# Numerical calculation of flute impedances and standing waves

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The purpose of this study was to investigate a numerical method for calculating impedances and standing wave patterns of flute structures. To this end, the physical dimensions of flute joints and tone holes were used to compute impedance and standing waves as a function of frequency for several different fingerings. Numerically computed resonance frequencies for head joint, middle and foot joints, and complete flute are compared to experimentally measured values. Computed pressure standing wave patterns for two fingerings of $A_6$ are compared to experimental values. Reasons for the observed discrepancies between the predicted and experimental frequencies are discussed.

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## INTRODUCTION

A knowledge of the acoustic impedance of flute structures may be of assistance in understanding their intonation, tonal quality, and interaction of the excitation with the bore. A method for numerically calculating impedances may be a useful supplement to other experimental and theoretical methods such as impedance measurements and perturbation techniques. Sufficient experimental data for musical instrument structures of interest must be available if a reasonable determination of the accuracy of a numerical method is to be made.

A successive impedance procedure was derived by Richardson (1929) for calculating resonance frequencies of bores with tone holes. Benade (1960) applied transmission line theory to bores having uniformly spaced tone holes and described a method for obtaining the positions of the tone holes of a cylindrical woodwind. Nederveen (1964) derived a method to calculate positions and diameters of tone holes for a clarinet. Nederveen and de Bruijn (1967) modified Nederveen's (1964) cylindrical-tube clarinet calculations to calculate the hole positions of the conical oboe. Nederveen and de Bruijn (1967) also included the effects of viscous and thermal losses at the walls. Nederveen (1969) compiled all these methods. Nederveen (1973) used his successive impedance approach to calculate flute passive resonance frequencies including the wall loss effects, which were compared with measured values. Coltman (1979) applied similar techniques to calculations of flute resonance length and distribution of acoustic pressure. Plitnik and Strong (1979) studied input impedances for the oboe and obtained curves in qualitative agreement with experimental curves. Lyons (1981) combined wave equation, lumped-line, and perturbation methods to obtain resonance frequencies of a recorder.

In order to apply successive impedance methods for calculating the input impedance of bores with tone holes it is necessary to have appropriate tone hole impedance data. Benade and Murday (1967) made measurements leading to empirical expressions for tone hole end corrections in terms of tone hole parameters. Nederveen (1973) used the Benade and Murday (1967) expressions for effective tone hole length in terms of tone hole parameters. Coltman (1979) measured reactances for various open tone hole and key combinations; Coltman also measured the cavity effects of closed holes. Keefe (1982a) carried out an extensive theoretical investigation of tone holes, the results of which were compared with his own experimental measurements (Keefe, 1982b) and those of Coltman (1979) and Benade and Murday (1967).

The current work employs a refined version of the Plitnik and Strong (1979) method to numerically calculate resonance frequencies and pressure distributions for a flute. The numerical results are compared to experimental results to check the validity of the approach. The paper consists of six sections: numerical method, resonance frequencies of the complete flute, resonance frequencies of the middle and foot joints, resonance frequencies of the head joint, pressure and power distributions, and a discussion.

## I. NUMERICAL METHOD

The flute bore is represented by a series of short cylindrical sections, as shown in Fig. 1. The impedance at one end of a section can readily be written in terms of the impedance at the opposite end and the dimensions of the section. The impedance at the far end of the bore is taken to be the radiation impedance at a particular frequency $f$. This becomes the load impedance from which the input impedance (at the same frequency) is calculated for the first section along the bore. The input impedance for the first section then becomes the load impedance for the second section, and the process is continued until the last section is reached at the embouchure end of the bore. Whenever a tone hole is encountered, the appropriate impedance (which depends on whether the hole is opened or closed) is added to the net bore impedance at the center of the tone hole. When the input impedance of the final section is computed, the input impedance of the entire bore is known at one frequency $f$. By repeating the above process at many different frequencies, a plot of impedance

  

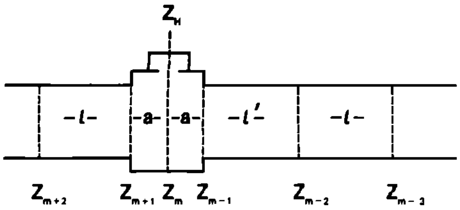


FIG. 1. Approximation of flute bore with contiguous cylindrical sections. Maximum section length is $l$; a shortened section length $l'$ precedes the closed tone hole. The section on either side of the tone hole center has a length equal to the hole radius $a$. The bore is enlarged and a small volume is placed in parallel with the bore to represent the effects of a closed tone hole.

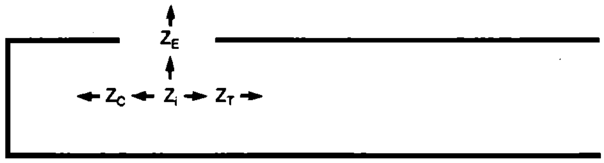


FIG. 2. Schematic of embouchure hole region of flute showing embouchure hole impedance $\mathbf{Z}_E$, cork cavity impedance $\mathbf{Z}_C$, and tube impedance, $\mathbf{Z}_T$. $\mathbf{Z}_i$ is the impedance seen at the inside "end" of the embouchure hole.

versus frequency can be constructed. The cross-sectional areas of the bore and the position, size, and nature of the tone holes are determined by direct measurement and are input as data to a computer program. The method is basically that of Plitnik and Strong (1979), and further details can be obtained there. [This is an analysis program, in contrast to the synthesis procedures discussed by Benade (1960) and Nederveen (1964).]

Viscous and heat conduction losses at the bore walls were incorporated by Plitnik and Strong (1979), but no correction was made for the decrease in wave speed at low frequencies caused by these losses. The decrease in wave speed due to losses is incorporated here and has the primary effect of lowering the fundamental mode frequency by a greater fractional amount than it does the higher modes. Losses at the bore walls are arbitrarily chosen to be 1.3 times those for the hard wall case and those at tone hole walls are arbitrarily chosen to be 2.6 times the hard wall case, to account for increased losses in a bore with tone holes.

Plitnik and Strong (1979) expressed the inertance of open tone holes in terms of length correction formulas from Benade and Murday (1967). The measured length corrections of Coltman (1979) are used here. Plitnik and Strong (1979) represented a closed tone hole as a shunt compliance having a volume equal to that of the physical volume of the hole. The compliance was positioned on a smooth continuation of the bore at the center of the tone hole, where it was sensitive to pressure antinodes. At pressure antinodes, such a compliance produces an effective lengthening of the bore, which lowers the modal frequencies. Nederveen and van Wulfften Palthe (1963) have pointed out that flow streamlines penetrate to some extent into the enlargement of the bore at a closed tone hole. This tone hole enlargement of the bore, which results in a lowered bore inertance, can be represented with a negative series inertance in addition to the shunt compliance. At flow antinodes, such an inertance produces an effective shortening of the bore, which raises the modal frequencies. A closed tone hole for these calculations is represented in Fig. 1. Two sections of the bore on either side of the tone hole center are enlarged slightly to represent series inertance sensitive to volume flow and shunt compliance sensitive to pressure. An additional volume at the hole center represents an additional compliance sensitive to pressure and is necessary because the inertance and compliance "volumes" are different. Experimental parameters of Coltman (1979) for closed tone holes are used to obtain the series and shunt volumes for the calculations. Reactance effects of the tone hole pads are incorporated to the extent that they are incorporated in the Coltman (1979) parameters.

The manner in which normal mode frequencies were calculated can be seen by referring to Fig. 2. The impedance at the inside "end" of the embouchure hole is given by the parallel combination of the embouchure hole impedance $\mathbf{Z}_E$, the cork cavity impedance $\mathbf{Z}_C$, and the flute tube impedance $\mathbf{Z}_T$ as

$$\mathbf{Z}_i = (\mathbf{Z}_C\mathbf{Z}_T\mathbf{Z}_E)/(\mathbf{Z}_E\mathbf{Z}_C + \mathbf{Z}_E\mathbf{Z}_T + \mathbf{Z}_C\mathbf{Z}_T). \tag{1}$$

A normal mode for a parallel combination such as this may be defined as that condition in which the sum of the flows at the junction is zero. This condition requires that the input admittance at the junction be zero or that $\mathbf{Z}_i$ be infinite. As is conventionally done for the case in which losses are present, a normal mode was defined as that condition in which the reactive part of the impedance vanishes near an impedance maximum. However, the normal mode frequencies so obtained are equal within less that 0.1% to those obtained from taking $\mathbf{Z}_i$ maximum.

## II. FLUTE RESONANCE FREQUENCIES

Dimensions for the body and tone holes of a Powell flute #1578 were used in the calculations so the results could be directly compared with the experimental data of Coltman (1966) for the same instrument. The dimensions of the embouchure hole were those of a round-cornered rectangle 12.5×10.8 mm. In the experiments (Coltman, 1966), the embouchure hole was covered to an extent of 5.8 mm, leaving an uncovered gap of 5.0 mm width. For the calculations an open embouchure area of 55.6 $\text{mm}^2$ was used; this was obtained by subtracting 3.45 $\text{mm}^2$ for each of the two rounded corners from the rectangular area of $12.5\times5=62.5$ $\text{mm}^2$. It was assumed that the open area was circular, with an equivalent diameter of 8.42 mm; this places the "computational" embouchure hole edge 4.21 mm from its center, as compared with the 6.25 mm in the actual instrument. The effective length of the embouchure hole was adjusted (to 7.1 mm) until the calculated frequency was equal to the measured frequency for the note $C_4$. This should be a reasonable place to match frequencies because all tone holes are closed for $C_4$ and any discrepancies in the experimental excitation method should be minimal.

Calculated frequencies for the first and second mode of notes in the first octave are shown in Table I. Deviations from an equal tempered A440 scale of calculated and measured frequencies also appear in Table I. Octave stretching

TABLE I. Calculated first- and second-mode resonance frequencies (in Hz) for first octave of the flute. Deviations (in cents) from equal tempered A440 scale for calculated and measured values are shown. Octave stretching (in cents) between first and second modes for calculated and measured values is shown. Measured deviations and octave stretchings are from Coltman (1966).

| Note | First mode | Second mode | Calculated deviation | Measured deviation | Calculated stretching | Measured stretching |
|---|---|---|---|---|---|---|
| $C_4$ | 255.6 | 520.0 | −40 | −40 | 29 | 30 |
| $D_4$ | 287.8 | 582.5 | −34 | −30 | 20 | 19 |
| $E_4$ | 324.1 | 652.6 | −29 | −20 | 11 | 15 |
| $F_4$ | 344.6 | 693.8 | −23 | −15 | 11 | 12 |
| $G_4$ | 390.8 | 789.6 | −5 | −3 | 17 | 17 |
| $A_4$ | 440.4 | 893.6 | 2 | 1 | 24 | 22 |
| $B_4$ | 496.6 | 1009.8 | 9 | 11 | 28 | 22 |
| $C_5$ | 529.0 | 1075.0 | 18 | 24 | 27 | 22 |

between first and second modes also are shown for calculated and measured frequencies.

The fundamental frequency components of notes $D_5$ through $C_6$ are fed primarily by the interaction of the air jet and the second mode of the air column. For notes $D_6$ through $G_6$ the fundamental frequency is similarly fed by higher modes of the air column. Calculated "feeding" mode frequencies for notes $D_5$ through $G_6$ on the flute appear in Table II, along with deviations from an equal tempered A440 scale for calculated and measured frequencies. Tone holes 1–7 and 14 were open for $D_6$, 1–4 and 10 were open for $E_6$, 1–5 and 11 were open for $F_6$, and 1–7 and 13 were open for $G_6$. (This description is in terms of tone holes numbered from 1 at the bottom of the foot joint to 16 at the top of the middle joint. The G# holes are numbered 8 and 9 in this description.)

## III. MIDDLE AND FOOT JOINT RESONANCES

Calculations were made for the cylindrical portion of the flute, consisting of the middle and foot joints and stopped at the tuning slide. The length of the tuning slide used in the calculation was adjusted (to a final value of 30.3 mm) until the calculated first-mode frequency agreed with the measured value for the all-holes-closed configuration normally used to play $C_4$. Calculated and measured deviations from odd integral multiples of the first-mode frequency are shown for note fingerings $C_4$, $E_4$, $G_4$#, and $C_5$ in Table III. No discrepancies are shown for the first mode of each note since it serves as a reference for the higher modes. The calculated first-mode frequencies for $E_4$, $G_4$#, and $C_5$ differ from the measured values by −13, 1, and 7 cents, respectively.

TABLE II. Calculated resonance frequencies (in Hz) for the second and part of the third octave of the flute. Deviations (in cents) from equal tempered A440 scale for calculated and measured values (Coltman, 1966) are shown.

| Note | "Feeding" mode | Calculated deviation | Measured deviation |
|---|---|---|---|
| $D_5$ | 588.0 | 2 | 1 |
| $E_5$ | 652.6 | −17 | −5 |
| $F_5$ | 693.8 | −11 | −3 |
| $G_5$ | 789.4 | 12 | 18 |
| $A_5$ | 894.0 | 27 | 22 |
| $B_5$ | 1010.3 | 38 | 33 |
| $C_6$ | 1075.0 | 46 | 48 |
| $D_6$ | 1196.0 | 30 | 38 |
| $E_6$ | 1346.0 | 35 | ... |
| $F_6$ | 1429.0 | 39 | 41 |
| $G_6$ | 1609.0 | 44 | 47 |

## IV. HEAD JOINT RESONANCE FREQUENCIES

Fletcher *et al.* (1982) have described a method for characterizing the acoustic properties of flute head joints by measuring the quantity $F_n = f_n/(2n-1)$. In this expression, $f_n$ is the frequency of the $n$th impedance maximum as seen from the end of the short (301.5 mm) cylindrical tube attached to the head joint. The pattern of $F_n$ is quite sensitive to the head joint geometry and so provides a further means of checking computational accuracy.

A calculated $F_n$ curve for an Armstrong head joint with a completely open embouchure hole is shown in Fig. 3, along with the corresponding measured $F_n$ curve. The effective area of this embouchure hole was taken to be 111.2 mm$^2$ and the embouchure hole effective length was adjusted (to 9.7 mm) in the calculations so the second impedance frequency matched that of experiment.

TABLE III. Deviation (in cents) of higher mode frequencies from odd integral multiples of first-mode frequency for flute stopped at the tuning slide. Measured data are from Coltman (1966).

| Fingering and first mode (Hz) | Mode | Calculated deviation | Measured deviation |
|---|---|---|---|
| $C_4$ | 1 | ... | ... |
| 172.7 | 2 | −2 | −1 |
| | 3 | −4 | −6 |
| | 4 | 7 | 11 |
| | 5 | 7 | 7 |
| $E_4$ | 1 | ... | ... |
| 238.6 | 2 | −26 | −20 |
| | 3 | −13 | −8 |
| | 4 | −29 | −22 |
| $G_4$# | 1 | ... | ... |
| 343.5 | 2 | −24 | −9(?) |
| | 3 | −44 | −28 |
| $C_5$ | 1 | ... | ... |
| 531.0 | 2 | −16 | −9 |

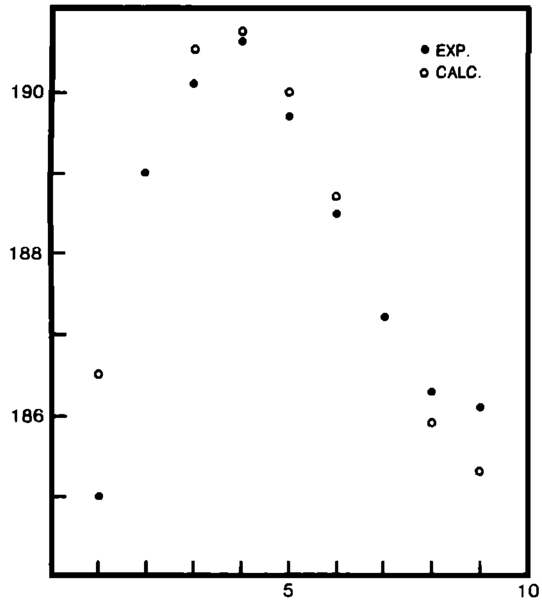

FIG. 3. Experimental and numerical signatures for an Armstrong head joint.

## V. PRESSURE AND POWER DISTRIBUTIONS

The pressure distribution in a tube with a series of open and closed tone holes may be quite sensitive in some instances to a particular fingering. Two alternate $A_6$ fingerings for a flute are known to produce rather different playing characteristics; with the D# key closed $A_6$ is more difficult to sound than with the D# key open. Numerical calculations of pressure distributions at resonance frequencies were made for the two alternate $A_6$ fingerings (tone holes 1–5, 8–10, and 14 were open for one fingering; the D# key closed tone hole number 3 for the other fingering). Experimental probe tube measurements were made on a flute excited sinusoidally by a small loudspeaker in the vicinity of the embouchure hole, which was covered to an extent similar to that of a player's lip when sounding $A_6$ (the outside diameter of the probe was 2.5 mm). The probe was moved in and out of the flute to obtain rough measures of pressure maxima and minima and their positions relative to the cork position.

Relative pressure was calculated at successive points along the bore of the flute. Pressure was calculated at resonance frequencies and at neighboring frequencies of interest. The embouchure hole was taken to have an area of 38 $mm^2$ and a length of 7.8 mm for the calculations to be consistent with the experimental setup. This gave resonances at 1780 Hz for the open D# fingering of $A_6$ and at 1800 Hz for the closed D# fingering. Figure 4 shows a calculated standing wave for pressure at a frequency of 1780 Hz for $A_6$, with the D# key open. There is some penetration of the wave beyond the first open tone hole and into the open tone hole region. The pressure wave is quite consistent with Fig. 6 from Coltman (1979) and is in reasonable agreement with the probe tube data when considering the roughness of these data. (Crosses in Fig. 4 show the probe tube data and circles at the bottom show positions of open holes.)

Power at each point in the bore was calculated as

$$\text{Power} = \text{pressure} \times \text{flow} = (p^2/2)(\mathrm{Re}\,\mathbf{Z})/|Z|^2. \quad (2)$$

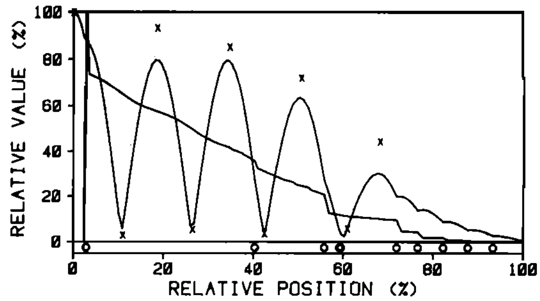

FIG. 4. Calculated pressure and power distributions in flute bore when fingered for $A_6$ with open D# key. Resonance frequency was 1780 Hz. Circles mark positions of open holes. Experimental values are shown by crosses.

The power is the rate at which the acoustical wave supplies energy to some point in the bore. Relative power is the fractional part of the power at the input impedance transmitted to some point in the bore. Relative power curves are calculated as a percentage of the power at the embouchure hole, where the relative power is set equal to 100%. (If the input impedance were defined at some other place in the bore the relative power would be maximum at that point and falling off in either direction.) A relative power curve at a frequency of 1780 Hz is shown in Fig. 4.

Plots of pressure and power at frequencies of 1750 and 1810 Hz produced patterns very similar to those in Fig. 4 and are not shown here. The standing wave is not very sensitive to frequency because the impedance looking into the open tone hole portion of the bore at the C# tone hole (number 14) mismatches the characteristic impedance of the tube at all the frequencies. The sound power reflection coefficient looking into the lower portion of the bore at the C# tone hole is defined as

$$\mathrm{RC} = |(\mathbf{Z}_B - \mathbf{Z}_C)/(\mathbf{Z}_B + \mathbf{Z}_C)|^2, \quad (3)$$

where $\mathbf{Z}_B$ is the impedance looking into the lower part of the bore and $\mathbf{Z}_C$ is the characteristic impedance of the cylindrical portion of the flute. The power reflection coefficient so defined is shown in Fig. 5 and indicates that over 80% of the power is reflected at frequencies of 1750–1810 Hz, which accounts for the similar standing wave patterns.

Figure 6 shows a calculated standing wave for pressure at the resonance frequency of 1800 Hz for $A_6$ with the D# key closed. Clearly, more of the wave has propagated into the lower portions of the bore due to a better impedance

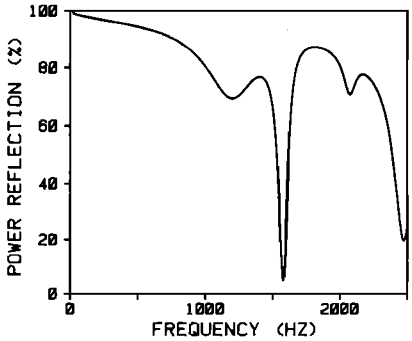

FIG. 5. Calculated power reflection coefficient looking into open hole portion of bore at C# hole for open D# key fingering of $A_6$.

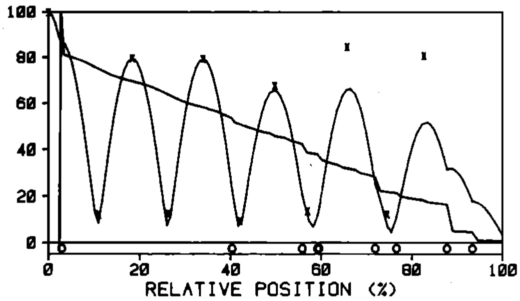


FIG. 6. Calculated pressure and power distributions in flute bore when fingered for $A_6$ with closed D# key. Resonance frequency was 1800 Hz.

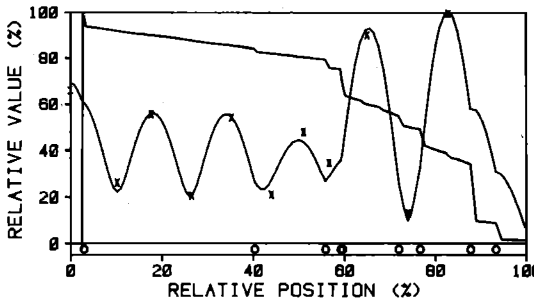


FIG. 8. Calculated pressure and power distributions in flute bore when fingered for $A_6$ with closed D# key. Calculation was for a frequency of 1740 Hz near a minimum in the power reflection curve.

match. Figure 7 shows the power reflection coefficient at the C# tone hole for the closed D# key fingering. It might be anticipated that even more penetration would result at a lower frequency, where the power reflection coefficient is least. Figure 8 shows a standing wave at a frequency of 1740 Hz, at which power reflection is least. Figure 8 is consistent with Fig. 7 from Coltman (1979). More power penetrates into and is lost from the lower portion of the bore in the latter case. The standing wave at the embouchure hole is too low to be useful for sustaining oscillation in this "worst" case, as noted by Coltman (1979), and $A_6$ with this fingering is nearly impossible to sound.

## VI. DISCUSSION

We now consider discrepancies between the experimental data and the numerical calculation, and some factors that may have contributed to them. Backus (1974) has pointed out that the external excitation method has some limitations when used to excite woodwinds. Backus was able to obtain good frequency data for the normal modes of a clarinet, but the method does not provide quantitative impedance amplitude data. However, Coltman's flute data (1966), with which the numerical data are compared, involve only normal mode frequencies and so the external excitation data Coltman obtained should be adequate for the comparisons made here.

For the complete flute, calculated frequency deviations were all within 12 cents of the measured deviations. (Frequency deviations for both calculated and measured cases are with reference to an equal tempered A440 scale. Frequencies were calculated to within a tenth of a percent, which means that deviations must be 2 cents or larger to be significant.) The largest discrepancies between experimental and numerical results were for $E_4$ and $E_5$, where the calculated values were 9 and 12 cents lower, respectively, than the measured values. This is consistent with the first-mode frequency for the flute stopped at the tuning slide, where the calculated value is 13 cents lower than the measured value. One way to explain the lower calculated values would be in terms of a key rise that is too small. The C# and D keys were calculated with 2.5 mm rises and the D# and E keys were calculated with 2.0 mm rises, which are thought to be consistent with the experimental setup. At most, key rise discrepancies of a few tenths of a millimeter could account for about half of the normal mode frequency discrepancies. Another possible explanation might be that the measured E tone hole position used in the calculations was incorrect. Similar arguments might be tried for some of the other notes, but an explanation of the discrepancies would be even less convincing because the deviations are smaller than for $E_4$ and $E_5$.

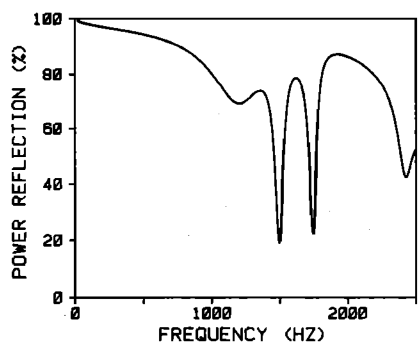


FIG. 7. Calculated power reflection coefficient for closed D# key fingering of $A_6$.

In the note $G_4$# for the flute stopped at the tuning slide, the calculated third-mode deviation was 16 cents lower than the measured value. No obvious explanation is available. It might generally be expected that the stopped flute calculations should be more accurate than the full flute calculations because imprecision in specifying and modeling the embouchure hole is not present. However, the full flute and stopped flute calculations seem to show similar discrepancies relative to their measured counterparts.

The calculated octave stretching clearly shows the same trends as the measured octave stretching. However, the differences tend to be smaller for the low notes than for the high notes.

Imprecision in specification of the embouchure hole may have been a major contributing factor to discrepancies between experimental and numerical values for several different comparisons. The "length" of the embouchure hole is difficult to specify via measurements because it is tapered on the underside and because the "length" of any cover (plastic plate for example) over the hole is poorly defined. A further complication is that the "rectangular" embouchure hole is represented as a circular hole of equal area and centered at the embouchure hole center. Experimental methods are available for characterizing the embouchure hole (Benade and French, 1965; Coltman, 1966; Fletcher *et al.*, 1982). However, for the calculations, a measurement was made of the embouchure area in each case and the embouchure length was adjusted to provide a single calculated frequency coinciding with a corresponding experimental value. This procedure resulted in effective embouchure lengths smaller than would be anticipated from the experimental results just cited. We have no good explanation for these results, but remark that Nederveen (1973) found a similar effect in which the measured "embouchure correction" had to be reduced signficantly to bring measured and calculated frequencies into agreement.

In the numerical method used here each tone hole was treated as if it were an isolated tone hole and no interaction among tone holes was taken into account. Keefe (1983) has pointed out that both internal and external interaction may occur for toneholes. Keefe noted that coupling is most pronounced between adjacent tone holes when the spacing between hole edges is small compared to the main bore diameter. Keefe measured a significant increase in effective tone hole length in a two-hole experiment, but found no significant increase in effective tone hole length for tone holes in a lattice. Keefe noted that the most important consequence of tone hole interactions is an increase in viscous and thermal losses. We conjecture that the increase in effective tone hole length due to interactions is probably of the order of errors in specifying key height.

One might have expected to find good agreement between calculation and experiment for the head joint plus cylindrical tube. The trends in the two sets of values are clearly similar. In fact, the major discrepancy seems to be in the frequency differences between normalized first and second modes. Experimentally the difference is about 4 Hz, but numerically it is only about 2.5 Hz. Three major factors might influence the normalized first- and second-mode difference. The combined cork distance and embouchure hole effective length should change very little in the frequency range of 100–500 Hz. It should tend to increase the difference but play a negligible role in this case. Viscous and thermal effects on sound speed might be expected to contribute a difference of about 1 Hz, as deduced from Fig. 2 in Fletcher *et al.* (1982). The head joint taper is the major factor and lowers the first mode relative to the second mode. From Fig. 10 in Benade and French (1965) we deduce that the tapered head joint appears about 5 mm longer at 170 Hz than at 500 Hz, which is about 1% of the nominal 500-mm cylindrical tube used in the calculations. This should contribute a difference of less than 2 Hz. The total difference between the normalized first and second modes by this analysis should be less than 3 Hz; the experimental value is about 4 Hz and the numerical is about 2.5 Hz.

In the standing wave patterns for pressure the positions of experimental and numerical maxima and minima agree within about twice the imprecision factor of the measurements. The heights of the pressure maxima follow the same trends for experiment and calculation, with the final two peaks in Fig. 6 as possible exceptions. One would not expect complete agreement because it is difficult to model losses well in the calculation.

The numerical method employed here produced frequencies good to about the nearest 2 cents. However, tone hole and embouchure hole representations used probably make the calculations good to no more than about the nearest 5 cents. Discrepancies between numerical and experimental values were as much as 16 cents, which may have been due in part to improper key rise and tone hole position specifications. The trends in the numerical data generally followed those in the experimental data. Coltman (1976) has noted that flutists can repeat a pitch with a standard deviation of 6 cents. Hence, if the numerical method is good to 5 cents accuracy it is probably a useful tool for exploring flute (and other woodwind) structures.

The current results, when comparing measured and calculated frequencies for the passively excited Powell flute, are generally a bit tighter ( − 5 to + 12 cents) than similar results for a Reiner flute ( + 35 to + 75 cents) reported by Nederveen (1973). Comparisons between measured and calculated frequencies for a passively excited Bressan alto recorder ( − 2 to + 20 cents) have been reported by Lyons (1981).

Whether the numerical method is useful as an analytical tool for studying woodwind structures may depend on how well the passive resonances relate to the "blown" resonances for such structures. In particular for the flute, the usefulness of the method may depend on the extent to which the excitation mechanism is a perturber of the system. Nederveen (1973) holds the view that the contracting head joint is necessary to counteract frequency shifts due to the blowing mechanism. Coltman (1966) holds the view that the contracting head joint is necessary to counteract frequency shifts due to increasing lip coverage. If the former view is predominantly correct, then the numerical method sketched herein has some basic deficiencies. However, if the latter view if predominantly correct, then the numerical method may have some value as an analytical tool.

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