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AIR FLOW AND SOUND GENERATION IN MUSICAL WIND INSTRUMENTS

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INTRODUCTION

Over the past two decades or so, interest in musical acoustics appears to have been increasing rapidly. We now have available several collections of reprinted technical articles (Hutchins 1975, 1976, Kent 1977), together with a large number of textbooks, of which those most suitable for citation in this review are by Olson (1967), Backus (1969), Nederveen (1969), and Benade (1976). The mathematical foundations of the subject were laid primarily by Lord Rayleigh (1896) and are well treated in such standard texts as Morse (1948) and Morse & Ingard (1968).

This review covers a much more restricted field than this preliminary bibliography might suggest. Among all the varieties of musical instruments I concentrate on those capable of producing a steady sound that is maintained by a flow of air, and even within this family I am interested not so much in the design and behavior of the instrument as a whole but rather in the details of the air flow that are responsible for the actual tone production.

Although musical instruments function as closely integrated systems, it is convenient and indeed almost essential for their analysis to consider them in terms of at least two interacting subsystems, as shown in Figure 1. The first of these is the primary resonant system, which consists of a column of air, confined by rigid walls of more or less complex shape and having one or more openings. Such a system is generally not far from linear in its behavior and it can be treated, at least in principle, by the classical methods of acoustics. The second subsystem is the airdriven generator that excites the primary resonator. This subsystem is generally

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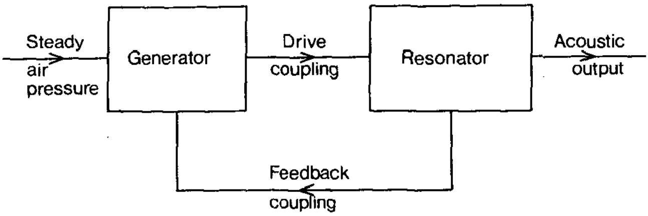

Figure 1 System diagram for a musical wind instrument.

highly nonlinear, either intrinsically or through its couplings with the resonator system, and it is this nonlinearity, as we shall see below, that is responsible for the stability of the whole system as well as for much of its acoustic character.

AIR COLUMNS

The acoustical behavior of an air column of arbitrary cross section is well understood provided the cross section is a slowly varying function of position (Eisner 1967, Benade & Jansson 1974, Jansson & Benade 1974). Columns enclosed in tubes of exactly cylindrical or exactly conical shape are particularly simple to analyze, as are a few other special shapes (Morse 1948, pp. 233–88, Benade 1959, Nederveen 1969, pp. 15–24), but the detailed shapes of the bores of real wind instruments usually differ significantly from these idealized models.

The quantity of major importance for our discussion is the acoustical impedance Z_p (defined as the ratio of acoustic pressure to acoustic volume flow) at the input to the resonator where the driving force from the generator may be supposed to act. Various instruments have been developed to measure this impedance (Benade 1973, Backus 1974, Pratt et al 1977) following early work by Kent and his collaborators. Because of the phase shifts involved, Z_p is usually written as a complex quantity, and the measuring system can be arranged to yield either the real part, the imaginary part, or simply the magnitude of either the impedance or its inverse, the admittance.

For a narrow cylindrical pipe of cross-sectional area A the acoustic-wave propagation velocity has very nearly its free air value c, and the major losses are caused by viscous and thermal effects at the walls, comparatively little energy being lost from the end (if open) provided its circumference is much smaller than the acoustic wavelength involved. If we denote angular frequency by ω then the propagation number is $k = \omega/c$, and we can generalize this to allow for wall losses by replacing k by $k - jk'$, where $j = \sqrt{-1}$ and $k' \ll k$. The loss parameter k' increases with

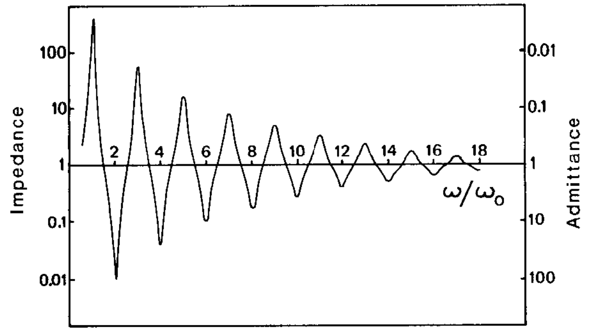

Figure 2 Acoustic input impedance Z_p, in units of $\rho c/A$, for a typical cylindrical pipe open at the far end. Z_p is plotted on a logarithmic scale so that the acoustic admittance $Y_p = Z_p^{-1}$ is obtained simply by inverting the diagram. Typically $\rho c/A \sim 10^6$ Pa m^{-3} s $\equiv 1$ SI acoustic megohm.

frequency like $\omega^{1/2}$. If the pipe is open at the far end then the input impedance is very nearly

$$Z_p \approx j(\rho c/A) \tan [(k - jk')l], \tag{1}$$

where ρ is the density of air and the effective pipe length l exceeds the geometrical length by an end correction equal to 0.6 times the radius. The form of this expression is shown in Figure 2, which is plotted on a logarithmic scale so that the magnitude of the input admittance $Y_p = Z_p^{-1}$ can be seen by simply imagining the picture to be inverted.

The impedance Z_p shows peaks of magnitude $(\rho c/A)$ coth $k'l$ at frequencies ω_0, $3\omega_0$, $5\omega_0$, ..., and the admittance Y_p shows peaks of magnitude $(A/\rho c)$ coth $k'l$ at frequencies $2\omega_0$, $4\omega_0$, ..., where ω_0 is given by $kl = \pi/2$ or $\omega_0 = \pi c/2l$. For a pipe of finite radius these resonances are slightly stretched in frequency, but this need not concern us here.

For our present purposes we need consider only two types of excitation mechanism: the air jet of flute-type instruments, whose deflection is controlled by the velocity of the acoustic flow out of the pipe mouth as shown in Figure 3, and the reed or lip-reed generator, whose opening is

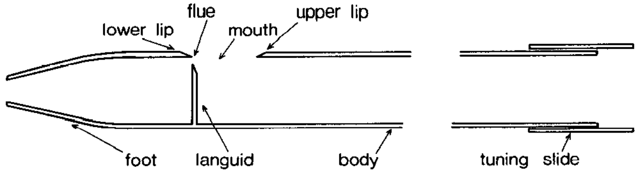

Figure 3 Section through a typical organ flue pipe showing its main constructional features.

controlled by the acoustic pressure inside the pipe at the mouthpiece as shown in Figure 4. We can call these respectively velocity-controlled and pressure-controlled generators. A velocity-controlled generator clearly transfers maximum power to the pipe resonator when the acoustic flow at the mouth is maximal and thus at the frequency of an admittance maximum. A pressure-controlled generator transfers maximum power when the acoustic pressure is maximal and thus at the frequency of an impedance maximum. I examine these statements in greater detail below and also note small modifications to allow for finite generator impedance.

The consequences of this behavior are easily seen. Flutes and open-ended organ pipes overblow to produce the notes of a complete harmonic series $2\omega_0$, $4\omega_0$, $6\omega_0$, ... based on the fundamental $2\omega_0$, while clarinets, which also have nearly cylindrical pipes, produce the odd harmonics ω_0, $3\omega_0$, $5\omega_0$, ... only (to a first approximation at any rate). We can also have flutelike systems in which the far end of the pipe is stopped rather than open, giving an input impedance like (1) with tan replaced by cot and a characteristic curve effectively inverted relative to Figure 2. A velocity-controlled air-jet generator leads to possible sounding frequencies ω_0, $3\omega_0$, $5\omega_0$, ... for such a system, while a reed generator fails to operate because of back pressure.

In the case of instruments like the oboe, bassoon, or saxophone, which are based upon an approximately conical pipe, the impedance maxima for the pipe lie at frequencies $2\omega_0$, $4\omega_0$, $6\omega_0$, ... with l equal to the complete length of the cone extended to its apex (Morse 1948, pp. 286–87, Nederveen 1969, p. 21). Such instruments produce a complete harmonic series like the flute rather than an odd-harmonic series like the clarinet.

Finally, the geometry of the brass instruments, with their mouthpiece cup, partly cylindrical, partly conical tube, and flaring bell, is adjusted

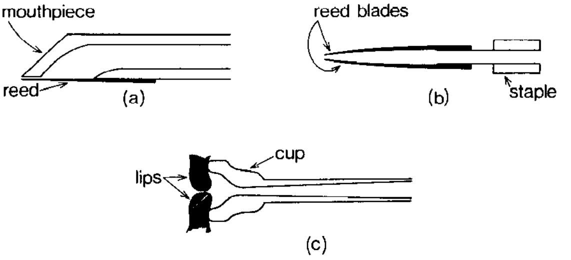

Figure 4 Reed configuration in (*a*) single-reed instruments like the clarinet or saxophone, (*b*) double-reed instruments like the oboe or bassoon. Note that in each case the blowing pressure tends to close the reed aperture ($s = -1$). For a lip-valve instrument like the trumpet (*c*), the blowing pressure tends to open the lip aperture ($s = +1$).

by the designer so that their impedance maxima follow a progression like 0.8 ω_0, $2\omega_0$, $3\omega_0$, $4\omega_0$, ... (Backus 1969, pp. 215–23, Benade 1973). This progression is musically satisfactory, with the exception of the lowest mode, and the flared bell of the instrument produces a generally more brilliant sound than the straight conical horn used in some now-obsolete instruments such as the cornett, ophicleide, and serpent (Baines 1966).

I do not pursue here details of the ways in which the fundamental reference frequency ω_0 for the air column is varied in different instruments to produce the notes of the modern chromatic scale (Backus 1969, pp. 223–27, Benade 1960a,b, Nederveen 1969). The important thing for this analysis is that, for every fingering configuration of a musical wind instrument, there is an impedance curve for the air column that displays a succession of pronounced maxima and minima. For musically useful fingerings the successive maxima will often have smoothly graded magnitudes and frequencies that are in nearly integer relationships, but this is by no means universally true in the case of the woodwinds, particularly in the upper register.

Because the acoustical behavior of the air column is closely linear, we can consider each possible normal mode (corresponding to an impedance maximum or an admittance maximum as the case may be) quite separately and characterize it by a resonance frequency n_i, a resonance width determined by $\kappa_i = k'/k$ at $\omega = n_i$, and a displacement amplitude x_i. A complete description then involves superposition of these driven modes.

SYSTEM ANALYSIS

A formal analysis of the system shown in Figure 1, including the nonlinearities that control its behavior, was first put forward by Benade & Gans (1968) for the case of musical instruments, though of course much of the basic work dates back to the early days of electronic circuits (Van der Pol 1934), and the general theory is of interest to mathematicians (Bogoliubov & Mitropolsky 1961). Since then the major formal developments of the nonlinear analysis of musical instruments have been in the work of Worman (1971) and Fletcher (1974, 1976a,c, 1978a) and in several unpublished papers by Benade.

Suppose that the pipe resonances are at angular frequencies n_i when the generator is attached but not supplied with air (thus terminating the pipe with a passive impedance that is generally either much larger or smaller than $\rho c/A$). Let x_i be the acoustic displacement associated with the ith mode; then, because the resonator is linear, x_i satisfies an equation of the form

$$\ddot{x}_i + \kappa_i \dot{x}_i + n_i^2 x_i = 0, \quad (2)$$

where κ_i is the width of the resonance.

If we refer to Figure 1, the individual pipe-mode amplitudes x_j influence the air-driven generator with coupling coefficients α_j, which may be directly related to either acoustic velocity or acoustic pressure, and cause it to produce a driving force $F(\alpha_j x_j)$, which depends nonlinearly upon all the influences $\alpha_j x_j$. This force F then drives each individual mode i through a second coupling coefficient β_i according to

$$\ddot{x}_i + \kappa_i \dot{x}_i + n_i^2 x_i = \beta_i F(\alpha_j x_j). \quad (3)$$

In general, the α_i and β_i will be complex, in the sense of involving a phase shift. The careful formal development of this approach (Fletcher 1978a) involves a distinction between air-jet and reed-driven instruments in interpretation of the x_i, but this need not concern us here.

If the instrument is producing sound in a quasi-steady state, then it is reasonable to assume that x_i has the form

$$x_i = a_i \sin(n_i t + \phi_i), \quad (4)$$

where both the amplitude a_i and the phase ϕ_i are slowly varying functions of time. Clearly a nonzero value of $d\phi_i/dt$ implies an oscillation frequency ω_i, given by

$$\omega_i = n_i + d\phi_i/dt, \quad (5)$$

which is close to but not exactly equal to the free-mode frequency n_i.

It is now easy to show (Bogoliubov & Mitropolsky 1961, pp. 39–55, Fletcher 1976a,c) that

$$da_i/dt \approx (\beta_i/n_i)\langle F(\alpha_j x_j)\cos(n_i t + \phi_i)\rangle - \tfrac{1}{2}\kappa_i a_i \quad (6)$$

$$d\phi_i/dt \approx -(\beta_i/a_i n_i)\langle F(\alpha_j x_j)\sin(n_i t + \phi_i)\rangle \quad (7)$$

where the brackets $\langle\ \rangle$ imply that only terms varying slowly compared with n_i are to be retained.

Several things are immediately clear from these equations. If the blowing pressure is zero, then $F = 0$, which implies that $\omega_i = n_i$ and that the amplitude a_i decays exponentially to zero. For nonzero blowing pressures, in general $\omega_i \neq n_i$, and because F is a nonlinear function of the x_j, the terms $\langle \ldots \rangle$ in (6) and (7) will contain slowly varying components with frequencies $\omega_i \pm \omega_j \pm \omega_k \pm \ldots$. The only situation in which a steady sound can be obtained occurs when all the blown frequencies ω_i are integer multiples of some common frequency ω_0. This is the normal playing situation for an instrument and is generally achieved after an initial transient occupying about 40 cycles of the fundamental frequency involved (Richardson 1954, Strong & Clark 1967, Fletcher 1976a). Once achieved, and this depends on the amount of nonlinearity present, the mode-locked regime is usually stable (Fletcher 1976a, 1978a). Clearly the nonlinearity of F is also largely responsible for most if not all of the harmonic structure of the sound spectrum.

Musical instruments can often be played in several different mode-locked regimes for a given tube configuration and thus for a given set of pipe resonances—one has only to think of the complex fanfares that can be played on horns and trumpets without valves. In general terms we can see that this flexibility can be achieved if the generator F itself has a resonant or phase-sensitive response that can be adjusted by the player so as to concentrate F in a narrow frequency range. This is one of the aspects of generator behavior that I investigate below. The remainder of this review, in fact, is concerned with the physical nature of different generator systems and with the air flows responsible for their operation.

Before leaving the general question of system behavior I should point out that there is a fundamental difference between the structure of the internal frequency spectrum of the instrument, which is what we calculate when we find the amplitudes x_i of the internal modes, and the structure of the spectrum radiated by the instrument, which is what our ears detect. For a simple cylindrical pipe with an open end of radius r, standard acoustic theory (Olson 1967, p. 85) shows that the radiation resistance at the open end varies as ω^2 for frequencies below the cut-off ω^* (which is given by $\omega^* r/c \approx 2$), while for frequencies above ω^* the radiation resistance is nearly constant. Thus the radiated spectrum below ω^* has a high-frequency emphasis of 12 dB/octave relative to the internal spectrum, while above ω^* the two are parallel.

The situation is similar for the more complex geometry of brass and woodwind instruments, except that for brass instruments ω^* is determined by the precise flare geometry of the horn (Morse 1948, pp. 265–88), while in woodwind instruments with finger holes ω^* is determined largely by the transmission properties of the acoustic waveguide with open side holes in the lower part of the instrument bore (Benade 1960b).

AIR-JET INSTRUMENTS

An essentially correct, though qualitatively expressed, theory of the operation of air-jet instruments was put forward as early as 1830 by Sir John Herschel (Rockstro 1890, pp. 34–35), but this was later neglected because of preoccupation with the related phenomena of edge tones and vortex motion in jets (Curle 1953, Powell 1961), which were made visible in fine photographic studies like those of Brown (1935). While it is certainly true that edge-tone phenomena are in some ways analogous to the action of an air jet in an organ pipe, the mechanisms involved differ importantly in the two cases (Coltman 1976). Similarly, while vortices are undoubtedly produced by the jet in an organ pipe, their presence seems to be an incidental second-order effect rather than a basic feature of the mechanism, and a complete theory including all aspects of the aerodynamic motion will inevitably be extremely complex (Howe 1975). Our best present understanding is as set out below.

The basic geometry of an air-jet generator is illustrated for the case of an organ pipe in Figure 3. A planar air jet emerges from a narrow flue slit (typically a few centimeters in length and a millimeter or so in width) and travels across the open mouth of the pipe to impinge more or less directly on the upper lip. Acoustic flow through the pipe mouth and associated with the pipe modes deflects the jet so that it blows alternately inside and outside the lip, thus generating a fluctuating pressure that serves to drive the mode in question. The blowing pressure in the pipe foot is typically a few hundred pascals (a few centimeters, water gauge), giving a jet velocity of a few tens of meters per second and hence a transit time across the pipe mouth that is comparable with the period of the acoustical disturbance, so that phase effects are certainly important. The discussion below is in terms of the organ pipe geometry, but other instruments of the air-jet type behave similarly.

Wave Propagation on a Jet

The work of Rayleigh (1879, 1896, pp. 376–414) provides the foundation for understanding the behavior of a perturbed jet. He treats the case of a plane inviscid laminar jet of thickness $2l$ moving with velocity V through a space filled with the same medium, and he shows that a transverse sinuous disturbance of the jet with angular frequency $n = \omega + j\omega'$ and propagation number k, such that the displacement has the form

$$y = A \exp\left[j(nt \pm kx)\right], \tag{8}$$

satisfies the dispersion relation

$$(n+kV)^2 \tanh kl + n^2 = 0. \tag{9}$$

Solving for n and substituting back in (8) shows that the wave on the jet propagates with velocity

$$u = V/(1+\coth kl) \tag{10}$$

and grows exponentially with time or with the distance x traveled by the wave as exp (μx), where

$$\mu = k(\coth kl)^{1/2}. \tag{11}$$

Equations (10) and (11) show that, at frequencies low enough that the wavelength λ of the disturbance on the jet is much greater than l, so that $kl \ll 1$, the propagation velocity $u \approx klV \approx (lV\omega)^{1/2}$, while the growth parameter $\mu \approx (k/l)^{1/2}$. At the other extreme when $\lambda \ll l$, we find $u \approx V/2$ and $\mu \approx k \approx 2\omega/V$.

Rayleigh realized that these results are somewhat unrealistic since the behavior of μ for large ω predicts catastrophic instability for the jet in this limit. He correctly identified the origin of this catastrophe in the velocity profile assumed and went on to investigate jet behavior for jets with smoother velocity profiles (Rayleigh 1896, pp. 376–414). He showed that instability ($\mu > 0$) is associated with the existence of a point of inflection in the velocity profile, and that μ is positive in the low-frequency limit, increases with increasing frequency to a maximum when $kb \approx 1$, b being some measure of the jet half-width, and then decreases to become negative for $kb \geqq 2$.

Further advance did not come until Bickley (1937) investigated the velocity profile of a plane jet in a viscous fluid, showing it to have a form like $V_0 \operatorname{sech}^2(y/b)$, and until Savic (1941) examined the propagation of transverse waves on such a jet in the inviscid approximation. This and more recent work has been summarized by Drazin & Howard (1966). If b is the half-width parameter defined by Bickley and J is the flow integral defined by

$$J = \int_{-\infty}^{\infty} [V(y)]^2\, dy, \tag{12}$$

where y is the dimension transverse to the jet, then the best numerical calculations indicate

$$u \approx 0.95\,(bV\omega)^{1/2} \quad \text{for} \quad 0 < kb \leqq 0.4, \tag{13}$$

$$u \approx 0.65\,(J\omega)^{1/3} \quad \text{for} \quad 0.4 \leqq kb \leqq 2. \tag{14}$$

Equation (13) is close to Rayleigh's result while (14) is close to that of Savic. The amplification factor μ varies approximately as

$$\mu b \approx 0.74\,[1 - \exp(-3kb)] - 0.37\, kb, \tag{15}$$

this being an approximate fit to the calculated points in the range $0.1 \leqq kb \leqq 2$, which confirms Rayleigh's investigation in that it has a maximum near $kb \approx 0.6$ and becomes negative for $kb > 2$.

Few extensive experimental studies of wave propagation on jets appear to have been made, though the work of Brown (1935) on vortex motion, Sato (1960) on instability, Chanaud & Powell (1962) on edge tones, and Coltman (1976) on organ pipes bears on the problem. Most useful is a recent study by Fletcher & Thwaites (1978) for low-velocity jets as found in organ pipes. This work confirms a propagation velocity close to that given by (14) at low frequencies, but shows that the wave velocity saturates to a value μ_∞, given in SI units by

$$u_\infty \approx 50\,J \tag{16}$$

for higher frequencies. Fletcher and Thwaites conjecture, on the basis of dimensional analysis, a form

$$u \approx (J/v)\,f(v\omega^{1/3}J^{-2/3}) \tag{17}$$

for the wave velocity u, where J is given by (12), v is the kinematic viscosity of air, and the function $f(z)$ has a form rather like

$$f(z) = c_1 z/(1+c_2 z), \tag{18}$$

where $c_1 \approx 0.7$, $c_2 \approx 1000$. This reduces to (14) if the kinematic viscosity is set to zero and to (16) if ω becomes large. An interesting thing about these experimental results and their theoretical counterparts is that, since J is constant along the jet irrespective of viscous spreading, u is similarly constant. Other observations confirm this.

The experiments give rather less specific information about μ but agree with (15) to good approximation for $kb \leqq 0.6$. The experiments also suggest that no amplification occurs for $kb \geqq 2$. In the range $0.6 \leqq kb \leqq 2$, however, the experimental value of μ is nearly constant rather than decreasing smoothly to zero as predicted by the inviscid theory.

It is perhaps important to note that the validity of many of the theoretical calculations is restricted to wave amplitudes less than the jet half-width b, while the experiments are all made for amplitudes larger than b. It is not known how important this distinction may be. The theoretical and experimental results are also limited, as yet, to the laminar-flow regime, while many musically important jets operate at Reynolds numbers high enough to be turbulent.

Acoustic Perturbation of a Jet

We have now to consider the way in which the acoustic modes of a pipe resonator interact with a jet and induce the formation of traveling waves upon it. The experimental studies of Brown (1935) show that the perturbation takes place just at the point where the jet emerges from the flue into the acoustic field, and Rayleigh's discussion, with new values substituted for u and μ, shows how this occurs.

A jet flowing in the x direction in open air would be simply moved backwards and forwards by an acoustic velocity field $v \cos \omega t$ acting transversely to the jet plane, the displacement amplitude being (v/ω) sin ωt. Such a displacement would not induce any wave motion on the jet. When, however, the jet emerges into the field from a slit located along the z-axis in the plane $x = 0$, the jet displacement is constrained to be zero when $x = 0$. This is equivalent to superposing a local displacement $-(v/\omega) \sin \omega t$ on the general displacement produced by the acoustic field. Such a localized displacement does produce waves traveling in the $\pm x$ directions on the jet, and the resultant displacement is (Fletcher 1976c)

$$y(x,t) = (v/\omega)\{\sin \omega t - \cosh \mu x \sin [\omega(t - x/u)]\}. \tag{19}$$

The form of $y(x,t)$ is rather complicated for $\mu x < 1$, which means within a few jet-widths of the slit, but it then takes the form of a rapidly growing wave with constant propagation velocity. Extrapolation of (19) to y values greater than the jet half-width is justified by studies of the behavior of real jets in air, as seen from schlieren photographs or from studies of smoke-laden jets (Cremer & Ising 1967, Coltman 1968). Additional nonlinearities ultimately occur, however, and the jet breaks up into a double street of vortex rings (Brown 1935) when the displacement amplitude exceeds the wavelength on the jet.

Quite recently Coltman (1976) has studied phase relations in acoustic disturbance and propagation on a jet in more detail. Although this is not immediately obvious, the behavior he finds is in quite reasonable agreement with the predictions of (19), while the residual disagreements are at least qualitatively accounted for in terms of an expected slight decrease in u near the jet flue (Fletcher & Thwaites 1978).

Jet Drive of a Pipe

The basic mechanism by which a jet influences the modes of a pipe is fairly straightforward. Clearly the jet is a low impedance source, since in the absence of air flow the mouth of the pipe is effectively open. The mouth impedance is, however, not exactly zero because of the end correction at the mouth. Now when the jet blows momentarily inside the pipe lip it provokes acoustic motion of the air column both through simply adding to the acoustic flow velocity and also through building up a local pressure that can drive the acoustic flow. These two views of the dominant mechanism involved were put forward initially by Cremer & Ising (1967) and by Coltman (1968) respectively.

More recently, Elder (1973) has given a more careful discussion, which integrates these two approaches. This has been extended by Fletcher (1976b) and some of the implied phase relations have been checked experimentally by Coltman (1976). In the case treated by Elder, the flow of the jet was assumed to be varied by modulating the flow velocity to match an applied signal. Such a model, as discussed by Fletcher, leads to a large amount of harmonic generation and thus complicates the resulting equations. Fletcher's assumption was that the jet flow is varied by changing its cross section, an assumption better approximating the situation in a real pipe and at the same time leading to the simplest possible equations. The third alternative, in which the blowing pressure follows the applied signal, does not seem to have been examined in detail. Once the physical assumptions are made it is further assumed that the jet interacts with the air in the pipe over a mixing length Δx, which does not enter the final result provided it is very small compared with the sound wavelength involved.

In terms of Fletcher's model, the final result for the acoustic flow U_p into the pipe produced by a time-varying flow U_j at frequency ω in the jet is given by (Fletcher 1976b,c)

$$Z_s U_p \approx [\rho(V + j\omega\Delta l)/A_p]\, U_j, \tag{20}$$

where A_p is the cross sectional area of the pipe, ρ the density of air, Δl the end correction at the mouth, and Z_s the acoustic impedance of the pipe and mouth in series. This equation is, in fact, equivalent to our symbolic Equation (4) for one of the pipe modes.

In (20), both versions of the driving force appear. The term $\rho V U_j/A_p$ can be written $\rho V^2(A_j/A_p)$, where A_j is the cross section of the jet, and it represents the pressure built up by the jet on entering the pipe, as discussed by Coltman (1968), while the second term $j\omega\Delta l V(A_j/A_p)$ is the velocity-drive term discussed by Cremer & Ising (1967). In practice for most simple jets the second term dominates over the first, so that the driving term on the right of (20) is 90° in advance of the jet flow U_j in phase.

Incidentally it is clear from (20) that the amplitude of the pipe flow U_p is a maximum when the series impedance Z_s, corresponding to the pipe with both its end corrections, is a minimum. This confirms our view of the jet as a low-impedance velocity-controlled generator and shows that the pipe modes with which we are concerned are those giving impedance minima when measured from just outside the pipe mouth.

Nonlinearity

While some of the terms neglected in (20) lead to slight nonlinearity of behavior, the major nonlinear mechanism arises from quite a different cause. We have already seen from (19) that a simple sinusoidal acoustical influence on the jet leads to a similarly sinusoidal deflection of the jet tip where it strikes the pipe lip. The velocity distribution across the jet is, however, of the form $V_0 \operatorname{sech}^2 (y/b)$ and its width b is finite, so that the jet flow U_j into the pipe varies, as a function of jet deflection, in a highly nonlinear manner, saturating for deflections completely into or completely out of the pipe lip. The extent of the linear region is proportional to the effective jet width $2b$.

It is clear that, once the jet becomes fully switched from inside to outside the pipe during each cycle of the fundamental, U_j will contain considerable amplitudes of harmonics of all orders. This has been considered in some detail by Fletcher (1974, 1976c). The steady regime has also been discussed in detail by Schumacher (1978a) using a powerful integral equation approach and computer symbolic manipulation to keep track of the coefficients involved.

System Implications and Performance

If we refer to Figure 1, it is clear on general grounds that the total phase shift around the feedback loop must be zero for the system to operate. If we take the acoustic flow $v \cos \omega t$ into the pipe mouth as reference, it follows from (19) that the propagating-wave part of the jet displacement at $x = 0$ is 90° in advance of this in phase. If the distance from the flue to the pipe lip is d, then the transit time for the jet wave introduces a phase lag of $\delta = \omega d/u$, where u is the wave velocity on the jet. Finally, from our discussion of the dominant term in (20), there is a further advance in phase of nearly 90° in the jet interaction coefficient. Thus, if the phase loop is to close at the resonance frequency where Z_s in (20) is real, we must have $\delta \approx \pi$, implying a phase shift of 180° or just half a wavelength along the jet.

If this condition is not precisely met, then the oscillation frequency must shift slightly away from resonance so that the extra phase shift can be accommodated in the impedance factor Z_s. Thus suppose that the blowing pressure is raised so that the jet velocity increases and the phase lag δ along the jet decreases to $\pi - \varepsilon$. The right side of (20) is then an amount ε in advance of the reference, (which is essentially U_p) in phase. This can be accommodated if the impedance Z_s has a phase ε with respect to U_p, which requires that it be slightly inductive. Since the pipe behaves at an impedance minimum like a series resonant circuit, this implies that the increased blowing pressure must cause the sounding frequency to rise slightly. This is exactly what happens in practice.

Air Jet Impedance

At this point it is helpful to change our viewpoint slightly, following Coltman (1968), and to define an effective impedance Z_j for the air-jet generator as viewed from inside the pipe mouth and with allowance made for the unblown pipe-mouth correction, considered to be in series with it. Because of our viewpoint from inside the pipe, if an acoustic flow U_p into the pipe causes the jet to produce an acoustic pressure p inside the pipe lip, then

$$Z_j = -p/U_p. \tag{21}$$

Now the acoustic power delivered to the pipe by the jet is $U_p^2 \operatorname{Re}(-Z_j)$ and the acoustic dissipation in the pipe is $U_p^2 \operatorname{Re}(Z_s)$, where Z_s is the impedance of the pipe and mouth in series, so that the condition for stable or growing oscillations is that

$$\operatorname{Re}(Z_j+Z_s) \leqq 0. \tag{22}$$

Since $\operatorname{Re}(Z_p) > 0$, this implies that we must have $\operatorname{Re}(Z_j) < 0$ and that pipe oscillation is favored at the impedance minima or admittance maxima of the pipe with its mouth end correction taken into account. This latter is just the conclusion reached from (20).

For stable oscillations we must have equality in (22), which is achieved by the effect of nonlinearities on the magnitude of Z_j, which decreases steadily with increasing U_p. Since the flow U_p must be continuous through the system, we must also have

$$\operatorname{Im}(Z_j+Z_s) = 0, \tag{23}$$

which implies that if Z_j is inductive so that $\operatorname{Im}(Z_j) > 0$, then $\operatorname{Im}(Z_s) < 0$, and from (1), using an effective length l to include the mouth correction, the sounding frequency must be lower than the resonance frequency of

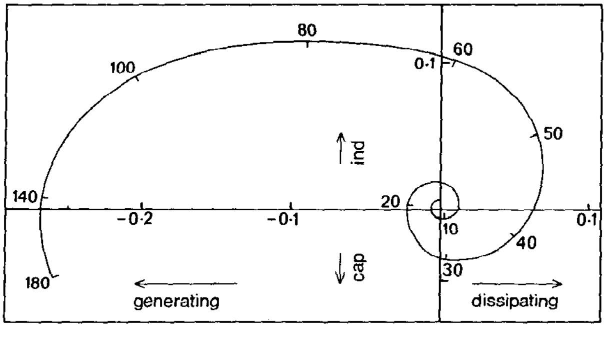

Figure 5 Measured complex acoustic impedance Z_j of a jet-driven acoustic generator with flutelike geometry, as a function of blowing pressure shown in pascals as a parameter (1-cm water gauge = 100 Pa), other parameters being normal for a flute jet. Measurement frequency is 440 Hz and impedance is given in SI acoustic megohms (1 Ω = 1 Pa m^{-3} s). (After Coltman 1968)

the complete pipe. If Z_j is capacitative, then the sounding frequency will be above the resonance frequency.

In an elegant series of experiments, Coltman (1968) measured Z_j as a function of blowing pressure for a flutelike jet system. One of his measured curves is shown in Figure 5. The spiral form of the curve is caused by the varying phase shift for waves traveling along the jet, while the magnitude of Z_j is a rather complicated function of the amplification factor μ and the interaction expression (20) as functions of jet velocity and thus of blowing pressure. Qualitatively similar curves are to be expected for organ pipe jets.

The design and voicing of organ pipe ranks to produce optimum attack and sound quality is an art, the practical results of which conform fairly generally with the expectations derived from the theory (Mercer 1951, Fletcher 1976c). The much more complex situation of performance technique on flutelike instruments is also well accounted for (Coltman 1966, Fletcher 1975). In particular, the flute player adjusts the blowing pressure and air-jet length (increasing the first and reducing the second for high notes) in such a way that the phase relations requisite for stable oscillation are satisfied only in the vicinity of the particular resonance peak corresponding to the fundamental of the note he wishes to sound. An experienced player, for example, can easily take a simple cylindrical pipe with a side hole cut in its wall and the near end closed with a cork, thus producing a flutelike tube with an impedance curve like that in Figure 2, and blow steady notes based upon each of the first six or more impedance minima.

The loudness of sound produced by the instrument is controlled almost entirely by varying the player's lip aperture and hence the cross section of the jet, since blowing pressure must be set fairly closely to meet the phase requirements on the jet. It is still possible, however, for the player to vary the sounding frequency of a note by varying the blowing pressure, though frequency control is more usually achieved by altering the lip shape and hence the end correction at the mouthpiece. This and more subtle aspects of performance technique can also be understood on the basis of the theory (Fletcher 1975).

REED AND LIP-DRIVEN INSTRUMENTS

Common instruments of the woodwind reed family include the clarinet, which has a single-reed valve, as shown in Figure 4(*a*), driving a basically cylindrical pipe, the saxophone, which has a similar reed driving a conical pipe, and the oboe and bassoon, which each have a double reed, as in Figure 4(*b*), driving a conical pipe. In all these cases the application of blowing pressure tends to close the reed opening, and we say that the reed strikes inwards. In brass instruments like the trumpet or trombone the player's lips form a type of reed valve as shown in Figure 4(*c*) but in this case the blowing pressure tends to open the lip aperture and we say that the reed strikes outwards. We have already discussed the complex geometry of brass instrument horns. In what follows we perforce ignore the large amount of careful and detailed work that has been done on the shape of the air column and the behavior of finger holes (Backus 1968, 1974, Benade 1959, 1960b, 1976, Nederveen 1969) and concentrate on the way in which sound is produced by the reed generator, using this term to include lip reeds.

In its essentials the behavior of a reed system coupled to a pipe was first correctly described by Helmholtz (1877, pp. 390–94), and it was he who clearly made the distinction between reeds striking inwards and outwards. He showed that an inward-striking reed must drive the pipe at a frequency that is lower than the resonant frequency of the reed, viewed as a mechanical oscillator, while an outward-striking reed must drive the pipe at a frequency higher than the reed resonance. Work since that time has concentrated largely on the clarinet reed, with important advances in understanding (Backus 1961, 1963, Nederveen 1969, pp. 28–44, Worman 1971, Wilson & Beavers 1974). There has been relatively little work on details of sound generation in brass instruments (Martin 1942, Benade & Gans 1968, Backus & Hundley 1971, Benade 1973). Our discussion is based largely on a recent paper by Fletcher (1978b), which incorporates this earlier work and at the same time makes possible a unified treatment of all types of reeds.

Reed Generator Admittance

Just as with the air-jet generator, it is helpful to define an impedance Z_r, or in this case more conveniently an admittance $Y_r = Z_r^{-1}$ for the reed generator as viewed from inside the mouthpiece of the instrument. If p is the mouthpiece acoustic pressure and U the acoustic volume flow through the reed into the pipe at some frequency ω, then

$$Y_r = -U/p. \tag{24}$$

An acoustic pressure p thus leads to acoustic power generation $p^2 \operatorname{Re}(-Y_r)$ by the reed and power dissipation $p^2 \operatorname{Re}(Y_p)$ in the pipe, where Y_p is the input admittance of the mouthpiece and pipe measured at the reed position. If sound generation is to occur then we must have

$$\operatorname{Re}(Y_r + Y_p) \leqq 0, \tag{25}$$

by analogy with (22) for the velocity-controlled air jet. When stable oscillation is achieved (25) must become an equality, with nonlinear effects reducing $|Y_r|$ at large amplitudes, while the frequency is determined by

$$\text{Im}\,(Y_r + Y_p) = 0. \tag{26}$$

Oscillation is thus always favored near frequencies for which Re (Y_p) is a minimum, that is at the impedance maxima of the pipe. We thus expect a clarinet to produce a series of odd harmonics, a saxophone, oboe, or bassoon to produce a complete harmonic series, and a brass instrument of good design to produce a harmonic series that is complete except for the "pedal" note based on the fundamental.

To evaluate the admittance Y_r of the reed generator we must examine the way in which the reed opening and the flow U through it vary with blowing pressure p_0 and with internal mouthpiece pressure p. We can formulate this in such a way that it applies to reeds striking in either direction. Because the blowing pressure p_0 is relatively high, the flow U through the reed opening is determined largely by Bernoulli's law, except that we should recognize that, because of the peculiar geometry of the reed opening, there may be slight deviations from the simplest expected behavior. At very low frequencies we can therefore write

$$U = B'b\xi^{\alpha}(p_0 - p)^{\beta}, \tag{27}$$

where b is the breadth of the reed opening (assumed constant) and ξ the height of the opening, which varies according to the net pressure acting on the reed; α and β are constants which for simple Bernoulli flow would have the values $\alpha = 1$, $\beta = 1/2$; and B' is another constant which for simple Bernoulli flow would equal $(2/\rho)^{\frac{1}{2}}$ where ρ is the density of air. In fact, from measurements on a clarinet mouthpiece and reed, Backus (1963) found $\alpha \approx \frac{4}{3}$, $\beta \approx \frac{2}{3}$, which values differ appreciably but not very significantly from those expected for simpler geometry. We therefore retain the general form (27).

When the mouthpiece pressure p fluctuates at a normal acoustic frequency we must also include the impedance of the mass of air that must move in the gap at the reed tip. If the effective length of this small channel is d, then the acoustic inertance of this mass of air is $\rho a/b\xi$, and we can rewrite (27) in the form

$$p_0 - p = B\xi^{-\alpha/\beta}U^{1/\beta} + (\rho d/b\xi)(dU/dt), \tag{28}$$

where we have written B for $(B'b)^{-1/\beta}$. There should really be an additional term in (28) to allow for viscous losses in the reed channel, but we neglect this for simplicity since it is generally small.

We must now recognize that the reed opening ξ will vary in response to both the blowing pressure p_0 and the mouthpiece pressure p, with the reed system behaving like a mechanical resonator of mass m, free area a, resonant frequency ω_r, and coefficient of damping κ, which perhaps is provided largely by the player's lips. The appropriate equation for reed motion is then

$$m\left[\frac{d^2\xi}{dt^2}+\kappa\omega_r\frac{d\xi}{dt}+\omega_r^2(\xi-\xi_0)\right]=sa(p_0-p), \tag{29}$$

where ξ_0 is the reed opening with the lips in position but no blowing pressure applied. The parameter s has the value -1 for an inward-striking reed generator and $+1$ for an outward-striking lip generator. Clearly, for the inward-striking case, if p_0 exceeds $p_0^c=(m\omega_r^2/a)\xi_0$, the reed will be forced closed by the static blowing pressure and no sound generation can take place. No such blowing pressure limit applies to outward-striking reeds.

Further analysis of the system involves substitution of a Fourier series for each of the acoustic variables ξ, p, and U in (28) and (29) and examination of the resulting mode equations. Because (28) is quite non-linear, there is a good deal of mixing between different modes, and this is important to the behavior of the instrument. Retaining only the linear terms, however, gives us considerable insight into the small-signal behavior.

Even in the linear case the formal result for the reed admittance Y_r is

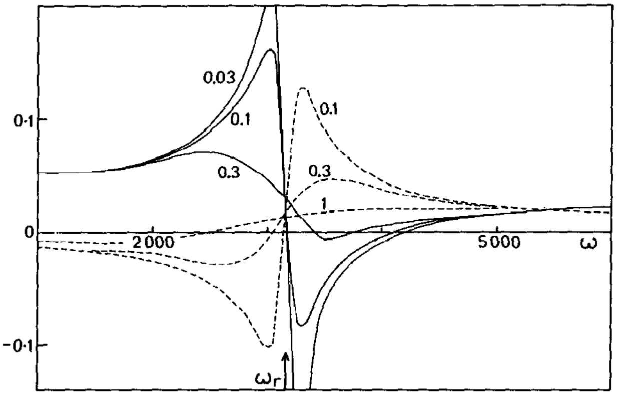

Figure 6 Calculated real part of the acoustic admittance Y_r in SI acoustic micromhos ($1\ \Omega^{-1}=1\ m^3\ \text{Pa}^{-1}\ s^{-1}$) for typical reed-valve generators above the critical blowing pressure p_0^*. Broken curves refer to woodwind-type reeds ($s=-1$) and full curves to lip reeds ($s=+1$). The generator resonance frequency is ω_r and its damping coefficient κ, is given as a parameter. (After Fletcher 1978b)

complex, and its meaning is not transparent (Fletcher 1978b). It is therefore better to look at the results of typical calculations. Figure 6 shows the real part of Y_r plotted as a function of frequency for both cases $s = \pm 1$ and for a blowing pressure somewhat less than the closing pressure p_0^c for the $s = -1$ case. Clearly from Figure 6 Re(Y_r) is negative in the case $s = -1$ only for ω less than the reed resonance frequency ω_r, while Re(Y_r) is appreciably negative in the case $s = +1$ only for a small frequency range just above ω_r. The behavior in the two cases is thus very different. In fact it is not very hard to see why this is so. In the $s = -1$ case with $\omega < \omega_r$, the reed behaves in a springlike manner and always tends to open when the mouthpiece pressure rises. In the $s = +1$ case with $\omega > \omega_r$ the reed behaves like a mass load and thus moves out of phase with the mouthpiece pressure, once again opening the reed aperture as the mouthpiece pressure increases. A plot of the static behavior (27) for $s = -1$ clearly indicates a negative resistance region between some critical pressure p_0^* and the closing pressure p_0^c, and it is in this region that the instrument operates in either case, the phase shift in reed motion referred to above effectively cancelling the sign of s.

Another informative plot is given in Figure 7, which shows the behavior of the complex reed admittance Y_r for the two interesting cases $s = -1$, $\omega < \omega_r$ and $s = +1$, $\omega > \omega_r$. The critical pressure for operation p_0^* is apparent in each case, as well as the closing pressure p_0^c when $s = -1$. We also see, remembering we are dealing with admittances now rather than impedances, that a woodwind reed with $s = -1$ presents a capacitative impedance to the pipe near one of its impedance maxima so that the sounding frequency must be slightly below the pipe resonance frequency to match the imaginary parts of the admittances as required by (26).

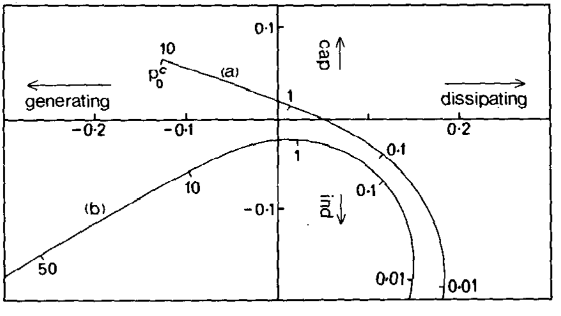

Figure 7 Calculated complex acoustic admittance Y_r in SI acoustic micromhos for typical reed-valve generators as a function of blowing pressure p_0, shown in kilopascals as a parameter (1-cm water gauge = 0.1 kPa). (*a*) A woodwind-type reed with $s = -1$ and $\omega = 0.9\,\omega_r$; note that the reed closes for $p > p_0^c$. (*b*) A lip-valve with $s = +1$ and $\omega = 1.1\,\omega_r$. (After Fletcher 1978b)

Conversely, for the brass instrument case $s = +1$, the generator impedance is inductive and the sounding frequency must lie a little above one of the resonance impedance maxima of the horn.

Nonlinearity and Performance

Inspection of Figure 6 for a woodwind reed shows that two possible modes of operation are possible. If the reed damping κ is very small, as could be achieved for example with a metal reed, then the pipe will sound at a frequency close to the reed resonance ω_r and associated with whatever pipe mode lies in this frequency range. This is the situation with the reed pipes of pipe organs, which are tuned by adjusting the resonance frequency of the reed. If, however, the damping is large so that κ approaches unity—a condition that can be achieved by the loading effect of the soft tissue of the player's lips—then the reed admittance has a nearly constant negative value for all $\omega < \omega_r$ and the pipe will sound at the frequency that minimizes Y_p and is thus at the highest of the pipe impedance maxima. This is essentially the playing situation in woodwind instruments, ω_r being as much as 10 times the fundamental frequency of the note being played.

Actually the nonlinearity of the reed behavior makes the situation rather more complex, as has been emphasized many times by Benade (1960a; 1976). Because all the pipe modes are coupled through the nonlinearity of the reed generator, the pipe impedance that is important is not just that at the frequency of the fundamental but rather a weighted average over all the harmonics of that fundamental. If the instrument is well designed then its resonance peaks will be in closely harmonic relation, the weighted impedance will be large, and the instrument will be responsive and stable. If, however, some of the resonances are misplaced, not only will the weighted impedance be lower, giving a less responsive instrument, but also the frequency at which the weighted impedance is greatest will depend on the harmonic content and thus on the dynamic level or loudness, giving the instrument an unreliable pitch.

Worman (1971) has examined the effects of nonlinearity in clarinetlike systems for playing levels small enough that the reed does not close, and he has shown that, within this regime, the amplitude of the nth harmonic within the instrument tube, or indeed in the radiated sound, is proportional to the nth power of the amplitude of the fundamental. This is, in fact, a very general result that applies to nearly all weakly nonlinear systems and so to all wind instruments in their soft-playing ranges, provided adjustment of lips or other playing parameters are not made. The result no longer holds in the highly nonlinear regime in which the reed closes. Schumacher (1978b) has also applied an integral equation approach combined with computer symbolic manipulation to this problem and has been able to obtain a steady state solution essentially complete to all orders. These extended results confirm the simpler approximations in general terms while introducing modifications in detail.

The onset of this highly nonlinear regime determines the maximum amplitude of the internal acoustic pressure, the peak-to-peak value of which is essentially equal to the difference between the minimum generation pressure p_0^* and the closing pressure p_0^c. Each of these pressures, and so the difference between them, increases linearly with the unblown reed opening ξ_0, so that to produce a loud sound the player relaxes his lip pressure and allows the reed to open. Relatively small changes in blowing pressure may also be made, largely to adjust tone quality, and the playing pressure used for clarinets and oboes does not vary much from 3.5 kPa and 4.5 kPa (35 and 45 cm water gauge) respectively over the whole of the dynamic and pitch range.

Because a woodwind reed operates well below its resonance frequency it behaves like a simple spring, so that its deflection accurately reflects the acoustic pressure variation within the mouthpiece. A study of the clarinet reed by Backus (1961) shows this deflection to have nearly square-wave form as we should expect. Instruments such as the oboe and bassoon may have a pressure wave of less symmetrical form (Fletcher 1978b).

Performance on brass lip-valve instruments is quite a different matter. Figure 6 shows that the lip valve has a negative conductance over only a small frequency range just above the lip resonance ω_r, so that playing must be based upon this regime and the lip resonance frequency adjusted so as to nearly coincide with the appropriate horn impedance maximum. In fact, skilled French-horn players can unerringly select between resonances lying only one semitone apart (6% in frequency), which implies that the damping coefficient κ for the lip vibrator must be less than about 0.1. Such a low value is probably not achieved by the lip tissue unaided by other effects, even under muscular tension, and it seems probable that the regenerative effect of the mouth cavity, fed by an airway of finite resistance, acts to decrease the effective value of the damping (Fletcher 1978b).

Because the lip valve responds only close to its resonance frequency, its motion is quite closely sinusoidal, just closing during each cycle (Martin 1942). Despite this, the sound of brass instruments is rich in upper harmonics, specially in loud playing (Luce & Clark 1967), and Benade's general principles on the alignment of resonances still apply. The primary cause of harmonic generation once again arises from the nonlinearity of the flow equation (27) and particularly from the fact that, when the lip aperture is wide, the instantaneous generator admittance may fall below that of the horn in magnitude, thus failing to satisfy (25) or, nearly equivalently, driving the pressure difference $p_0 - p$ below the critical value p_0^* (Backus & Hundley 1971). Because of this one-sided limiting effect and the fact that a brass instrument horn has a nearly complete harmonic resonance series, the mouthpiece pressure waveform has a general shape approaching that of a half-wave-rectified sinusoid.

However, this effect is probably not the only cause of harmonic generation at high sound levels, which may exceed 165 dB in a trumpet mouthpiece. One must certainly suspect an additional acoustic nonlinearity in the relatively narrow constriction connecting the mouthpiece cup to the main horn of the instrument (Ingard & Ising 1967). We must also remember the transformation function between internal and radiated sound-pressure spectra, which greatly emphasizes the upper partials of the sound.

Finally we should remark that, because increased blowing pressure tends to open rather than close the lip aperture in brass instruments, there is no limit (other than physiological) to the blowing pressure that can be used. The sound output is determined by a combination of lip opening and blowing pressure using the same general principles as set out for woodwind instruments, except that at the highest sound levels we may expect additional losses and inefficiencies because of increasing turbulence and other nonlinearities in flow through the lip aperture and mouthpiece cup.

CONCLUSION

Our review has shown the subtle variety of air flow responsible for sound production in musical wind instruments and has indicated the extent to which the behavior of the generators involved can be said to be understood. Clearly a great deal of work remains to be done.

Individual wind instruments typically have a dynamic range of 30 to 40 dB and acoustic output powers ranging from 10^{-6} W for the softest note on a flute to 10^{-1} W for a fortissimo note on a trumpet or tuba. The maximum efficiency with which pneumatic power is converted to acoustic power rarely exceeds 1% (Bouhuys 1965, Benade & Gans 1968). Within these limits, however, the experienced player can achieve an extraordinarily sensitive control of pitch, dynamic level, and tone color in the steady sound of his instrument and a comparable variety of vibrato and of attack and decay transients. It is both a challenge and a useful task to try to understand how this is done.

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