1e-8 (Fig 5b). With a few more trials we find that 3e-7 is a good match against the actual Littlemore data. This is shown in Fig 6 below.

To get a feel for the horological magnitude involved, the standard deviation of hourly rate samples for a perfect Littlemore are about 0.2 ms/day. With 3e-7 noise it is 1 ms/day.

To reinforce the value of simulation, remember that these playful explorations of tides and noise on a computer take only minutes and they tell us a great deal about seeing tides. By contrast Prof. Hall ran his pendulum for 250 days to obtain the same disappointing result.

Fig 6: simulated perfect pendulum with precisely 3e-7 WFM (rate noise):



Because Fig 6 (PSD of a simulated noisy clock) so closely matches Fig 2b (PSD of the actual Littlemore clock) we conclude that Littlemore is dominated by WFM noise at the level of 3e-7 per hourly sample.

For a world-class pendulum this is exceedingly high. Something is very wrong with Littlemore. It is almost as if, as Elvis would say, there's a *Whole Lotta Shakin' Goin' On*. The rate may be relatively constant on average over the long-term but it isn't constant over the short-term. And that short-term noise is hiding most of the tidal peaks that Hall expected to see clearly.

Conclusions

The signal (tides) and noise performance of Shortt and Littlemore have been compared using Fourier analysis, specifically PSD plots. Another clock comparison is added by creating a simulated theoretically perfect pendulum clock located as the same latitude as Littlemore (Oxford, UK). Then different levels of random noise are added to visually explore the ratio of signal to noise. Finally a noise level is found that causes the simulated PSD to visually match the actual Littlemore PSD.

This approach graphically shows the ramifications of signal to noise ratio in detecting tides in a high-precision pendulum clock, such as Shortt or Littlemore. The type of noise in Littlemore is WFM. The magnitude of random noise is approximately 3e-7 in hourly rate. This is enough to significantly cloud tides but not obscure them entirely.

To solve the riddle of Littlemore, one step is to understand the nature of tides. Another step is to quantify the noise in the pendulum clock. Something caused exactly this level of WFM noise in Littlemore (no more, no less). The next step, the next article in this series, will explore that clue.

It should be possible to determine what went wrong with Littlemore from Hall's documentation and from the raw data alone. For me, analysis and simulation are much faster and simpler than the alternative of building a faithful replica of the Littlemore clock and then debugging it.

A final comment – it would be incorrect to claim "Littlemore did not see tides". Rather, it saw some tides, but poorly; far worse than Hall or anyone expected. If nothing else, the progression of spectral plots in Fig 2a (Shortt), Fig 2b (Littlemore), Fig 3 (Perfect), and Fig 6 (Perfect with 3e-7 noise) tells the story.

Notes

v2

– Philip Woodward's spectral plots appear in *Woodward on Time*, 2006
– Much clearer images in Derek Roberts, *Precision Pendulum Clocks*, Vol 3, p 276-277
– For Stable32 (free software) see: http://www.stable32.com/
– Bryan Mumford: *Some thoughts on the Littlemore clock*, HSN 2004-5
– Tom Van Baak: *Lunar/Solar Tides and Pendulum Clocks (part 1)*, HSN 2006-1
– Alan Emmerson: *A Little More*, HSN 2009-1
– Robert Belleville: *Tidal Effects on Pendulums at Various Latitudes*, HSN 2015-2
– Duncan Agnew: *Another Look at the SH41 Data*, HSN 2018-5
– Copies of my HSN pendulum papers: http://leapsecond.com/hsn2006/
– Raw data and full-sized plots for this article: http://leapsecond.com/pend/