A class of chaotic bird calls?

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Evidence is presented that the basic vocalized sound produced by some cockatoos, specifically the Australian sulfur-crested cockatoo (*Cacatua galerita*) and the gang-gang cockatoo (*Callocephalon fimbriatum*), has a chaotic acoustic structure rather than the harmonic structure characteristic of most birdsongs. These findings support those of Fee *et al.* [Nature (London) **395**(3), 67–71 (1999)] on nonlinear period-doubling transitions in the song of the zebra finch (*Taeniopygia guttata*). It is suggested that syllables with chaotic structure may be a feature of the songs of many birds. © *2000 Acoustical Society of America.* [S0001-4966(00)04108-4]

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INTRODUCTION

Birdsong has been the subject of study for many years, from physiological, acoustical, and behavioral points of view. A comprehensive account is given in the classic book by Greenewalt[1] and there are many more recent papers to which we do not need to refer here. An excellent recent account of the relation between physiology and birdsong has been given by Suthers *et al.*[2]

Until recently, only three types of birdsong have been recognized, which we now discuss in turn. They might be termed simple voiced song, double-voiced song, and whistled song, respectively.

Simple voiced song is characterized by an acoustic spectrum consisting of a series of exact harmonics of a fundamental frequency F_0 (to use a notation common in human phonetics). The envelope of the spectrum of this song at any instant is characterized by a set of formant bands at frequencies $F_1, F_2, F_3, \ldots$ in which the amplitude of the harmonics of F_0 rises above the general declining trend with increasing frequency. The frequencies of the fundamental F_0 and of the formant bands F_n may vary with time under voluntary control by the bird, the extent of the variation being very different for different species of bird.

This simple voiced song is closely analogous to human speech and song generally, and to human vowel sounds in particular. The physiological interpretation is given in terms of a ''source-filter model,'' as for human speech. The avian syrinx acts as a self-excited oscillator, driven by air pressure from the lungs to vibrate at a characteristic frequency F_0 that is determined by the mass and tension of the syringeal membranes. This vibration in turn modulates air flow into the upper vocal tract at the characteristic frequency F_0. Because the flow equations for the syringeal valve are nonlinear, this flow U contains a complete spectrum of exact harmonics nF_0 of the fundamental, with amplitudes U_n generally decreasing steadily with increasing frequency. The acoustical properties of the upper vocal tract can be expressed in terms of the frequency-dependent acoustic impedance $Z(f)$ that it

presents to air flow from the syrinx, and the formant bands occur around the resonances of the vocal tract for which this impedance is a maximum, the acoustic pressure associated with the flow component U_n being $p_n = Z(nF_0)U_n$. Integration of these concepts into a calculable model gives good agreement with experiment.[3] The frequency F_0 of the fundamental can be varied by varying the muscle tension on the syringeal membranes, while the frequencies F_n of the vocal tract resonances can be controlled by changing tongue position and beak opening.[4] Details of the mathematical approach to calculations such as these have been given elsewhere.[5]

In the case of song-birds, account must be taken of the fact that they possess two independent syringeal valves, one located in each bronchus. There are, in fact, two possibilities that are exploited differently by different species. In one case, the bird simply closes off one syringeal valve during song and uses the other, perhaps permanently and perhaps on an alternating basis. In the second case, both syringeal valves operate at the same time, but their oscillations are locked into synchronism by the common oscillating pressure at the base of the trachea. Such frequency and phase locking is common in many types of oscillators, and requires only that their natural frequencies be not too far apart and that there exist a nonlinear physical coupling mechanism between them.[6] These conditions are easily met in the avian vocal system, where the two syringeal valves are at least nominally identical in structure, the flow through them is a nonlinear function of pressure, and the tracheal pressure oscillation provides an effective coupling mechanism.

The second type of song, which might be called double-voiced song, is one in which the frequencies of the two syringeal valves are controlled to be so different that locking is impossible. When this happens, each bronchus feeds a flow signal to the trachea at its characteristic oscillation frequency $F_0^{(1)}$ or $F_0^{(2)}$. Each of these flows contains harmonic components at integer multiples of its fundamental frequency but, in addition, there will be nonlinearly generated components at multiple sum and difference frequencies $nF_0^{(1)} \pm mF_0^{(2)}$, where n and m are positive integers. The amplitudes of these mixture terms decline approximately as x^{n+m}, where x is a

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quantity less than unity that depends upon the exact nature of the flow nonlinearity and is proportional to the amplitude of the fundamental of the oscillation. Songs of this type sound rather like pairs of notes played on musical instruments, provided the nonlinearity is not too great.

The third generally recognized type of song is termed whistled song. Analysis shows that it consists of an almost pure sine wave with no upper harmonics, and the frequency can often be changed rapidly in a sweep over a range of about a factor two. The physiological mechanism for production of such song has not been established. The fact that whistles are often interpolated within a voiced song suggests a common generation mechanism, and it has been suggested that this might be a retraction of the syringeal membranes while in oscillation so that they no longer completely close, leading to a great reduction in the harmonic content of the flow. Flow calculations, however, indicate that, while such a procedure would reduce the relative amplitude of higher partials in the flow, it would not eliminate them entirely, as appears to be the case in whistled song. It should also be noted that some human languages, notably those from central Africa, involve the interpolation of aerodynamically produced whistles amidst voiced speech.

An alternative explanation of whistled song is that it is produced by pure aerodynamic means without any vibrating surfaces, in much the same way as sound in flutes, organ pipes and whistles. In all these cases the source of sound is the interaction of an unstable air jet with a resonator. If the resonator has many modes, as in an organ pipe, then the sound has many harmonic overtones. If, however, it has but one mode, as in the case of the Helmholtz resonator of an occarina, which is a simple cavity vented by finger holes, then the sound output will approximate a pure sine wave. A simple biological example is human whistling, in which the resonator is the mouth cavity and the jet issuing through the aperture between the lips is subject to a varicose instability (or change in diameter) which feeds back to influence flow through the aperture and thus internal pressure. It is not clear what structures in the avian vocal system might be responsible for sound production by this mechanism. Possible candidates are structures at the base of the trachea, but another possibility involves the larynx, a raised tongue, and a further aperture between tongue and beak.

It is the purpose of the present note to present further evidence for a modified type of voiced utterance that might be called chaotic song. The possibility that some bird calls might be chaotic was raised by the present author some time ago[7] in relation to the Australian sulfur-crested cockatoo *Cacatua galerita*, and since then a pioneering study of a transition from normal to period-doubled and perhaps chaotic song in the case of the zebra finch *Taeniopygia guttata* has been published by Fee *et al.*[8,9] The difference between the calls reported here and those of the zebra finch will be discussed briefly in the conclusion to the present paper.

I. ANALYSIS OF CHAOTIC CALLS

To the ear, the calls of Australian cockatoos have the sound of a steady raucous screech, quite unlike the more melodious sounds of other birds. This observation was the
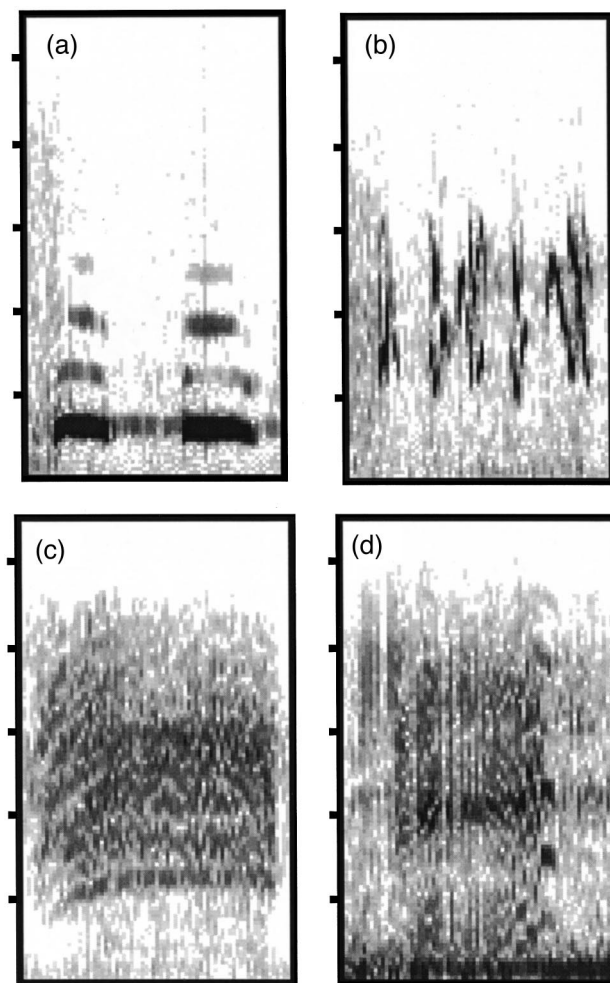

FIG. 1. Time-resolved spectrograms of the calls of the four birds in the study: (a) the boobook owl, (b) the Eastern rosella parrot, (c) the sulfur-crested cockatoo, and (d) the gang-gang cockatoo. In each case the duration of the sample is about 0.7 sec and the frequency range is 0 to 5.5 kHz.

stimulus of the present investigation. A good collection of the songs of Australian birds of the region near Canberra, recorded in their native habitat, is available,[10] and this was used for the required sound samples. The two cockatoos selected for study were the sulfur-crested cockatoo (*Cacatua galerita*) and the gang-gang cockatoo (*Callocephalon fimbriatum*). Each selected call was digitized at 32 kb/s and its waveform examined. A section of nearly constant amplitude and duration about 0.7 sec was then selected for detailed analysis. From an auditory point of view these sections of the call were constant in loudness and sound quality, and this constancy was confirmed by the nature of the oscillograph records of the waveforms, suggesting strongly that they are the result of processes that are ''stationary'' in a statistical sense. Two similar segments from the songs of birds with simple sound, namely the boobook owl (*Ninox novaeseelandiae*) and the Eastern rosella parrot (*Platycercus eximius*), were similarly studied for comparison purposes.

As an initial study, sound spectrograms of the four bird calls were made, and the results are shown in Fig. 1. The call of the owl, in Fig. 1(a), has a simply harmonic structure and no pitch change, while that of the parrot, in Fig. 1(b), consists essentially of a single simple tone with rapid frequency

variations. In contrast, the sulfur-crested cockatoo call in Fig. 1(c) has a very broad-band spectrum, though with traces of two independent voices and vestiges of quasiharmonic structure. The call of the gang-gang in Fig. 1(d) is broadly similar, but with a higher pitch and less obvious substructure. It appears reasonable to treat the major part of these latter two songs as being stationary in a statistical sense.

It was the assumption at the beginning of the study that the calls in question might have a chaotic structure, and the digitized records were therefore subjected to analysis on this basis. Such an analysis of time-series data to detect chaos and other interesting phenomena has been the subject of much detailed study,[11,12] and a set of computer programs[13] with the title ''Chaos Data Analyzer'' is available to carry out the analysis. In addition, a newly available suite of programs[14] for ''Visual Recurrence Analysis'' provides an independent and rather different approach. These programs were applied to the observational records with the results detailed below.

The essential feature of normal nonchaotic song is its predictability. As discussed above, the short-term spectrum consists of an assembly of pure sinusoidal tones that are generally in harmonic relationship, giving an exactly repeating waveform. When two syringeal sources produce sounds at unrelated frequencies the overall harmonic relationship is complicated by multiple sum and difference tones and the waveform no longer repeats. In either case, however, once the spectral composition of the sound is known, its future waveform can be predicted exactly. This statement applies, of course, only to short segments of song in which the bird does not deliberately change the defining parameters such as pitch and loudness.

At the other end of the scale comes random noise. Here the future waveform is entirely unpredictable and the signal can be described only in statistical terms.

Between these two extremes lies chaotic behavior, in which the oscillation is governed by well-defined and often simple laws but, because of nonlinearity in the basic vibration mechanism, the future waveform is unpredictable in the absence of knowledge of the precise initial conditions—a very small change in initial conditions makes an immense change in the exact future course of the oscillation.[12] Chaos is often studied by examining the behavior of the system when the parameters of the underlying differential equations are progressively changed. In a natural biological system such as birdsong such an approach is not possible; rather we must attempt to find something of the nature of the underlying equations by examining the sound output.[11]

A. Data analysis

The syrinx is, of course, a complex vibrating system, since the pressure-controlled vibrating valve, whether it be the syringeal membrane or some other structure, can support a large number of possible oscillation modes. It therefore turns out not to be possible to discover much about the exact nature of its vibration from the sound output, but a demonstration that it is indeed chaotic is relatively simple. The approach is to calculate the so-called Lyapunov exponents that describe the waveform. In essence this exponent mea-

TABLE I. Parameters computed from CDA program.

	Lyapunov exponent	Capacity dimension	Correlation dimension
Two nonlinearly coupled signals	0.0	1.4	2.2
Henon attractor	0.6	1.4	1.3
Lorenz attractor	0.1	1.7	2.0
Random noise	0.15	$2\rightarrow4$	$1\rightarrow6$
Boobook owl	0.06	1.5	1.9
Rosella parrot	0.02	2.5	2.1
Sulfur-crested cockatoo	0.3	2.2	3.2
Gang-gang cockatoo	0.2	2.3	3.8

sures the degree of sensitivity of the oscillation to its initial conditions and its deviation from predictability. First the waveform is mapped onto a phase space which, in the simplest case, plots the slope of the waveform at each point against the magnitude of the waveform at that point. For a wave consisting of exactly harmonic components, this phase map consists of a simple closed curve, while for more complex waveforms the map is correspondingly more involved. The Lyapunov exponent essentially measures the rate at which the distance between two points on the curve (related by the sampling time between them) increases as the waveform evolves. For a simple harmonic or multifrequency deterministic system, the distance between the points settles down to a constant value, on average, while for a chaotic system the distance increases steadily. The Lyapunov exponent measures the exponential rate of this increase in separation. Chaotic systems are characterized by moderate positive exponents and simple systems by negative or zero exponents. Random systems have large positive exponents.

The Chaos Data Analyzer programs[13] provide facilities for plotting phase-space maps, calculating the largest Lyapunov exponent, and otherwise examining time-series data. As a control, short segments of song from the boobook owl (*Ninox novaeseelandiae*) and from the Eastern rosella parrot (*Platycercus eximius*) were also analyzed. The boobook song was nearly sinusoidal, with a single dominant frequency, giving a simple closed-loop phase map and a largest Lyapunov exponent of 0.06 ± 0.03. The rosella song contained two dominant frequencies, had a more complex but still generally ring-shaped phase map, and a largest exponent of 0.02 ± 0.02. Both these exponents are essentially zero, as would be expected for deterministic signals. Further details of the analysis are given in Table I, which also shows the capacity dimension (Hausdorff dimension) and correlation dimension calculated from the data. For both of these songs, the correlation function had the form of a cosine function and did not decay in amplitude over the sample length, indicating a closely predictable behavior. The phase-space plot in both cases had the form of a broad elliptic ring, as would be expected for a basically sinusoidal signal modulated by other frequencies. Comparison with computed data for a signal consisting of two nonlinearly interacting unrelated harmonic oscillations, given in the first line of Table I, indicates close similarity and confirms the predictable nature of these songs, despite their irregular waveform. We return to discuss such songs in a later section.

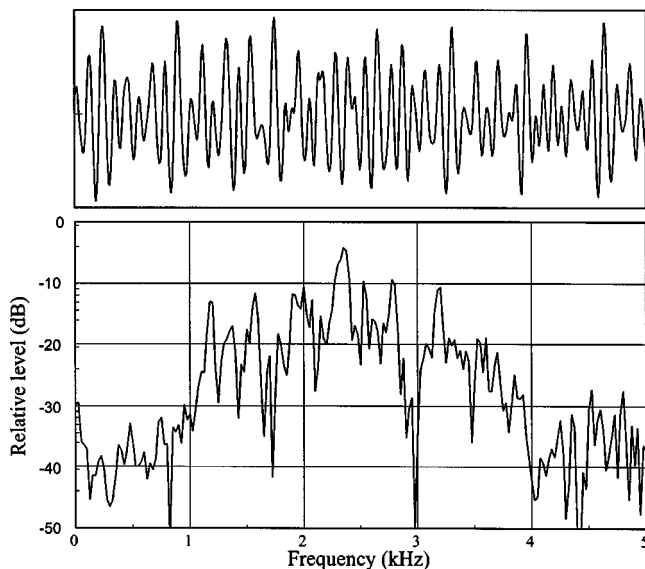The contrast with the calls of the sulfur-crested cockatoo

FIG. 2. (a) Waveform and (b) spectrum of a short sample of the song of the sulfur-crested cockatoo. Plots for the gang-gang cockatoo are very similar.

and the gang-gang cockatoo is marked. While the waveform of the song of the boobook owl and rosella parrot appears ''smooth'' in each case, that of the two cockatoos has a ''rough'' appearance, as shown for the case of the sulfur-crested cockatoo in Fig. 2(a). Each cockatoo call had a broad power spectrum peaked at about 2.5 kHz and with many subsidiary peaks, as shown for a short segment of the sulfur-crested cockatoo call in Fig. 2(b). This confirms the analysis provided by the spectrograms of Fig. 1. The quasiperiodic components are, however, of large amplitude. The largest Lyapunov exponent for the sulfur-crested cockatoo call was 0.28 ± 0.06 and that for the gang-gang 0.23 ± 0.05. As shown in Table I, these values are comparable with those computed for typical chaotic signals, such as those associated with the Lorenz and Henon systems, and indicate a lack of predictability. For each bird the correlation function has the form of a decaying cosine wave, as shown for the sulfur-crested cockatoo in Fig. 3, the decay rate being rather more rapid in the case of the gang-gang. In neither case, however, is the
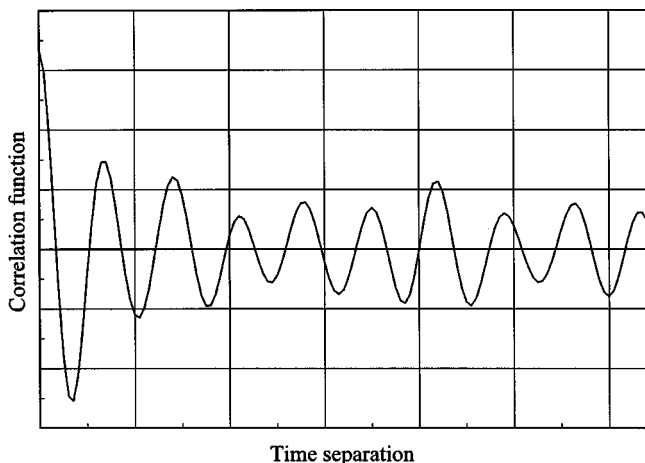
decay as rapid as for the Lorenz or Henon systems, and there appears to be significant sinusoidal residue.

From the data given in Table I it is clear that there is a close resemblance between the parameters for these two cockatoo calls and those for the well-known Lorenz and Henon chaotic attractors. These two cases, however, derive from analysis of well-defined and simple nonlinear differential equations and lead to phase-space maps that are fractal ''strange attractors.'' In the case of the cockatoo calls, the phase-space maps essentially filled uniformly an elliptical area, and no further information could be gained from them, even if the embedding dimension was raised. The explanations for this are probably that the call is not truly stationary in a statistical sense, and that it also contains noise, both of which features tend to blur any phase-space pattern, although it could also result from the dimensionality of the system being higher than 3. The further fact, however, that the computed correlation dimension, regarded as a function of the embedding dimension, rises to a gently sloping plateau value for an embedding dimension greater than about 3, distinguishes the call structure from simple band-limited noise, for which the correlation dimension rises smoothly to large values with increasing embedding dimension.
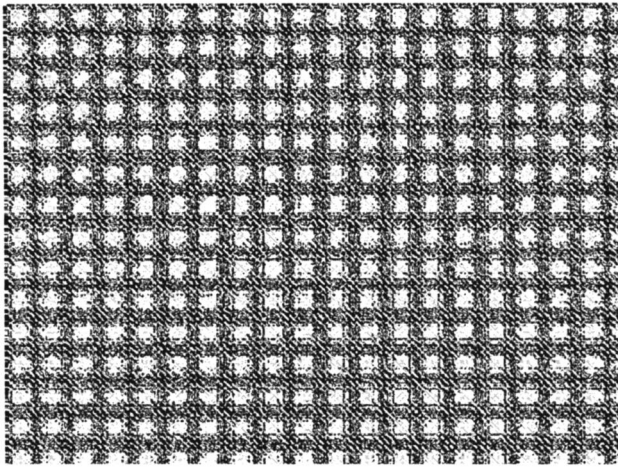
B. Visual recurrence analysis

As a second approach to understanding the nature of the cockatoo call, the technique known as visual recurrence analysis[14] was used. In this approach, the signal is sampled at a set of equally spaced times t_i and at each time the following N sample values are used to define an N-dimensional vector Y_i associated with the time t_i. A color-coded matrix plot is then made of the Euclidean distance between all pairs of vectors, such that the (i,j)th element of matrix is the distance between the vectors Y_i and Y_j. Although it is difficult to draw any quantitative conclusions from such a plot, it does show up patterns in the data in a very clear manner— a well-correlated signal gives a repetitive pattern, a chaotic signal has an irregular but definite pattern, and simple noise has a pattern that is irregular at all scales.

Figure 4(a) shows a recurrence plot for a signal consisting of two incommensurate nonlinearly interacting harmonic signals, as discussed in the next section, and Fig. 4(b) a plot to a similar scale for a random noise signal. These figures set the range of variation to be expected. Figure 5(a) then shows the recurrence plot for the Lorenz attractor, while Fig. 5(b) shows a plot to a similar scale for the cockatoo call. The similarity between Figs. 5(a) and 5(b) is clear, as is their difference from the plots of Fig. 4. We again conclude that the call of the cockatoo likely has a chaotic nature.

II. PHYSICAL MODEL

It is important to seek the reasons why the cockatoo calls might be chaotic, or at least the mechanism that might lead to this result. For the present this is largely speculation, but as such is necessary as a guide to further studies. In particular it is important to distinguish chaotic song from the type of nonlinearly mixed two-voice song described in the Introduction.

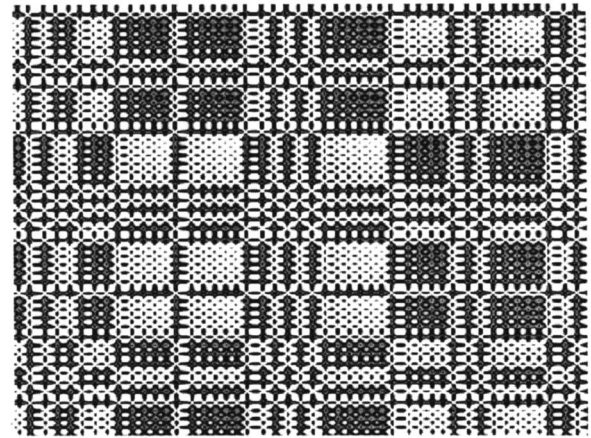FIG. 3. Computed correlation function for the song of the sulfur-crested cockatoo.
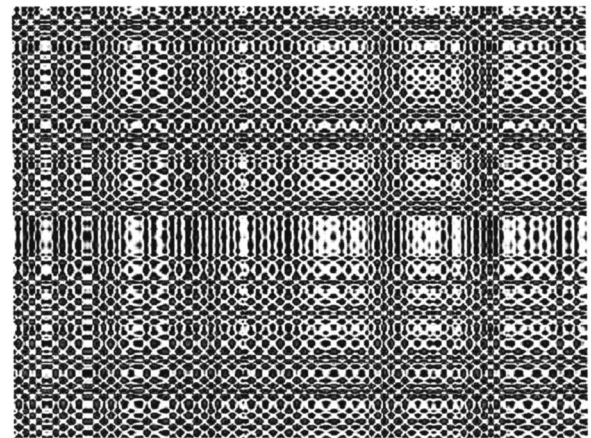
(a)
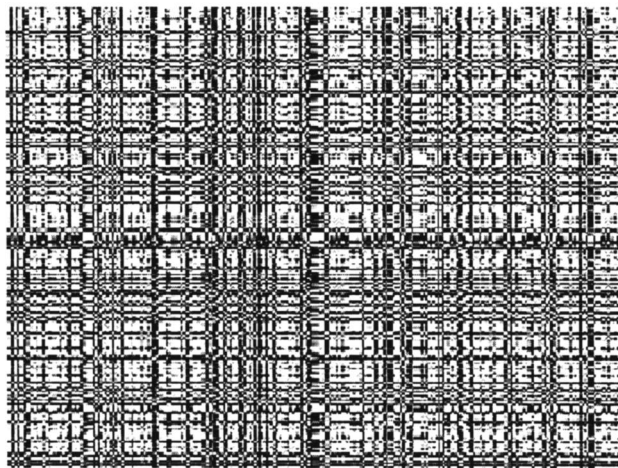
(b)

FIG. 4. Visual recurrence analysis plot of (a) two nonlinearly coupled harmonic signals, as defined by Eqs. (1)–(3), and (b) a random noise signal. (The original plots were in color.)

(a)

(b)

FIG. 5. Visual recurrence analysis plot of (a) the Lorenz attractor time series, and (b) the song of the sulfur-crested cockatoo. The recurrence plot for the Henon attractor is qualitatively very similar to that for the Lorenz attractor. (The original plots were in color.)

Although the largest Lyapunov exponent is significantly different from zero, the waveform in Fig. 2(a) has at least one strong quasiperiodic component, which is also visible on the spectrogram in Fig. 2(b). In addition, the correlation function of Fig. 3 also has a periodic structure, although one that decays toward zero with time. These facts suggest that the mechanism might be one in which the syringeal valves in the two bronchi of the bird oscillate simultaneously and are coupled by a strong interaction through the sound pressure at the base of the trachea. The fact that the bird has a very loud cry, with an estimated radiated power of at least 100 mW, implies an acoustic pressure at the base of the trachea that is perhaps as high as 1 kPa (154 dB). Such an acoustic pressure is comparable to the blowing pressure below the syrinx and can therefore strongly influence the vibration of the syringeal valves. Furthermore, the airflow through these valves is a highly nonlinear function of pressure drop across them, and the membrane vibrations themselves are probably quite non-linear because they are likely to contact the wall once in each cycle.

With two nonlinear vibrating sources and a strong non-linear coupling between them, there is an adequate number of degrees of freedom for chaotic oscillation, rather than simply for the generation of multiple sum and difference frequencies. At the same time, the natural frequencies of the two syringeal valves may well contribute quasiperiodic components to the overall oscillation, as observed.

On the other hand, it is possible that the two nonlinear membrane vibrations simply couple nonlinearly to produce a multitude of multiple sum and difference frequencies that gives the appearance of chaos. To test this, a synthetic signal $y(t)$ was created from two harmonic signals $x_1(t)$ and $x_2(t)$ combined nonlinearly. Specifically,

$$x_1(t) = \sin(\omega_1 t) + a_1 \sin(2\omega_1 t + \phi_1)$$
$$+ a_2 \sin(3\omega_1 t + \phi_2), \tag{1}$$

$$x_2(t) = \sin(\omega_2 t) + a_1 \sin(2\omega_2 t + \phi_3)$$
$$+ a_2 \sin(3\omega_2 t + \phi_4), \tag{2}$$

$$y(t) = [x_1(t) + x_2(t)] + b_1[x_1(t) + x_2(t)]^2$$
$$+ b_2[x_1(t) + x_2(t)]^3, \tag{3}$$

where the fundamental frequencies ω_1 and ω_2 have no simple integer relationship ($\omega_2 = 1.3824\omega_1$), and the amplitudes a_n and b_n are given as $a_1 = 0.3$, $a_2 = 0.1$ and $b_1 = b_2$

=0.1. The waveform and spectrum of this signal showed patterns broadly similar to those recorded for the cockatoo cries. As shown in Table I, however, the largest Lyapunov exponent in this case is essentially zero, the actual value given by the CDA program being -0.01 ± 0.03. The correlation function is irregular but shows no decline in amplitude with time, while the visual recurrence plot, shown in Fig. 4(a), has an obvious regularly repeating structure. We conclude that a simple nonlinear mixing of two periodic signals, as might indeed be achieved in the syrinx, cannot explain the nature of the observed sound signal.

The final possibility to be investigated is that the signal consists of a nonlinearly combined pair of harmonic signals, as just discussed, with a significant admixture of random noise. A signal with this property was constructed, the ratio of peak noise to peak signal being either 0.1:1, 0.3:1 or 1:1. The CDA program was then used to determine the largest Lyapunov exponent for the signal and also the behavior of capacity and correlation dimensions with increasing embedding dimension. For noise ratios of 0.1:1 and 0.3:1, the largest Lyapunov exponent was less than 0.01, indicating nonchaotic behavior. For a noise ratio of 1:1, the calculated Lyapunov exponent was 0.2, but the capacity and correlation dimensions increased steadily with increasing embedding dimension, as for an essentially random-noise behavior. It can be concluded that the bird cry is not adequately simulated by such signal.

III. CONCLUSIONS

The conclusions to be drawn from this brief examination of the calls of these particular birds appear clear. The songs of birds such as the boobook owl and the rosella parrot are basically sinusoidal, but with rapid modulations in the case of the rosella. Such songs are quite normal variations of basic voiced or whistled song. The basic repetitive ''carrier'' is provided by vibration of the syringeal membranes, and its amplitude and frequency are modulated at a slower rate by oscillatory changes in the tension of the supporting muscles.[2] These songs can therefore be classified as ''normal'' in the present context. This conclusion is supported by the facts that the largest Lyapunov exponent is approximately zero, that the phase-space plot is essentially an elliptical ring, which is perhaps the projection of a torus in a space of higher dimension, and that the correlation function extends with undiminished amplitude over a long time period.

For cockatoos, specifically the sulfur-crested and the gang-gang cockatoos, on the other hand, we conclude that the call structure is very different, and in fact chaotic. This conclusion is supported by the fact that the largest Lyapunov exponent is in each case in the range 0.2–0.3, a range typical of chaotic behavior rather than random noise or nonlinearly coupled periodic signals, and by the visual appearance of the recurrence mappings. The power spectrum does have several dominant frequencies, which presumably represent major oscillation modes of the syringeal structures, but the amplitudes and phases of these modes vary in a chaotic fashion. The chaotic nature of these calls is audibly recognizable in their harsh screeching character.

The investigation of chaotic oscillations in simple systems, by which is generally meant systems with a small number of degrees of freedom or dynamic variables, is now a well-established subject.[12] The case of birdsong is likely to be more complex because the primary oscillating system, the syrinx, has many degrees of freedom corresponding to wave-like motions on the membranes or other vibrating structures, and because there may often be two vocal sources involved. This makes detailed analysis much more difficult.

This study thus supports the detailed work of Fee and colleagues[8] on the song of the zebra finch. Their analysis showed the existence of abrupt period-doubling transitions in the song, a feature that is characteristic of one major route to chaos.[15] Their numerical analysis showed that such a transition could be derived for the case of a simple model of airflow through a compliant constriction, without requiring two independent oscillatory sources. It is also interesting to note that the double reeds of woodwind instruments such as oboes or bassoons show a period-doubling transition when blown vigorously and not fitted to the instrument — a test carried out when the reed is being adjusted by the player.

While the vocal utterances of these Australian cockatoos appear to be entirely chaotic in structure, and thus an extreme case, it seems likely that shorter syllables with highly nonlinear or even chaotic structure may be part of the vocal repertoire of many birds.

ACKNOWLEDGMENT

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