Cutoff frequencies and cross fingerings in baroque, classical, and modern flutes[a]

Joe Wolfe[b] and John Smith[c]
School of Physics, University of New South Wales, Sydney 2052, Australia

(Received 14 June 2002; accepted for publication 4 August 2003)

Baroque, classical, and modern flutes have successively more and larger tone holes. This paper reports measurements of the standing waves in the bores of instruments representing these three classes. It presents the frequency dependence of propagation of standing waves in lattices of open tone holes and compares these measurements with the cutoff frequency: the frequency at which, in an idealized system, the standing waves propagate without loss in such a lattice. It also reports the dependence of the sound field in the bore of the instrument as a function of both frequency and position along the bore for both simple and ''cross fingerings'' (configurations in which one or more tone holes are closed below an open hole). These measurements show how ''cross fingerings'' produce a longer standing wave, a technique used to produce the nondiatonic notes on instruments with a small number of tone holes closed only by the unaided fingers. They also show why the changes from baroque to classical to modern gave the instruments a louder, brighter sound and a greater range. [DOI: 10.1121/1.1612487]

PACS numbers: 43.75.Ef, 43.75.Qr [NHF]

I. INTRODUCTION

The tone holes of woodwind instruments are used to reduce the effective length of their bore. An open tone hole provides a low inertance shunt between the bore and the external radiation field so, at sufficiently low frequency, the acoustic pressure inside the bore near an open tone hole is small. Consequently the bore behaves, at low frequencies, like a simple tube whose end is a little way beyond the first open tone hole. This extra length or end correction is frequency dependent: at higher frequencies, the impedance of the inertive shunt is larger, and so the standing wave in the bore propagates past the first open tone hole with an increasing relative amplitude as the frequency increases. As the relative amplitude of the standing wave propagating beyond the first open tone hole increases, the length of the end correction increasingly depends upon whether tone holes further down the bore are open or closed. This allows what musicians call cross fingering: the closing of tone holes down stream from the first open tone hole so as to change (usually to flatten) the pitch of the note played. In older instruments, cross fingerings are used in all registers. In modern orchestral woodwinds, there is a tone hole for each semitone interval and cross fingerings are principally used in the high registers, although they are also used in the other registers for pitch and timbre adjustments and for contemporary techniques such as multiphonics and microtones.

The tone holes of woodwind instrument became larger, in relation to the bore, from the baroque to classical to modern periods. This change was particularly pronounced in the flute. The larger tone holes had several effects: they made the instruments louder and brighter in timbre and they allowed them to play higher notes. They also eliminated or reduced the use of cross fingering in all but the highest registers. These effects are explained below.

Many aspects of the acoustics of wind instruments are well understood, and reviews are given by Fletcher and Rossing (1998), Nederveen (1998) and others. The effects of individual tone holes have been studied in detail theoretically (Dubos *et al.*, 1999; Keefe, 1982a; Strong *et al.*, 1985; Nederveen *et al.*, 1998) and experimentally (Coltman, 1979; Keefe, 1982b). Benade (1960, 1976) derives approximate theoretical expressions for the cutoff frequency of an array of open tone holes: the frequency above which the standing waves propagate significantly past the first open tone hole (see Sec. III). He also explains how the frequency-dependent propagation past a single open tone hole allows cross fingering to flatten the pitch, and why the effect is greater in the second register than in the first.

Figure 1 is a sketch of two of the standing waves in the spirit of Benade's description. [Figure 1(a) combines features of Figs. 21.1, 21.10 and 22.12 in Benade (1976).] The pressure of the standing wave does not fall to zero at the first open hole, because the inertance of the air in the open hole is not zero. Rather, it penetrates into the lattice, where it is attenuated along the bore. Extrapolating the standing wave past the open hole gives the end effect. Because the impedance of the open tone hole is greater at higher frequency, the standing wave pressure at the open hole is greater, so higher modes propagate further into the lattice and so have a larger end effect. Figure 1(b) is an analogous sketch for a cross fingering, showing the larger end effect due to the lower attenuation under the closed tone holes.

Benade's explanations are in good qualitative agreement with the observed effects on real instruments. However, we know of no experimental studies of the end effects in musical instruments that examine the standing waves as functions of both position and frequency.

The purpose of the current study is to measure these standing waves in examples of flutes from different eras and

[a]This work has not previously been submitted for publication. A brief account was presented at the International Congress on Acoustics, Rome (2001), ''Tone holes and cross fingerings in wood wind instruments,'' J. Smith and J. Wolfe, Session 8.09, pp. 14–15.
[b]Electronic mail: j.wolfe@unsw.edu.au
[c]Electronic mail: jrs@newt.phys.unsw.edu.au

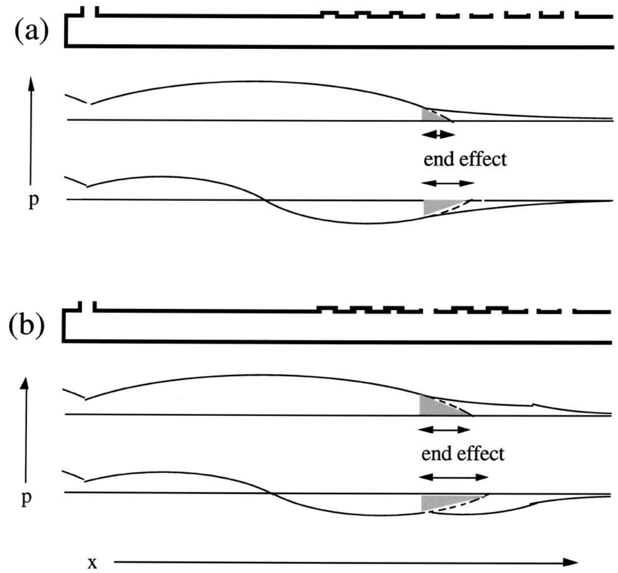

FIG. 1. (a) is a sketch, after Benade (1976), of the acoustic pressure *p* of the standing waves for the first two harmonics in the bore of a flute for a simple fingering. (b) shows an analogous sketch for a cross fingering. The dotted line indicates the standing wave in a simple tube with a length that gives the same resonant frequency as the flute with this fingering.

in particular to measure their propagation beyond the first open tone hole, and how it varies with frequency. We also measure the effects of cross fingering and show some of the effects produced when the size of tone holes is changed. We chose flutes from the baroque, classical, and modern eras. In the modern flute, the ratio of tone hole diameter to bore diameter approaches the upper practical limit. In the baroque flute, the diameter ratio is smaller by a factor of nearly 2. The classical instrument has intermediate parameters. For the three instruments studied, acoustic impedance spectra measured at the embouchure and sound files and spectra of played notes have been published previously (Wolfe *et al.*, 2001a).

II. MATERIALS AND METHODS

A. The instruments

The modern flute is a production line instrument (Pearl PF-661, open hole, C foot), the same instrument studied previously (Smith *et al.*, 1997; Wolfe *et al.*, 2001b). The classical and baroque flutes were reproductions made by T. McGee of Canberra, Australia. The classical instrument is based on a large-hole Rudall and Rose (R&R #655 from the Bate collection in Oxford) but has been rescaled to play at A = 440 Hz. The baroque flute is an unscaled replica of an instrument made by J. A. Crone in Leipzig in about 1760. It plays at A = 415 Hz. The measurements were made at room temperature and humidity (for the different instruments, measured on different days, this varied from 21 °C to 26 °C and 53% to 58% relative humidity, values lower than those in an instrument under playing conditions). The modern Boehm flute is nearly cylindrical, but the head joint tapers to be slightly narrower at the embouchure end. The classical and baroque flutes have cylindrical head joints and approximately conical bodies, narrowing towards the foot. The cone angle is greater in the baroque flute, so its bore is on average smaller than that of the classical instrument, which is in turn narrower than that of the modern flute. Table I gives some of the dimensions of the three instruments.

B. Measurement of impedance and standing waves

The measurements of acoustic impedance were made as described previously (Smith *et al.*, 1997; Wolfe *et al.*, 2001b). The reference for calibration was a semi-infinite cylindrical waveguide, whose impedance was assumed to be real and to have a value given by $\rho c/S$, where ρ is the density of air, c is the speed of sound and S is the cross-sectional area. A compromise is made among frequency range, frequency resolution, and dynamic range. For the experiments measured over the range 0.2 to 3 kHz, the fre-

TABLE I. A table of sizes of holes, positions of their centers, and bore diameters for the three flutes studied. The key naming convention gives numbers to the three long fingers of each hand or the name of the note played when the key is closed. The holes "tr2" and "tr1" are used for trills and as register holes on the Boehm flute. "emb" refers to the embouchure hole, and its stated diameter is that of a circle having the same area.

	Baroque			Classical			Modern		
Hole	Diameter	Bore	Position	Diameter	Bore	Position	Diameter	Bore	Position
cork		18.2	−18.2		19.0	−17.5		15.7	−17.5
emb	8.5	18.2	0	11.1	19.0	0	11.5	17.5	0
tr2							8.0	19.0	201.0
tr1							7.7	19.0	218.3
1	6.6	16.4	229	7.7	17.1	230	7.0	19.0	234.8
B				5.5	16.8	249	13.2	19.0	267.2
A♯							13.2	19.0	286.7
2	6.4	15.8	267	10	16.5	268	13.2	19.0	307.4
3	5.4	15.3	304	7.5	15.8	303	13.2	19.0	330.6
G							13.3	19.0	352.8
F♯							14.2	19.0	377.3
1	5.5	14.6	363	9.0	14.6	361	14.2	19.0	402.9
2	5.6	14.0	398	10.9	14.0	393	14.2	19.0	430.3
E				8.0	13.9	412			
3	4.5	13.3	435	6.4	13.1	429	14.2	19.0	459.0
D	6.2	12.1	497	11.6	12.4	478	15.5	19.0	490.6
C♯				12.3	11.7	514	15.5	19.0	524.2
C				9.7	11.0	548	15.5	19.0	557.8

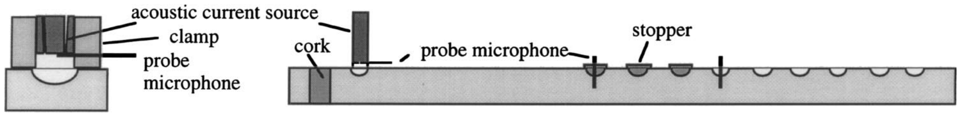

FIG. 2. A schematic of the technique we used to measure the standing waves, with a broad band acoustic current source and probe microphones that can be placed at the embouchure or at closed or open tone holes. The details at the embouchure are enlarged in the sketch at left.

quency spacing was 2.5 Hz. For those over the range 0.2 to 12.5 kHz, the frequency spacing was 10 Hz.

The calibration procedure for measurements of the standing wave in the bore was as follows (Fig. 2). An acoustic current was synthesized from frequency components ranging from 0.2 to 3 kHz in 2.5-Hz increments. This was input to the embouchure hole via a short pipe of diameter 7.8 mm with a length (6 mm) that was chosen so that its impedance approximates the radiation impedance that normally loads the instrument at this point when it is played (Smith *et al.*, 1997; Wolfe and Smith, 2001). The effect of this impedance may be removed using the transfer matrix for a cylindrical waveguide, and this has been done for the high-frequency measurements reported in Fig. 3, which aims to explain the shape of the impedance spectrum rather than to predict playing frequencies. For the other figures, it has been included. Fletcher and Rossing (1998) analyzed the momentum of the region in which the air entering from the jet mixes with the air in the bore and concluded that the embouchure radiation load impedance is effectively in series with the impedance of the bore when it acts on the jet. A probe microphone, outer diameter 1 mm, was placed on the upstream side of this short pipe, at the position of the acoustic current source. The pressure spectrum at this point was measured, and the spectrum of the output acoustic current was adjusted so that the pressure signal measured by this microphone was independent of frequency.

To maintain the same geometry after calibration, the original probe microphone located at the embouchure was replaced with a second microphone of the same type. The original probe microphone was now used to measure the sound pressure in the bore via the tone holes. Because we report only ratios of the pressure in the bore to the pressure at the embouchure, and these have been both measured using the same microphone, any intrinsic frequency dependence of the microphone response will cancel.

In order to perturb minimally the system under study, we used a probe microphone with a small outer diameter (1 mm). However, the frequency response of a long probe microphone with a small diameter decreases rapidly when its length is increased because of viscous losses. This imposes a short upper limit on the useful length of the probe microphone, and makes it difficult to insert usefully along the axis of the instrument. For this reason, measurements were only taken at tone holes. The tone holes are separated by a few cm, which is rather smaller than the wavelengths studied, so this spacing is adequate for the study. When the tone holes were closed (either by stoppers made to size, or by keys with central holes into which stoppers were inserted), the probe of the microphone was passed through a hole along the axis of the stopper. In all cases, the end of the probe microphone was on the axis of the bore.

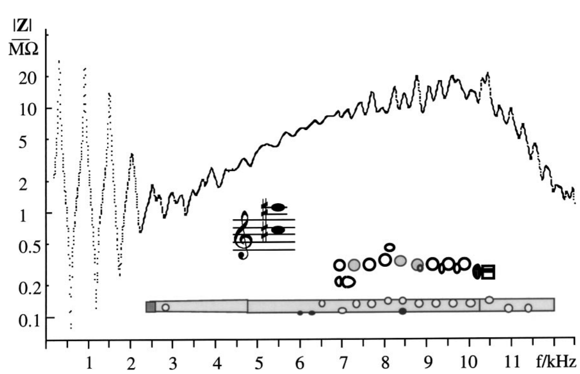

FIG. 3. The input impedance spectrum (in MPa.s.m^{-3} or MΩ) of a modern flute for the fingering used to play C♯5 and C♯6. The upper inset shows a fingering diagram such as a flutist would recognize. The lower inset shows a schematic of the keys. For these notes, all tone holes are open except for two small trill or register keys and one of two similarly placed holes that are used as alternates.

C. Measurement of pitch

To measure the pitch change produced by cross fingerings, three experienced flutists were asked to play each of the flutes, with each fingering, in each of two registers. They were told that the fingerings were not standard, and that they played at various pitches between about F4 and G4 in the first register and F5 and G5 in the second. They were asked to blow normally (*mezzo forte*) for a note in the first register and then to blow normally (*mezzo forte*) for a note in the second register. For these measurements, the temperature and humidity of the air were presumably rather higher than ambient, but this was not measured. For each flutist/fingering/register combination, three examples were recorded using a digital tape recorder. The fundamental frequency was determined from a digital Fourier transform, with windows of 2^{16} points, sampling at 44.1 kHz, and averaging over one or more seconds.

III. RESULTS AND DISCUSSION

The physical measurements are made at room temperature and humidity, rather than at the elevated temperatures and humidities of playing conditions, so the speed of sound is expected to be about 1% less than that under playing conditions. The baroque flute is tuned to A = 415 Hz, so it plays about one semitone (6%) flatter than the classical and modern instruments. This study, however, is primarily concerned with the relative pitch changes due to cross fingering or between registers, rather than with absolute pitch.

A. Input impedance spectrum of the flute

Figure 3 shows the input impedance spectrum for a modern flute, for a fingering in which all of the tone holes that are normally used are open. In this plot only, the radiation impedance at the embouchure is not included. This fingering is conventionally used to play C♯5 and C♯6 (554 and 1108 Hz), with the player changing embouchure and jet speed to select the note. It can also play G♯6 (1661 Hz), so it plays the first three notes in a harmonic series, which correspond to the first three minima in $\mathbf{Z}(f)$, approximately equally spaced by 550 Hz. These correspond to resonances of a length of bore about half the length of the flute. This is a little more than the distance to the first open hole, beyond which the standing wave is attenuated strongly. The resonances associated with this harmonic series become weaker with increasing frequency because of increasing viscothermal losses at the walls.

We show in the next section that the calculated cutoff frequency f_c for this instrument is about 2 kHz. At frequencies well above that of the cutoff, the frequency spacing between minima is 270 Hz. These resonances correspond to standing waves along the whole length of the instrument (see especially between 7 and 12 kHz). At such high frequencies, the inertive reactance of the tone holes is so high that the standing wave propagates along the tone hole lattice almost as though the tone holes were closed. This effect is seen more clearly in instruments with lower f_c (discussed later, Figs. 4 and 5).

Above about 3 kHz, the resonances of the bore become rather weak. This can be explained qualitatively by saying that they are effectively in parallel with a Helmholtz resonator formed by the air in the embouchure hole (the mass) and the volume of air that lies between that hole and the sealed end of the flute (the compliance). The expected frequency of this Helmholtz resonator is of the order 3 kHz (the value is approximate because the wavelength is no longer very much larger than the dimensions of the air forming the compliance). The broad maximum between 7 and 11 kHz may correspond to the resonance of the air in the embouchure riser. This is a truncated cone, 5 mm long. A waveguide model for a flanged, truncated cone with this length and the radii of Table I has a broad maximum of about 10 MΩ at around 9.5 kHz.

B. Cutoff frequencies

Benade (1976) applied transmission line theory to the section of bore with open tone holes, and derived the following approximate expression for the "cutoff" frequency f_c of this "open tone-hole lattice:"

$$f_c = 0.110 \frac{b}{a} \cdot \frac{v}{\sqrt{s \cdot t_e}}, \tag{1}$$

where b and a are the radii of the tone hole and the bore respectively, v is the speed of sound, s is half the distance between the centers of adjacent tone holes, and t_e is the effective length of the tone hole, being approximately its geometric length plus 1.5 b. In this model, waves with frequencies below f_c will be attenuated as they propagate along the lattice. However, frequencies above f_c will propagate through the lattice essentially without loss.

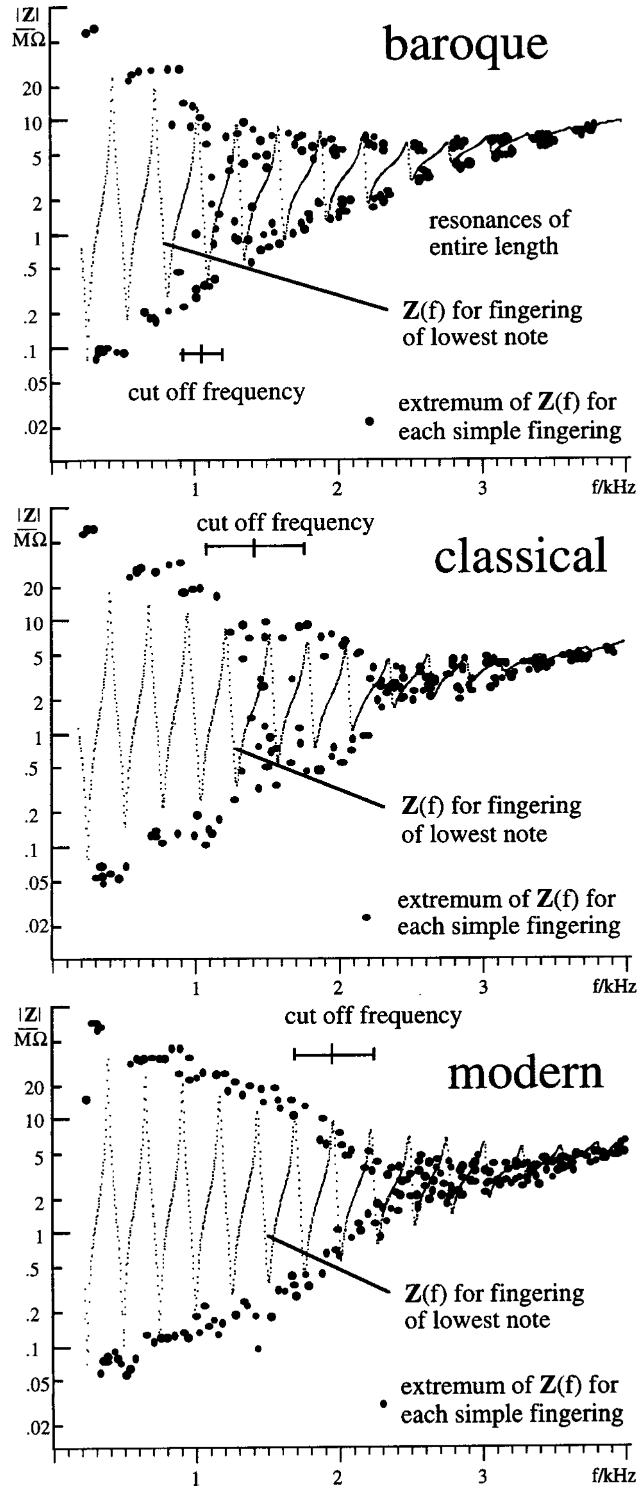

FIG. 4. The effects of the filtering by the open hole lattice, for the three instruments studied. The measured impedance spectrum Z_l of the lowest note is indicated by the faint dots that form an almost continuous line. Superimposed are the extrema of the impedance spectra for all simple fingerings (large dots). Cutoff frequencies calculated from Benade's approximate expression are shown. In practice, the cutoff frequency varies from hole to hole, so means and standard deviations are shown.

The three instruments studied have different size bores and tone holes, so the cutoff frequencies of the lattice of open tone holes are different: lowest for the small-holed baroque flute and highest for the modern instrument. For the baroque and classical flutes, in which adjacent holes usually

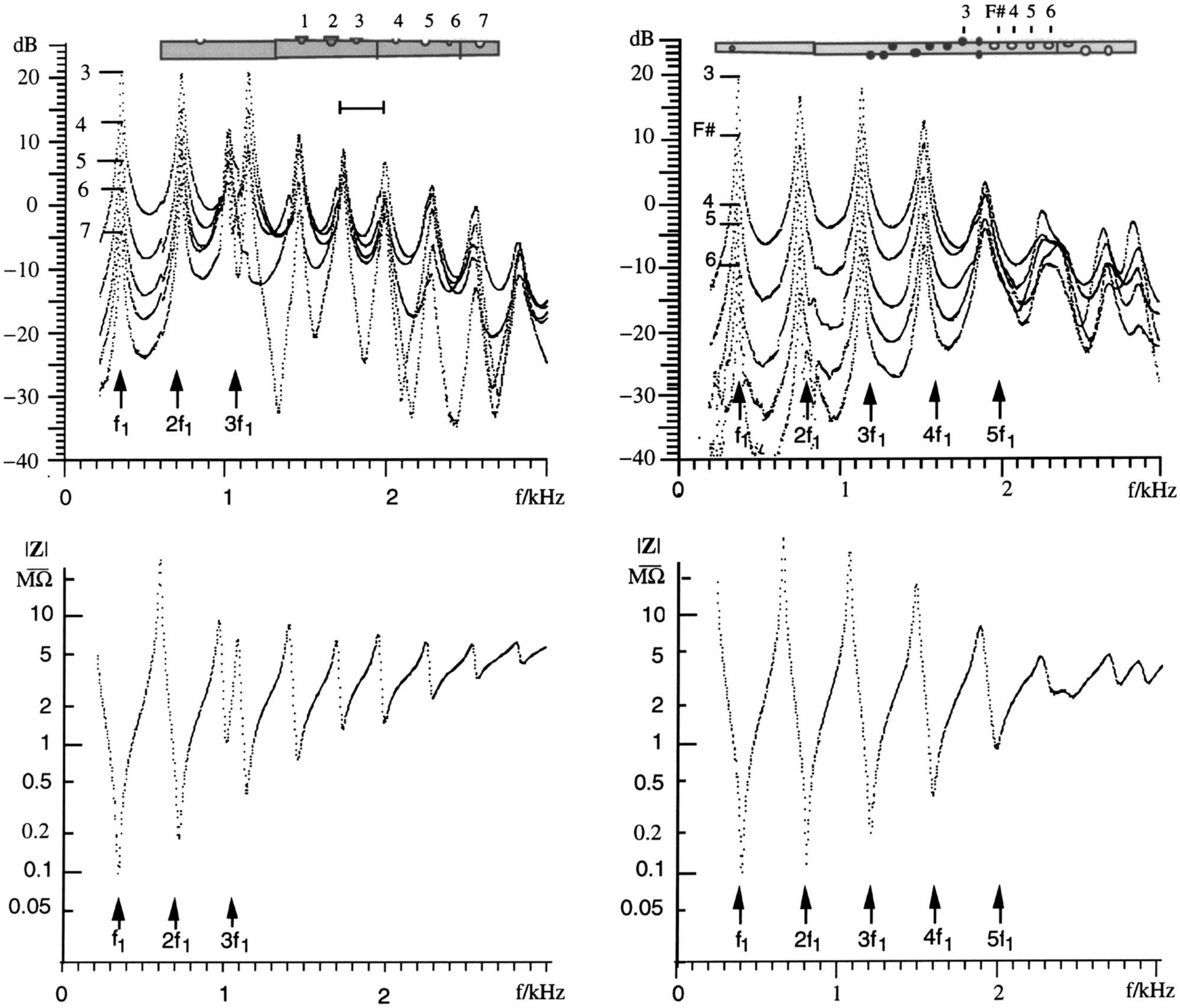

FIG. 5. The upper figures show the sound pressure spectrum in the center of the bore of a baroque flute (left) and a modern instrument (right) for the fingering XXX-OOO for the note G4 or G5. Results are expressed as $p_{\text{bore}}/p_{\text{emb}}$. Different curves were obtained on the bore axis at the positions of different tone holes, as indicated by the numbers on the curves and on the sketch of the instruments. The lower figures show the acoustic impedance spectra measured at the embouchure for these fingerings. The scale bar at the upper right of the figure for the baroque flute indicates the harmonic spacing of resonances of the entire bore. (To allow closer inspection, the individual curves that superpose here are printed separately at http://www.phys.unsw.edu.au/~jw/Crossfingeringfigures.pdf).

have different sizes, the cutoff frequency also depends upon the fingering. Some typical values are given in Table II.

The cutoff frequency predicted by Eq. (1) has been widely and successfully used. Due to approximations in the model Benade used, however, one would expect it to become increasingly imprecise around the cutoff frequency. The transmission line theory used assumes an infinitely long lattice with distributed or continuous components, whereas in a real instrument the tone holes and their separation are of finite size. In the waveguide model the dimensions of the elements in the lattice of open tone holes are always much smaller than the wavelength but this is not the case for the real instrument. Further, the model will not be applicable to sections of bore with a small number of tone holes of different size at higher frequencies. Accordingly we present in the Appendix a derivation of Eq. (1) for tone holes of finite size

TABLE II. Values for the cutoff frequency, calculated using Benade's expression, Eq. (1), and typical values of the relevant parameters. The effective length of the tone hole includes an end effect at each end. For the classical flute, only the tone holes used in the diatonic scale are included in the means. For the modern instrument, all holes except the trill holes are included.

Flute	Bore radius a	Tone hole radius b	Tone hole length t	Effective length t_e	Interhole spacing 2s	Cutoff frequency f_c/Hz
baroque	7.5 ± 1.7	2.8 ± 0.3	5	9.2 ± 1	41 ± 10	1030 ± 300
classical	7.5 ± 1.5	4.3 ± 1.1	5	11.5 ± 1	40 ± 10	1430 ± 500
Boehm	9.5 ± 0	6.7 ± 1.1	2.5	12.6 ± 1.2	27 ± 5	2050 ± 400

and spacing, and some calculations of the impedance and gain of an open tone hole lattice using a simple empirical model.

The cutoff frequency is one of several effects that determine the envelope of the impedance spectra of the flute. For each of the three instruments, the data for all possible simple fingerings (i.e., fingerings that are not cross fingerings) are summarized in Fig. 4. The extrema become successively weaker with increasing frequency, in part because of greater wall losses at high frequency, and in part because the air upstream from the embouchure hole acts as the reservoir of a Helmholtz resonator in parallel with the bore. This is seen most clearly in the impedance spectrum Z_1 for the fingering for the lowest note, in which there are no open tone holes. (The complete impedance spectrum is shown for this fingering only.) For all other simple fingerings, there is a lattice of open tone holes, and the extrema become weak at a frequency near the calculated f_c.

For the baroque and classical flutes, for frequencies above the calculated f_c, the extrema of Z for simple fingerings have an envelope similar to that of Z_1. On the baroque flute, above about 2 kHz, the extrema of simple fingerings tend to cluster near those of Z_1. In other words, for frequencies well above f_c, all fingerings behave approximately as though all holes were closed! This effect is successively less noticeable on the classical and modern flutes, whose tone holes are successively bigger and more numerous, and therefore less negligible, even at high frequency.

C. Standing waves for simple fingerings

To illustrate the effects of cross fingering, we chose one simple and one cross fingering for detailed study. [Measurements for other fingerings are given by Wolfe *et al.* (2001a).] The simple fingering chosen was that for the note G, in which the tone holes or finger holes controlled by the left hand are closed and those for the right hand are open. This is often represented by wind players as XXX-OOO, the characters representing the three largest fingers of the two hands, X being closed and O being open. This choice of fingering allows us to measure the standing waves in both the open and closed parts of the bore, via the finger holes. On all instruments, this is the standard fingering for G4 and G5.

The effect of the cutoff frequency can be seen in both the impedance spectrum and in the standing wave spectra of notes with simple fingerings. Figure 5 shows these spectra for the modern and baroque flutes for the fingering XXX-OOO, used on both instruments for G4 and G5. The impedance spectra are measured at the embouchure hole and include an impedance representing the embouchure radiation impedance, as described above. The standing wave spectra are measured at the tone holes. Only one spectrum measured inside the closed region of the bore is shown, so as not to complicate the figure further.

Below f_c, the flute behaves approximately as an open tube terminated at the position of the first open hole, plus an end correction which increases with frequency because of the greater penetration into the open tone hole lattice by the waves with higher frequency. Below f_c, the impedance minima and the peaks in the standing waves are approximately harmonically related. This is shown in the figures by the arrows, which have been drawn at harmonics of the fundamental frequency. For the baroque flute, the first two resonances fall below f_c. For the modern instrument, the first five resonances fall below f_c. On the modern instrument, the fingering for G4 can be overblown to sound G5, D6, G6, and B6 with intonation error less than the variability among players. The classical flute (data not shown in this figure, but see Figs. 4 and 6) has intermediate cutoff frequency and intermediate behavior: the first four impedance minima are approximately harmonic.

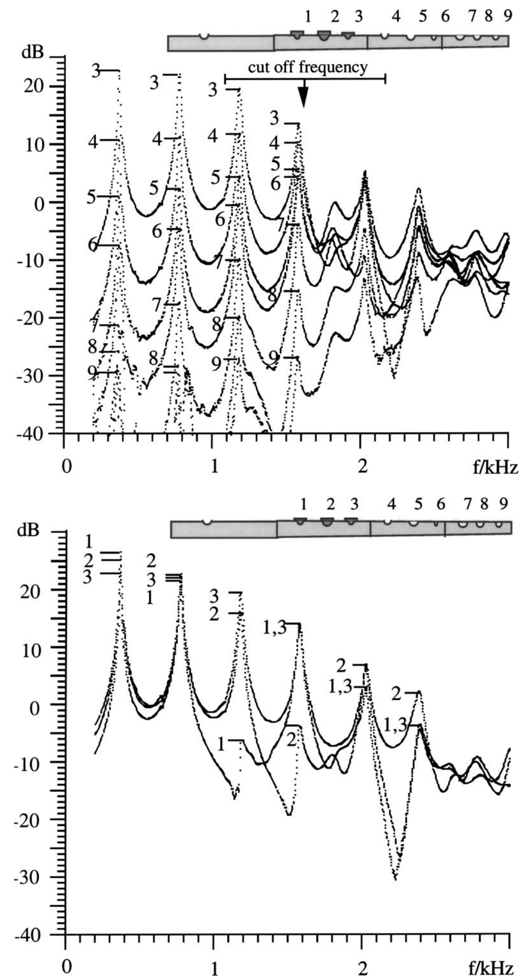

FIG. 6. The sound spectrum in the center of the bore of a classical flute for the fingering XXX-OOO for the notes G4 or G5. Results are expressed as $p_{\mathrm{bore}}/p_{\mathrm{emb}}$, the ratio of measured pressures at positions in the bore to that at the embouchure. Different curves were obtained on the bore axis at the positions of different tone holes, as indicated by the numbers on the curves and the sketch. They therefore represent displacement along the bore, at positions given by Table I. The numbered horizontal dashes are added to show the maxima of the superposed curves.

The increase in cutoff frequency from baroque to classical to modern has important consequences for the sound produced. For most fingerings, the higher f_c increased the number of impedance minima that are in nearly harmonic ratios and thus the number of resonances that interact with harmonics of the air jet to produce the spectrum of the note played. For the same note and dynamic level, the sound spectra of the more recent instruments are richer in higher harmonics [spectra given in Wolfe *et al.* (2001b)]. This makes them ‘‘brighter’’ in timbre, and also considerably louder, because, for most of the range of the flute, the higher harmonics fall in a range where the ear is more sensitive than it is to the fundamental frequency.

At frequencies somewhat higher than f_c, the impedance of the open tone holes becomes sufficiently high that the wave propagates past them all the way down the bore. The resonances in this frequency range are therefore standing waves along the entire length of the instrument. Hence, above about 1.4 kHz for the baroque flute, the standing wave peaks and the impedance minima are approximately equally spaced in frequency, but the spacing is smaller. The average difference in frequency between the last six impedance minima is 278 Hz, which is similar to the frequency of the lowest note of the baroque flute (277 Hz: D4 in baroque tuning with A = 415 Hz). For the modern flute at frequencies above about 2.5 kHz, the minima are also more closely spaced, but the resonances are weak at these frequencies (cf. Figs. 3 and 4).

The various features of the standing waves are most clearly seen in the classical flute, which has an intermediate number of tone holes, of intermediate size. Data for the fingering XXX-OOO are shown in Fig. 6. At this stage, we restrict discussion to frequencies below the cutoff, which for this fingering on this instrument is about 1.6 kHz (discussed below).

Figure 6a shows the decay of the standing waves in the lattice of open tone holes. The pressure amplitude falls by several dB from one open hole to the next. This can be analyzed with a model in which the sections of bore are treated as waveguides and the tone holes as discrete elements. Using such a model, and using the typical values in Tables I and II, the attenuation between adjacent open holes is of order 10 dB and depends in a complicated way on frequency (see the Appendix for a calculation). The actual value varies from hole to hole, because of the nonuniform bore and hole size. For instance, the sixth hole is small and the seventh large, so the attenuation from hole 5 to 6 is less than that from hole 6 to 7. For the most distant open holes (8 and 9), the measured attenuation is less strong. The intensity in the bore here is comparable with that of the external sound field. Near and above the cutoff frequency, the spectra are complicated and there is no simple dependence upon position. The simple model fails in this region because the standing waves penetrate further and more closely approach the resonances of the complete tube, as discussed below.

Notice that, at frequencies below the cutoff, the pressure amplitudes of the resonances measured at hole 4 (the first open hole in the lattice) are of approximately equal amplitude. This is because the input acoustic current spectrum has been adjusted to produce equal pressure amplitude at the embouchure hole. At the subsequent open holes (5,6,7), the pressure signal increases with frequency because of greater penetration into the lattice.

In the closed part of the bore [Fig. 6(b)], the behavior below the cutoff frequency has some complications that are explained in terms of the standing waves of the first few resonances. One expects the pressure amplitude to fall monotonically in the last quarter wavelength upstream from the first open hole. This is observed: for the first resonance ($\lambda \cong 870$ mm), the amplitude falls monotonically from holes 1 to 4 (131 mm). For the higher resonances, this length is greater than $\lambda/4$, and so the pressure variation is not monotonic. On this flute, hole 1 can be used as a register hole to produce a note near the third harmonic, D6, and hole 2 can be used as a register hole to produce a note near the fourth harmonic, G6. (A register hole is one that is opened to allow standing waves with nodes near its position, but to disallow lower frequency resonances. It thus selects notes in higher registers, whence the name.) At hole 1 (register hole for the third resonance), the third resonance has a low value of sound pressure and a discontinuity. Similarly at hole 2 (register hole for the fourth resonance), the fourth resonance has a low value of sound pressure and a discontinuity.

Most of the spectra from different positions show strong peaks at the first four harmonics of the note G4, corresponding to standing waves with half wavelengths equal to integral fractions of a length equal to the distance from the embouchure hole to the first open hole plus about 80 mm (i.e., the length of the closed tube, plus end effects for the embouchure and for the tone hole lattice). The frequencies of these peaks correspond, to within 2 Hz, to the minima in the acoustic impedance spectrum measured at the embouchure. The flute can be blown to play notes with these frequencies. Intervening frequencies do not generate strong standing waves with this fingering (and the flute cannot be played at these frequencies without making large changes to the end effects). The good approximation to harmonic ratios may seem surprising, because the end correction in the tone hole lattice is expected to be frequency dependent. The closed air volume upstream from the embouchure and the shape of the bore are responsible for ‘‘correcting’’ this frequency dependence, and others related to playing technique (Benade, 1959; 1976; Fletcher and Rossing, 1998).

D. Standing waves for cross fingerings

There is no cross fingering that is equally similar across all three instruments. For the cross fingering, we chose XXX-OXX, which is discussed below. Results for the simple fingering XXX-OOO and the cross fingering XXX-OXX are compared in Fig. 7. The fingering XXX-OXX on the baroque flute plays a sharp F♯4 (for which it is an alternative fingering): i.e., closing the additional holes lowers the pitch by about half of an equal tempered semitone. Overblown, it plays approximately F5, i.e., about two semitones flatter than the XXX-OOO fingering. The note is unstable and difficult to play. On the classical instrument this fingering plays G4 about a quarter of semitone flat, and G5 about half a semitone flatter than the XXX-OOO fingering. The Boehm flute has more tone holes and it also has key linkages that are not present on the other instruments. The fingering XXX-OXX engages a clutch that closes the F♯ key. The cross fingering effect alone lowers the pitch by about 15 and 30 cents in the first and second registers, respectively. (See Table III.) In all flutes, the cross fingering produces notes in the third register in which the open hole operates as a register hole, so considerations of the cross fingering end effect are not relevant.

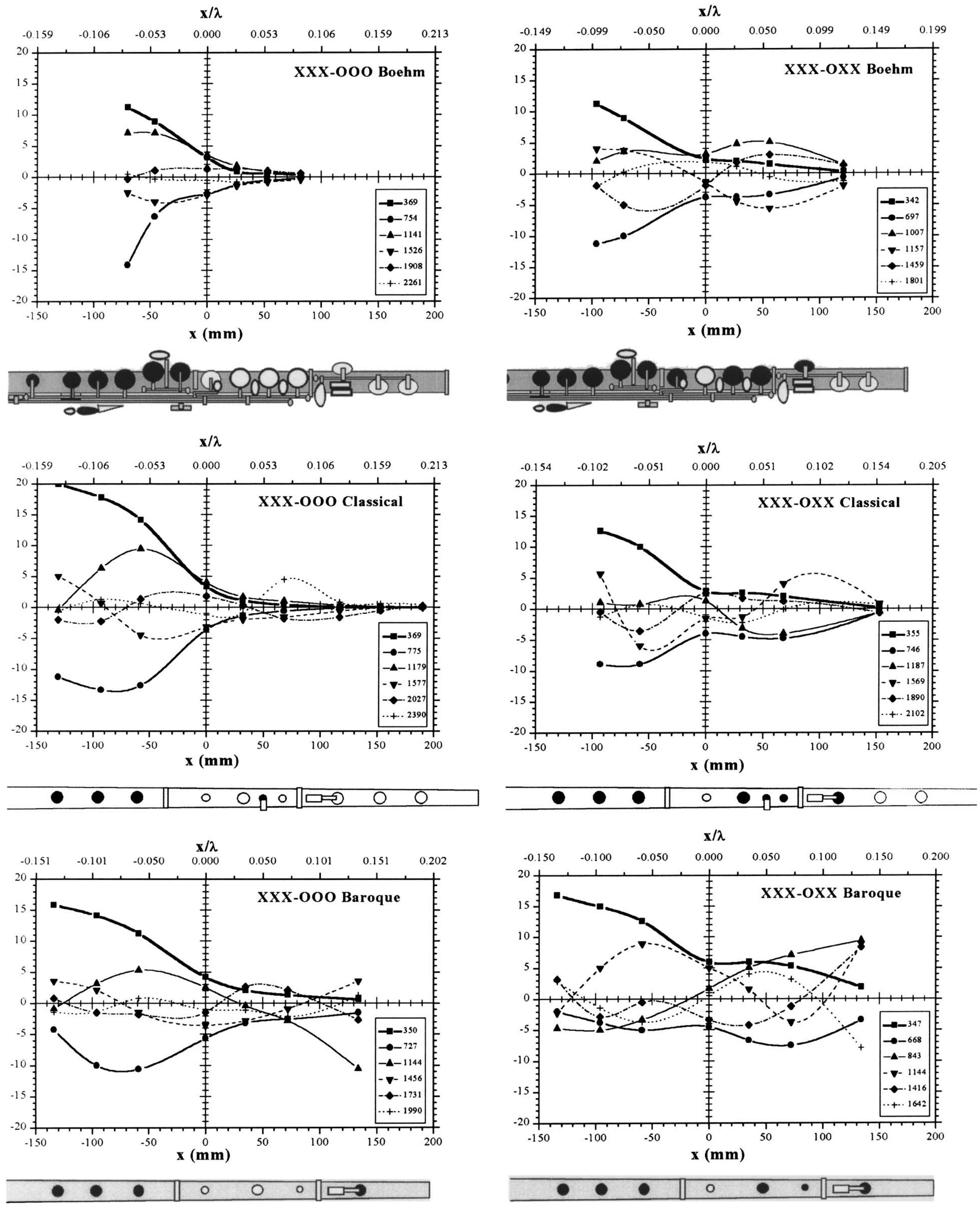

FIG. 7. Acoustic pressure in the bore for simple (left) and cross fingerings. The vertical axis is $p_{\mathrm{bore}}/p_{\mathrm{emb}}$ (linear scale): the ratio of pressure to that measured at the embouchure. Positions are measured with respect to the position of the first open hole and are shown in both mm (lower axis) and in fractions of a wavelength of the fundamental or lowest resonance (upper axis). The figures on the left show the simple fingering XXX-OOO and those on the right show XXX-OXX. The curve with the squares and heaviest line weight is that of resonance with the lowest frequency. The other curves are for next five resonances in increasing frequency. (The frequencies in Hz are given in the inserts.) The sketches of the instruments are drawn to scale in the horizontal direction so that the holes used to make the measurement line up with the data. Lines between data are a guide for the eye only.

The pressure at the peaks of the spectra measured at the tone holes (curves including those in Figs. 5 and 6) is plotted in Fig. 7 as a function of the position of the holes through which they were measured. The first few resonances—those falling below the cutoff frequency for each instrument—are very nearly in a harmonic series (two harmonics for the baroque, four for the classical, and five for the modern). For the simple fingering XXX-OOO, the shape of the fundamental

TABLE III. The pitch change, in cents ± standard deviation, due to the cross fingerings shown in Fig. 7. One equal tempered semitone = 100 cents.

	Baroque	Classical	Modern
First resonance	−50±20	−25±5	−15±5
Second resonance	−215±20	−55±5	−30±5

wave is simple: it decreases monotonically with distance along the bore, and is very small in the lattice of open tone holes. Attenuation in the open tone hole lattice is greatest in the modern instrument, which has larger holes, and least in the baroque instrument, as expected. For the modern instrument, the higher resonances also behave much as expected. These resonances have nodes in the closed part of the bore, so these curves are not monotonic: for example, the standing wave for the fourth resonance has a node near the hole at −90 mm [See also Fig. 6(b) and the explanation in the penultimate paragraph of section C.]

For the classical and baroque instruments, the greater penetration into the open keyhole lattice results in a complicated pattern, even for the simple fingering. For the baroque instrument, the fifth and higher resonances approximately resemble harmonic standing waves over the full length of the tube [as shown in Fig. 6(b)]. For the baroque flute, the pressure at the last hole downstream is comparable with that in the field outside the instrument.

For the cross fingering, the standing wave of the lowest resonance is substantially terminated by a single open hole in all instruments: the wave falls to a small amplitude at the open hole and its amplitude decreases almost monotonically in the downstream closed section. These curves resemble the sketch given by Benade (1976) to explain the effects of cross fingering.

For the modern flute, the second resonance also falls monotonically beyond the first open hole, but the third, fourth, and fifth all show local maxima inside the downstream tube. The third resonance plays a slightly flat D6 (with a little difficulty), the fourth resonance a sharp F♯6. For these cases, the open hole may be considered to act as a register hole. When the open hole is closed, the fingering is that for D4, whose fourth and fifth harmonics are near D6 and F♯6. In the cross fingering shown, the open hole is between one-quarter and one-fifth of the way along the pipe and thus the third and fourth standing waves of the cross fingering are closely related to the fourth and fifth resonances of the fingering for D4. All of these resonances may be played with the cross fingering, although the third is not stable.

For the classical and baroque flutes, all standing waves except the first increase in amplitude downstream from the open hole, and the third and higher resonances are similar to the fourth and higher harmonics of the D4 fingering: for these notes the open hole is sufficiently close to a pressure node and/or its inertive reactance is sufficiently high that its being open makes relatively little difference to the standing wave.

IV. CONCLUSIONS

The cutoff frequency of the instruments increases, as expected, with increasing tone hole diameter. Above the cutoff frequency, the standing waves penetrate strongly past the open tone hole. In the case of the instrument with the smallest holes (baroque flute), the standing waves above the cutoff frequency are little affected by the open holes and are close to the expected resonances for the complete length of the instrument, with all tone holes closed. With larger holes, the increasing cutoff frequency gives a larger number of impedance minima that are in approximately harmonic ratio. This is expected to contribute to the production of notes that are brighter in timbre and louder.

Below the cutoff frequency, the standing wave propagates past the first tone hole by an amount that increases with frequency and that decreases with increasing tone hole diameter. For both the cutoff frequency and the attenuation in the tone hole lattice, the observed values are in agreement with those of a simple empirical model. The observed behavior of the standing waves explains the observed effects of cross

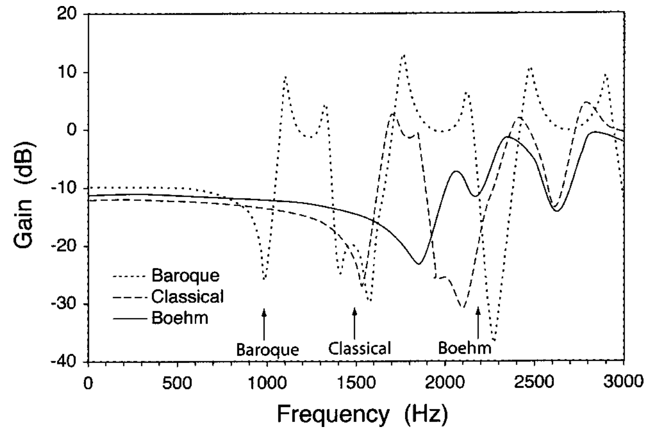

FIG. 8. The theoretical variation of the gain at the start of the tone hole lattice as a function of frequency for three different types of flute. This gain is defined as the ratio of the internal pressure at the second open tone hole to that at the first open tone hole. In this and in Fig. 9, the vertical arrows indicate the average value of the cutoff frequency calculated using dimensions for the first two open tone holes, for each instrument.

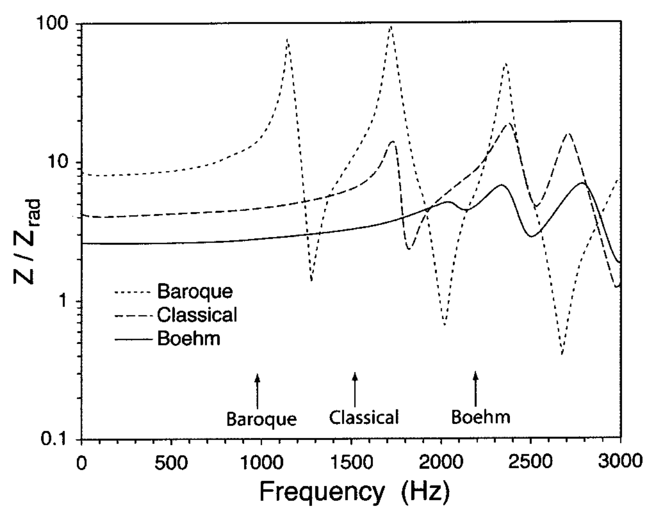

FIG. 9. The theoretical variation of the impedance downstream of the first open tone hole as a function of frequency for three different types of flute. Each curve has been normalized with respect to Z_{rad}, the radiation impedance for an unbaffled aperture with a diameter equal to that of the bore. Thus Z_{rad} is the impedance that would be measured if the flute were physically "cutoff."

fingering on the playing frequency and the impedance spectra for the different instruments and different registers.

ACKNOWLEDGMENTS

We thank Terry McGee, Mark O'Connor, and Matthew Ridley for lending instruments, Attila Stopic and John Tann for technical assistance, and Neville Fletcher for comments on the manuscript. This work was supported by the Australian Research Council.

APPENDIX: THE CHARACTERISTIC FREQUENCY OF AN INFINITE TONE HOLE LATTICE WITH FINITE COMPONENTS

Two successive open tone holes in an infinite, cylindrical bore are separated by a distance L. Looking downstream from one open tone hole, the acoustic impedance is given by the four terminal expression (Fletcher and Rossing, 1998):

$$Z_{\mathrm{in}}=Z_o\frac{Z_L\cos kL+jZ_o\sin kL}{jZ_L\sin kL+Z_o\cos kL},$$

where Z_o is the characteristic impedance of the bore, Z_L is the load impedance present at the next tone hole and other symbols have their usual meaning. Using lower case z for impedances nondimensionalized by dividing by Z_o, this becomes

$$z_{\mathrm{in}}=\frac{z_L+i\tan kL}{jz_L\tan kL+1}.$$

If the bore has radius a and the tone hole radius b, then the nondimensional radiation impedance z_{hole} at the tone hole is

$$z_{\mathrm{hole}}=j\frac{b^2}{a^2}\tan kt_e\equiv jR\tan kt_e,$$

where $R\equiv b^2/a^2$ and t_e is the effective length of the tone hole when the radiation impedance is considered.

z_L is the impedance of z_{hole} in parallel with the impedance of the bore beyond the hole. As the array is infinite, the latter impedance is z_{in}, so

$$z_{\mathrm{in}}=\frac{z_{\mathrm{in}}z_{\mathrm{hole}}+j(z_{\mathrm{in}}+z_{\mathrm{hole}})\tan kL}{jz_{\mathrm{in}}z_{\mathrm{hole}}\tan kL+(z_{\mathrm{in}}+z_{\mathrm{hole}})}.$$

Rearrangement gives a quadratic equation whose solution is

$$z_{\mathrm{in}}=j\tan kL\,\frac{1\pm\sqrt{1+4R\tan kt_e/\tan kL-4R^2\tan^2 kt_e}}{2(1-R\tan kt_e\tan kL)}.$$

If $t_e=0$, the tone hole is a short circuit and $z_{\mathrm{in}}=j\tan kL$, so we reject the solution with the negative sign.

In the cases of interest, $kt_e \ll 1$. kL is on the order of 1 for the characteristic frequencies, so $\tan kL\sim kL$, so

$$z_{\mathrm{in}}\sim jkL\,\frac{1+\sqrt{1+4(a/b)^2t_e/L-4(a/b)^4k^2t_e^2}}{2(1-(a/b)^2k^2t_eL)}.$$

Because $t_e<L$, the lower characteristic frequency is defined by $(a/b)^2k^2t_eL=1$, which gives

$$f_c\sim\frac{1}{2\pi}\frac{b}{a}\frac{c}{\sqrt{t_eL}}\cong 0.11\frac{b}{a}\frac{c}{\sqrt{t_es}},$$

where $s=L/2$. This is the same expression that Benade derives using transmission line theory.

Both the Benade model and the analysis given above fail at high frequencies. The behavior over the whole frequency range of interest may be determined explicitly using four terminal elements to represent the sections of the bore between tone holes. Calculations were made using an empirical model that relates the geometric parameters of the flute to its measured impedance (Botros *et al.*, 2002). Although this model was originally developed for the modern flute, we have also arbitrarily used it for the classical and baroque flutes, altering the bore and tone hole diameters appropriately. For simplicity the effects of energy losses along the bore have not been included in the results presented here. Figure 8 shows the calculated variation of acoustic pressure from one tone hole to the next. Figure 9 shows the calculated frequency dependence of the lattice. In the case of the baroque flute, the acoustic impedance above the cutoff frequency shows a regular series of maxima and minima, rather similar to that of a simple pipe. As observed above, in relation to Figs. 4 and 5, this is because the tone holes behave as though they were closed at these frequencies and so the downstream impedance does indeed approximate that of a simple pipe.

Benade, A. H. (**1959**). "On woodwind instrument bores," J. Acoust. Soc. Am. **31**, 137–146.

Benade, A. H. (**1960**). "On the mathematical theory of woodwind finger holes," J. Acoust. Soc. Am. **32**, 1591–1608.

Benade, A. H. (**1976**). *Fundamentals of Musical Acoustics* (Oxford U.P., New York), pp. 430–501.

Botros, A., Smith, J., and Wolfe, J. (**2002**). "The virtual Boehm flute—a web service that predicts multiphonics, microtones and alternative fingerings," Acoust. Austral. **30**, 61–65.

Coltmann, J. W. (**1979**). "Acoustical analysis of the Boehm flute," J. Acoust. Soc. Am. **65**, 499–506.

Dubos, V., Kergomard, J., Khettabi, A., Dalmont, J-P., Keefe, D. H., and Nederveen, C. J. (**1999**). "Theory of sound propagation in a duct with a branched tube using modal decomposition," Acustica **85**, 153–169.

Fletcher, N. H., and Rossing, T. D. (**1998**). *The Physics of Musical Instruments* (Springer-Verlag, New York), pp. 461–548.

Keefe, D. H. (**1982a**). "Theory of the single woodwind tone hole," J. Acoust. Soc. Am. **72**, 676–687.

Keefe, D. H. (**1982b**). "Experiments on the single woodwind tone hole," J. Acoust. Soc. Am. **72**, 688–699.

Nederveen, C. J. (**1998**). *Acoustical Aspects of Wind Instruments* (Northern Illinois Univ., DeKalb, IL), pp. 45–78.

Nederveen, C. J., Jansen, J. K. M., and van Hassel, R. R. (**1998**). "Corrections for woodwind tone-hole calculations," Acustica **84**, 957–966.

Smith, J. R., Henrich, N., and Wolfe, J. (**1997**). "The acoustic impedance of the Bœhm flute: standard and some non-standard fingerings," Proc. Inst. Acoust. **19**, 315–320.

Strong, W. J., Fletcher, N. H., and Silk, R. K. (**1985**). "Numerical calculation of flute impedances and standing waves," J. Acoust. Soc. Am. **77**, 2166–2172.

Wolfe, J., and Smith, J. (**2001**). Embouchures and end effects in air-jet instruments, Proc. International Congress on Acoustics, Rome, Session 8.06, pp. 8–9.

Wolfe, J., Smith, J., Tann, J., and Fletcher, N. H. (**2001a**). "Acoustics of baroque, classical and modern flutes: a compendium of impedance spectra, sound spectra, sounds and fingerings," JSV+. Electronic publication at http://journals.harcourt-international.com/journals/jsv/supplementary/suppindex.htm

Wolfe, J., Smith, J., Tann, J., and Fletcher, N. H. (**2001b**). "Acoustic impedance of classical and modern flutes," J. Sound Vib. **243**, 127–144.