Proceedings of the International Symposium on Music Acoustics
(Associated Meeting of the International Congress on Acoustics)
25-31 August 2010, Sydney and Katoomba, Australia

The low down on the double bass: looking for the effects of torsional modes

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PACS: 43.75.DE

ABSTRACT

The action of the bow produces torsional oscillations in a string, as well as the normal transverse motion [1]. The torsional modes have frequencies several times higher than, and not harmonically related to, the transverse modes. Via the strongly non-linear bow-string interaction, the torsional modes are driven at a harmonic of the translational modes [2]. The torsional mode frequencies for wound steel bass strings decreased weakly with increasing tension, while those of twisted gut increased weakly. When inexperienced string players bowed notes, the sound spectra of the relatively long starting transients contained strong components near the first torsional resonance. In another experiment, eight professional bassists both played and listened to notes on steel strings of two brands prepared with different degrees of inharmonicity between torsional and translational modes. As both players and listeners, their responses were highly concordant. They showed no clear preference for harmonically related modes. They did, however, show a clear preference for one of the brands. Perhaps because of the relatively small quality factors (~15-20) of the torsional resonances and the high frequencies at which they occur, it appears that any effects on playing caused by the coupling of inharmonic torsional modes are smaller than those due to other parameters controlled by string makers.

INTRODUCTION

Ideal Helmholtz motion

Helmholtz motion [3] is a simplified model of the motion of a bowed string in periodic motion, in which both bow and string are one-dimensional. This is illustrated in Figure 1.

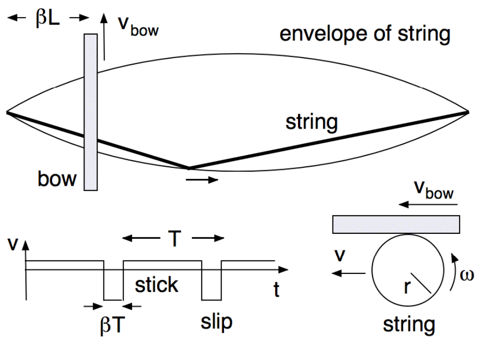


Figure 1. Helmholtz motion. The parabolic curves are the envelope traced by the Helmholtz corner (the kink in the string), which here travels anticlockwise. The graph shows the string speed v at the bowing point. The sketch at lower right shows a string with finite radius.

The string (length L) is bowed at L/β from one end by a bow moving at speed v_{bow}. The motion has period T. At the point of contact, the string travels with the bow ($v = v_{bow}$) for time $T - \beta T$, until the Helmholtz corner arrives from the distant end. It then slips in the opposite direction ($v = - v_{bow}/\beta$) for βT until the corner arrives from the close end.

In normal playing, the fundamental frequency of the sound is largely determined by the lowest member of a set of nearly harmonic translational resonances, with frequencies $nv_{trans}/2L$, where n is an integer, L the string length, and v_{trans} the speed of translational wave in the string. Helmholtz motion at frequency $v_{trans}/2L$ gives a first order approximation of the string motion, but see Figure 2 for some differences.

Complications and the present study

Many of the studies of the motion of a bowed string have considered a range of effects that cause a real string to depart from ideal Helmholtz motion [4-13].

In this paper, we examine some of the torsional properties of steel and gut strings of a double bass, how they influence string motion and sound, and how the relationship between torsional resonances and harmonics of the string vibration influence players' perceptions of the quality of the string.

TORSIONAL MOTION

Torsional complications to Helmholtz motion

On a string with finite radius r, the bow exerts a torque and so excites torsional motion with angular velocity ω, in which case $v_{bow} = v + r\omega$ during the stick phase.

A string whose torsional motion is restricted at the bridge and at the nut or finger is expected to support a set of nearly harmonic torsional resonances with frequencies approximately $nv_{tors}/2L$. However, v_{tors} is typically several times higher than v_{trans}, so the lowest torsional resonance occurs at a frequency several times higher than that of the string vibration, and in general there is no harmonic relation between the two frequencies [2,14,15]. The frequency of string vibration is proportional to the square root of the string tension, which is of course how strings are tuned. In contrast, the frequencies of the torsional resonances depend on the ratio of the torsional stiffness to the moment of inertia. Both of these depend weakly on the string tension, so the frequency of the torsional resonance depends only weakly on the string tension. This allows the possibility that the string may be tuned so that the first torsional resonance falls on or away from a harmonic of the translational motion.

Is torsional motion important?

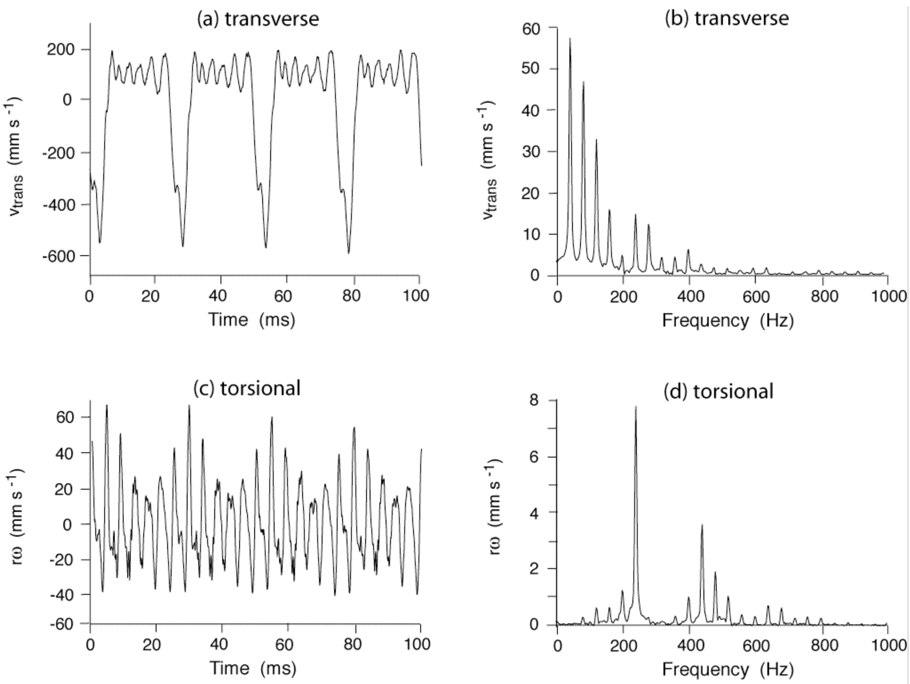
Torsional motion of the string exerts a torque on the bridge that is tiny compared to that exerted by translational motion. So the direct acoustic effect of torsional modes is negligible.

For a bowed string, however, the translational and torsional modes are strongly coupled because, during the stick phase, the bow speed $v_{bow} = v + r\omega$. The period at which the translational kink returns to the bow does not in general equal a whole number of cycles of torsional motion. Because the initiation of stick and slip phases depends on the relative speed of bow and string, one might expect that the noncomensurate motions might lead to aperiodicity or jitter, to which the ear is very sensitive [16].

The small amplitude of jitter in the sound of (expertly) bowed string instruments suggests that, in this condition, either (i) the amplitude of $r\omega$ is a tiny percentage of v during the stick phase or (ii) the two motions occur at harmonically related frequencies. In a previous study [2], we used magnetic induction to measure the translational and torsional speeds of a bowed string. That study showed that $r\omega$ may be a few tenths of v during the stick phase but that indeed the torsional mode is driven at a harmonic of the translational vibration, and not exactly at its own resonant frequency, as shown in Figure 2.

Torsional modes, strings and players

Our previous study [2] left a number of questions unanswered. First, it measured the string properties on a monochord whose bridge impedance was high. Consequently, the string lost little energy via sound radiation, which contributed to a high measured quality factor for translational waves. In this study, similar measurements were conducted on a bass (double bass) whose body radiates sound effectively, leading to a lower quality factor. Two different types of steel-wound strings and a multiply twisted gut string were used. We measured the quality factors of torsional resonances and the way in which the resonances depended on string tension, and thus on translational frequency.



Source: Bavu *et al* (2005), used with permission

Figure 2. Bavu *et al.* [2] measured the speeds of the string v (top) and $r\omega$ (bottom) of a string vibrating at 40 Hz. The time domain plots (left) show that, during the stick phase, $r\omega$ may be tens of percent of v. The frequency domain plots (right) show that the torsional modes here are driven at the 6^{th} and 11^{th} harmonics of the transverse vibration, close to the first and second torsional resonances at 240 and 472 Hz, but that the torsional motion is periodic with the frequency of 40 Hz. The string was bowed at 1/6 of its length. Speeds were measured at a point one fifth of its length from one end, which explains the weak fifth harmonic in translation. Concerning the ripples in the stick phase of v, plots of $v + r\omega$ showed much weaker ripples, suggesting that the ripples were strongly associated with torsional motion.

We were also interested in whether the harmonicity or otherwise of torsional resonances affects the ease with which a stable 'Helmholtz' regime could be established, and whether it affects the quality of the note produced. In principle, we argued, non-harmonic torsional resonances might lead to longer duration of the aperiodic starting transient. It might also change the spectrum of the quiescent translational motion and thefore the spectrum of the output sound conducted. For that reason, we conducted an experiment in which a panel of expert bassists rated the playability and quality of strings that had been tuned so that the first torsional resonance fell either very close to or far from the nearest harmonic of translational motion.

MATERIALS AND METHODS

Figures 3 and 4 show the experimental appraratus. A small coil (mass 165 mg) was glued to the string near either end, its plane was parallel to the string and the field provided by permanent magnets. One was driven by an oscillator and an amplifier to provide a torque on the string. The other, which measured the consequent torsional rotation, was connected via filters to one channel of an oscilloscope. A piezo-electric bass pickup in the bridge measured the translational signal. It was input to the other channel of the oscilloscope.

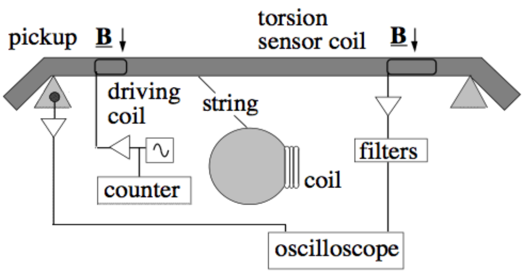


Figure 3. Two coils are glued to the side of the string, as shown in the inset. The magnetic field, provided in each case by permanent magnets, is perpendicular to the string and parallel to the plane of the coil. One coil (left) drives torsional waves and the other (right) detects them.

For the panel experiment, the bass was strung with 4 E1 strings and the tension in each string adjusted to produce the desired values. Numbering from what is usually the highest string, their values for first translational resonance, first torsional resonance and their ratio (= γ) are given in the table below.

Number	string	translation	torsion	ratio γ
1	Helicore	40.28 Hz	465 Hz	11.5
2	Helicore	41.87 Hz	294 Hz	7
3	Helicore	37.72 Hz	322 Hz	8.5
4	Flexocor	41.32 Hz	412 Hz	10

The eight subjects who assessed the strings on the experimental bass were the bass section of a professional symphony orchestra. (The bass and some of the apparatus were transported to their rehearsal venue for this part of the experiment. This allowed the translational and torsional resonance frequencies to be verified immediately before and after the measurements were made.)

Each player in turn bowed all four open strings and the remaining players assessed the sound for rapidity and quality of attack. Each player also played string #1 with the string stopped by their finger at the position to produce a note that was a musical fourth higher than the open string (about one quarter of the way along the string). The order in which the strings were bowed was varied from player to player. After playing each note, the player assessed the ease of starting and the quality of the attack.

For each note the players were asked to score the following:

- How well did it start ? very easily (5) -> not easily(1)
- How easy was it to bow ? very easy (5) -> not easy(1)

For each note the listeners were asked to score the following:

- How well did it start? very easily (5) -> not easily(1)
- How good was the attack? very easy (5) -> not easy(1)

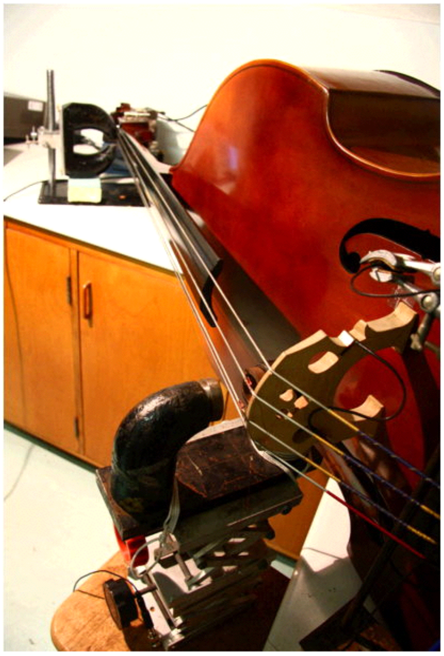


Figure 4. This photograph shows the apparatus for resonance measurements. Three different types of E1 string were studied: D'Addario Helicore orchestral medium H614, Pirastro Flexocor medium, both multi-core wound steel and an Efrano plain varnished gut E #450, a twisted, multistrand, gut string.

RESULTS AND DISCUSSION

Resonant frequencies of the torsional modes

Figures 5 to 6 show plots of the frequencies of the first two or three torsional modes versus that of the first translational resonance measured on the same string when the tension in the string was varied. The translational resonance, of course, increases with tension. Figure 5 shows that, for the wound steel string, the torsional mode frequencies *decrease* with increasing tension, which suggests that the effect on moment of inertia is less important than that on the torsional stiffness. Figure 6 shows that, for the gut string, the torsional mode frequencies *increase* with increasing tension.

These results mean that, when the strings are tuned, the ratio γ of torsional to translational frequencies is varied. So, by tuning the string, one can make γ an integer, or any other value.

Bandwidths of the torsional modes

An important observation of Bavu et al [2] was that the torsional modes were driven, not at their own resonance frequency, but at a harmonic of the translational Helmholtz motion (and thus at one of the higher translational resonances). For this reason, we measured the bandwidth of the torsional resonances.

Figures 7 and 8 show the amplitude of the torsional vibration as a function of frequency, when driven by an oscillating magnetic field of constant amplitude.

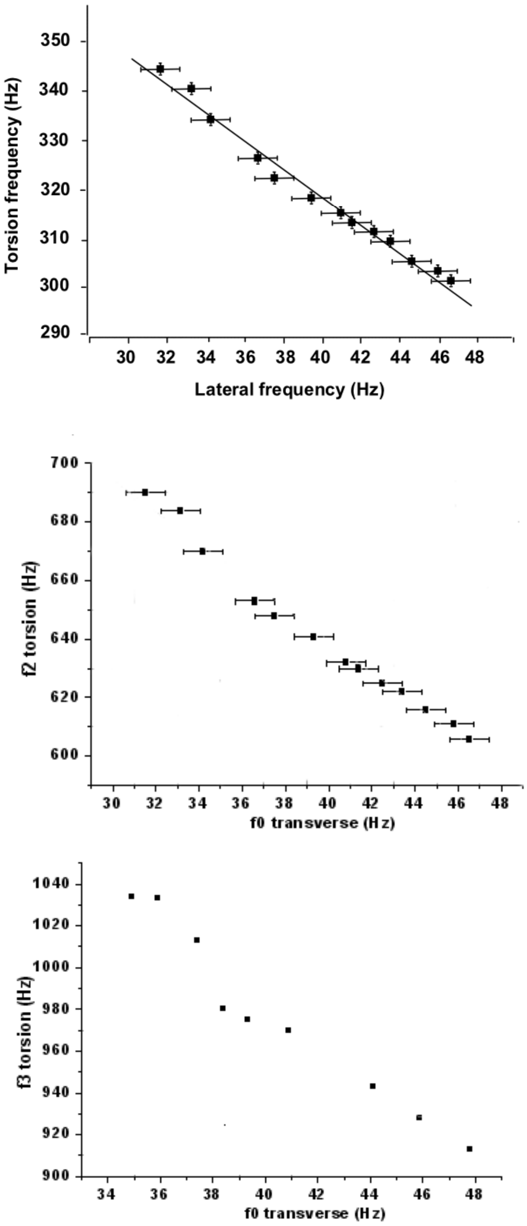


Figure 5. The frequencies of the first three torsional resonances are plotted against that of the first translational resonance for a wound steel E1 string (Flexocor) when the tension was varied.

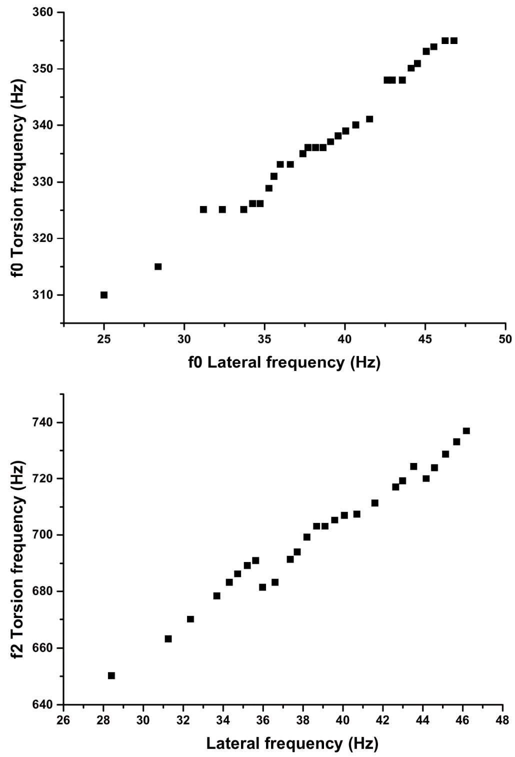


Figure 6. The frequencies of the first and second torsional resonances are plotted against that of the first translational resonance for a gut E1 string.

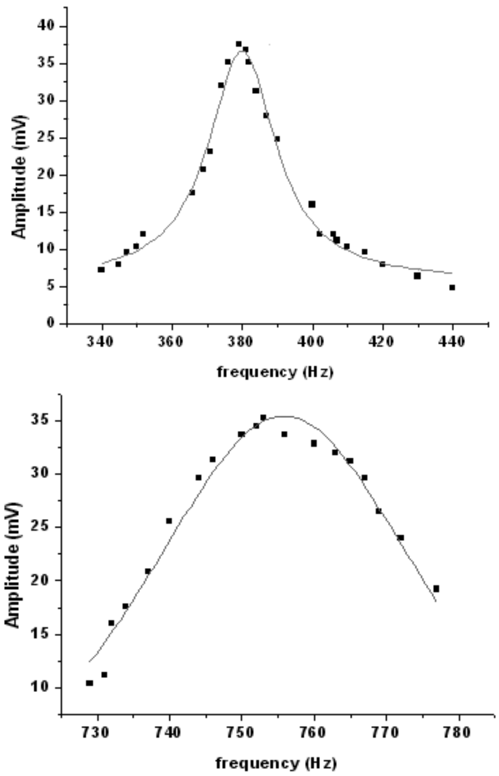


Figure 7. The amplitude of the first and second torsional modes of a wound steel E1 string (Flexocor) plotted against the frequency of the magnetic field producing the torsional motion (as shown in Figure 3). The continuous lines indicate the best fit to a Lorentzian.

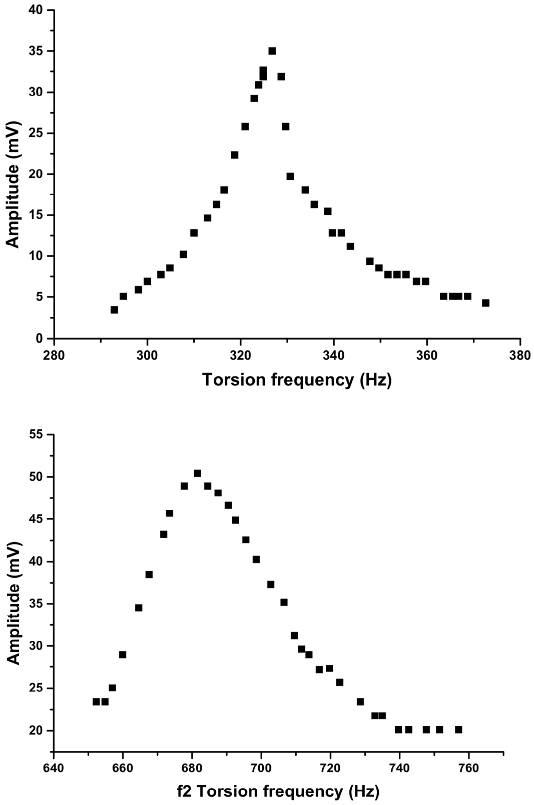


Figure 8. The amplitude of the first and second second torsional modes of a gut E1 string as a function of frequency.

Table 1 provides the results of fitting a Lorenztian function to the measurements on different strings. Helicore #1 and Helicore #2 were identical strings and provide some idea of the reproducibility of the measurements.

Table 1. Measured quality factor Q and bandwidth Δω for the first and second torsional modes.

String	Q_1	$\Delta\omega_1$	Q_2	$\Delta\omega_2$
Helicore #1	21.5	14.5	25.5	23
Helicore #2	21	16	27	25
Flexocore	16	23	13.5	56
Gut	18	18	16	43

In bowed motion, the harmonics of the open E1 string are spaced by 40 Hz, so the torsional resonance will never be further than 20 Hz from one of the translational harmonics.

Starting transients

For most of the experiments, the string was bowed by two of the authors. IW, a violist, did most of the bowing JS, a bassist, also bowed the string. Both are amateurs with considerable experience. The investigators felt they had more difficulty bowing the string when γ was far from being an integer.

Examples of bowing an 'inharmonically tuned' string are shown in Figures 9 and 10. It was tuned to 39.0 Hz, which produced a torsional resonance at 410 Hz for this string ($\gamma = 10.5$). Figure 11 shows a spectrum of the quiescent behaviour, once Helmholtz motion is established. As expected, the spectrum is harmonic.

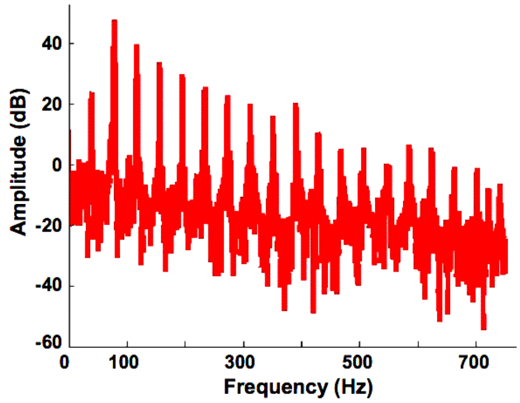


Figure 9: The spectrum of the steady-state bridge pick-up signal for the open string, bowed by the investigators in the lab.

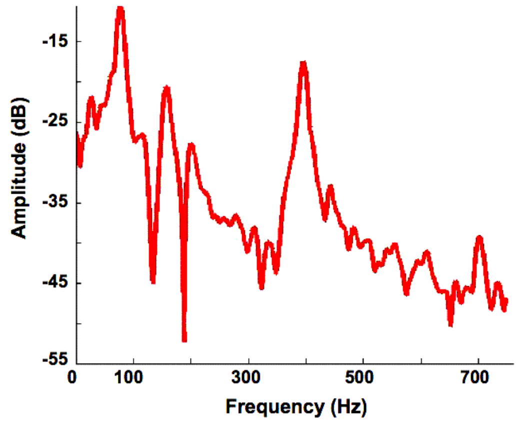


Figure 10. The spectrum of the starting transient for the same bowing gesture as shown in figure 9.

Figure 10 shows the spectrum of the starting transient for the same bowing gesture. This is not harmonic, and it shows a strong peak near the frequency of the first torsional resonance. Could it be that the difficulty of exciting harmonic motion was related to inharmonic torsional motion? To answer this, we conducted the experiment with professional bass players (described above).

Playing tests

The Kendall Coefficient of Concordance (W) was calculated for the two questions, and also for the combined scores. In each case agreement in responses was significant at the 1% level or better. This suggests that the players are using the same internal rules (that are not known to us) for their assessments. The average scores were:

Table 2. Average scores in the playing test.

String		Average score / 5	
	ratio γ	Starting	bowing
1	11.5	2.6	2.3
2	7	2.8	2.4
3	8.5	2.4	2.6
4	10	3.9	3.9
1 (stopped)		2.9	2.4

It is immediately apparent that the players expressed a strong preference for string four. This string was from a different maufacturer than the other three, but was also in the position where brain and bow would expect an E1 string to be located. It was also the string whose torsional resonances had the largest bandwidths. One might expect this to make it easier to establish Helmholtz motion with inharmonic torsional modes. However, one would also have expected that of strings 2 and 4, which in this experiment were tuned to have harmonic torsional modes.

Paired t-tests indicate that the only significant differences (at the 5% level) in the scores for starting occurred between strings 3 and 4, and between string 4 and the stopped string 1. For bowing, string 4 was significantly different (at the 1% level) from all the other strings, but there were no significant differences among strings 1-3.

There was also no significant difference in the scores of players when string 1 was played open or stopped at the fifth fret.

It thus appears that differences in playability due to differences in the ratio γ were not detectable in this experiment.

Listening tests

Each playing test was assessed by seven listeners (i.e. the othe bass players on the panel). The Kendall Coefficient of Concordance (W) was calculated for the two listening questions, and also their combined scores. Again the agreement in responses was significant at the 1% level or better. We now assume that the listeners are judging the strings rather than the player. Table 3 shows the average scores of the 8 listeners.

Table 3. Average scores in the listenng test.

String		Average score / 5	
	ratio γ	starting	attack
1	11.5	2.2	2.0
2	7	2.6	2.5
3	8.5	2.8	2.8
4	10	3.7	3.5
1 (stopped)		2.9	2.6

The scores for both ‘starting’ and ‘attack’ were significantly different (at the 1% level) between all pairs of strings, the exception being strings 2 and 3.

There was also a significant difference in the scores of listeners when string 1 was played open or stopped at the fifth fret. Because of the positions of the coils (Figure 3), we could not (in the same experiment) measure the torsional frequency of stopped strings. It is quite possible that players automatically improved the sound by careful adjustment of their finger on the stopped string.

Although the listening panel had more statistically significant preferences than the players, there was still no clear preference for strings whose torsional mode was tuned exaclyly to a harmonic of the translation mode.

CONCLUSIONS

The first two torsional modes of the wound steel and gut strings had Q values around 15 to 25. The torsional resonance frequencies of the wound steel strings decreased with increasing tension, whereas they decreased for the gut string studied.

When strings were tuned so that the first torsional mode fell halfway between harmonics of the translational motion, they could produce a starting transient with a sustained component near the torsional resonance.

When professional players bowed strings that were tuned so that torsional modes fell either directly on, or midway between, translational harmonics, there were no clear preferences.

ACKNOWLEDGMENTS

We thank our volunteer subjects for their participation.

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