

Dretske and Epistemic Closure

Anders J. Schoubye

University of Copenhagen

Department of Philosophy, Rhetoric, and Education

<http://www.legenden.dk>

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Abstract

These notes were intended to shed some light on Fred Dretske's notorious "Epistemic Operators" from 1970. The notes were prepared for a short presentation during a course in "Modality, Evidentiality, Epistemicity" at the Department of Linguistics, University of Copenhagen – March, 2004. This is also my first efforts in \LaTeX which is the reason why I've now spend five hours working on the layout on a simple fairly unimportant presentation.

1 Penetration and Deductive Closure

1.0.1 Penetration in Classic Sentential Logic

p: "Peter stole the car", q: "His father will be dissapointed"

1.	$p \rightarrow q$	premise	
2.	p	premise	
\therefore	q	conclusion	<i>modus ponens, 1, 2</i>

1.0.2 Penetration in First-Order Predicate Logic

"a": Anders, "F": _ is a fish, "G": _ has gills

1.	$(\forall x)(Fx \rightarrow Gx)$	premise	
2.	Fa	premise	
\therefore	Ga	conclusion	<i>modus ponens, 1, 2</i>

1.0.3 Penetration in Sentential Modal Logic

1.	$\Box(p \rightarrow q)$	premise	
2.	$\Box p$	premise	
\therefore	$\Box q$	conclusion	<i>modus ponens, 1, 2</i>

1.	$\Box(p \rightarrow q)$	premise	
2.	$\Diamond p$	premise	
\therefore	$\Diamond q$	conclusion	<i>modus ponens, 1, 2</i>

2 Fully Penetrating Operators

- It is true that p
- It is necessary that p
- It is possible that p

2.1 Consequences of Fully Penetrating Operators

- You cannot have reason to believe p unless you have reason to believe that q
- You cannot know p unless you know that q
- If you assert that p, then you assert that q
- If you hope that p, then you hope that q

Thus, “an operator operating on p is fully penetrating if it penetrates to every logical consequence of p”

3 Non-Penetrating Operators

- It is strange that p
- It is lucky that p
- It is accidental that p

3.1 Deductive Problems

“ I ”: Operator “It is lucky that...”

“ a ”: Constant “Anders”

“ F ”: Predicate “_ is talking about philosophy rather than linguistics”

1.	IFa	premise	
2.	$IFa \rightarrow I(\exists x)Fx$	premise	
\therefore	$I(\exists x)Fx$	conclusion	<i>modus ponens, 1, 2</i>

(ad 1.) “It is lucky that Anders is talking about philosophy rather than linguistics”

(ad 2.) “If it is lucky that Anders is talking about philosophy rather than linguistics, then it is lucky that *someone* is talking about philosophy rather than linguistics”

(ad \therefore .) “It is lucky that someone is talking about philosophy rather than linguistics”

However, clearly this does not follow. Even though it is lucky that Anders is talking about philosophy rather than linguistics, because his linguistic understanding is very limited, it does not follow that it is lucky that someone is talking about philosophy. For all we know, everybody would be better off if someone was talking about linguistics rather than philosophy.

4 Epistemic Operators

1. S knows that p
2. S sees that p

3. S can prove that p
4. S believes that p

We focus on the so-called *K-operator*. Throughout the following, we assume that knowledge is factive, such that S cannot know something which is false.

4.1 Notation

When an agent S knows that p, we formalize this in the following way:

1. $K_S p$

4.2 Knowledge and Deductive Closure

To test whether knowledge is closed under deductive closure, consider the following:

- | | | | |
|--------------|------------------------|------------|---------------------------|
| 1. | $K_S p$ | premise | |
| 2. | $K_S(p \rightarrow q)$ | premise | |
| \therefore | $K_S q$ | conclusion | <i>modus ponens, 1, 2</i> |

(ad 1.) S knows that Anders is giving a presentation of Dretske.

(ad 2.) S knows that if Anders is giving a presentation of Dretske then the listeners are bored out of their minds.

(ad \therefore) S knows that the listeners are bored out of their minds

Intuitively, it does seem as if the *K-operator* is a penetrating operator. Under reasonable constraints on rationality etc. the above inference seems fairly plausible.

4.3 Unknown Implications: $\neg K_S(p \rightarrow q)$

We must bear in mind that if an agent S knows that p, yet does not know that p entails q, then naturally, S cannot infer and thereby know q.

1. $K_S(2 + 2 = 4)$ premise
2. $(2 + 2 = 4) \leftrightarrow (a^2 + b^2 = c^2)$ premise
- $\therefore K_S(a^2 + b^2 = c^2)$ conclusion (*False*)

This should not be construed as proof that the *K-operator* is non- or semi-penetrating. For an operator to be non- or semi-penetrating, all logical consequences of the propositional variables must be known to S. This is not the case in the above argument.

5 Complications

5.1 Introducing a Disjunct (v-introduction)

In regards to the disjunctive introduction, we construe disjunctions as being inclusive. This entails that a disjunctive formula is false only if all disjuncts in the formula are false.

p “Anders is giving a presentation of Dretske”

p¹ “John F. Kennedy is giving a presentation of Dretske”

pⁿ *n* being a variable for any proposition introduced into the formula

1.	$K_S p$	premise	
\therefore	$K_S(p \vee p^1)$	conclusion	<i>v-introduction</i>
\therefore	$n K_S(p \vee p^1 \dots p^n)$	conclusion	<i>v-introduction</i>

What is the problem with this inference? Obviously, logically speaking, it does follow that these alternatives can be effectively introduced along side the true left disjunct. However, it does not follow, that any competent and rational agent does in fact know or even considers these alternative possibilities.

5.2 Introducing a Conjunct (&-introduction)

A formula containing a conjunction is true if, and only if, all conjuncts of the formula are true.

We presuppose that only one person (Shakespeare) wrote “Hamlet”. If not, then we would have to utilize the modal predicate calculus to formalize the argument. Therefore, the following presupposition holds:

$$\lceil (\exists x)Hx \ \& \ (\forall y)(Hy \leftrightarrow y = x) \rceil$$

p “Shakespeare wrote ‘Hamlet’ ”

p^1 “Chomsky wrote ‘Hamlet’ ”

p^2 “Peano wrote ‘Hamlet’ ”

p^3 “Montague wrote ‘Hamlet’ ”

p^n n being a variable for any proposition stating that someone other than Shakespeare in fact wrote “Hamlet”

1.	$K_S p$	premise
\therefore	$K_S(p \ \& \ \neg p^1 \ \& \ \neg p^2 \ \& \ \neg p^3 \ \dots \ \& \ \neg p^n)$	conclusion <i>\mathcal{E}-introduction</i>

As we saw earlier, the problem that arose with the introduction of disjuncts repeats itself for the introduction of negated conjuncts. Obviously, even though I *do* know that Shakespeare was the single unique author of “Hamlet”, it does not follow, that I then know that neither Chomsky, Peano, Montague wrote “Hamlet”. It could be entirely unknown to me who Chomsky, Peano, and Montague are.

5.3 A Tentative Example

Imagine that S know that Jim committed the murder. The detective then asks S if he knows, who committed the murder. S then replies:

“In fact I do. I know that either Jim, or John, or James, or Peter, or Robert committed the murder”.

This is entirely consistent with S’s knowledge. Given that Jim committed the murder, it is correct that either Jim, or John, or James, or Peter, or Robert committed the murder. However, this is obviously not the answer that one would normally give.

5.4 Dretske on Presupposition

“The general point may be put this way: there are certain presuppositions associated with a statement. These presuppositions, although their truth is entailed by the truth of the statement, are not part of what is operated on when we operate on the statement with our epistemic operators. The epistemic operators do not penetrate to these presuppositions”. (DRETSKE: 1970)

And,

“Hence, the first class of consequences that differentiate the epistemic operators from the fully penetrating operators is the class of consequences associated with presuppositions of a proposition”.
(DRETSKE: 1970)

6 Relevant Alternatives

Explaining a fact that p will depend on relevant alternatives. What counts as relevant alternatives depend on a contextual presupposition of the fact being explained. Explaining why Brenda didn't order dessert, could therefore be explaining why Brenda didn't order dessert and eat it. Or it could be explaining why Brenda didn't order dessert and throw it at the waiter etc.

In other words, we can identify the relevant alternatives by introspecting the contextual presupposition.

We can represent a tentative framework for relevant alternatives.

1. Relevant fact
 - (p) Brenda did not order dessert
2. Relevant alternatives
 - (a) She was not hungry
 - (b) She was on a diet
 - (c) She doesn't like desserts
 - (d) She was not mad at the waiter
 - (e) Her sick friend is dead

We explain the fact by considering possible alternatives to the actual state of affairs.

If we want to know why Brenda didn't order dessert and eat it, then alternatives (a), (b) and (c) are relevant. Alternative (d), and (e), are both irrelevant, although they cannot be ruled out logically. They are irrelevant because they do not explain why Brenda didn't order dessert and eat it.

So, if S knows that Brenda didn't order dessert because she wasn't hungry, S must then be able to rule out that Brenda didn't order dessert because she wasn't mad at the waiter. This is logically implied by knowing the reason why she didn't order dessert.

But, according to Dretske, the point is, that this is not a relevant alternative. This would not explain the fact that Brenda didn't order dessert and eat it. So, S is not required to rule out this alternative, even though it is logically implied.

This is a very controversial statement and why will become clear in a few moments time.

6.1 Skeptical Hypothesis

There are many varieties of skeptical scenarios or hypothesis, but perhaps the most well-known kind is that of Pyrrhonian (Cartesian) skepticism.

In short, a skeptic will claim that it is impossible to gain and have knowledge. He does so by introducing hypothetical scenarios, which must be ruled out, in order to have knowledge. If this cannot be done, the skeptic argues, then we do not have knowledge. Such skeptical hypothesis usually include evil demons tricking us into believing that everything we see is real, when it is in fact an illusion, or scenarios where we are merely brains in vats being stimulated by a supercomputer to have perceptual sensations etc.

The skeptic argument can thus be formalized in the following way:

p “I have two hands”

q “I am a brain in a vat”

1.	K_{Sp}	premise	
2.	$K_{Sp} \rightarrow K_S \neg q$	premise	
3.	$\neg K_S \neg q$	premise	
\therefore	$\neg K_{Sp}$	conclusion	<i>modus tollens, 2,3</i>

In the above argument, the conclusion is reached by modus tollens. However, we could just as well perform a modus ponens, and thereby reach,

\therefore	$K_S \neg q$	conclusion	<i>modus tollens, 2,3</i>
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... thereby beating the skeptic opposition. However, intuitions usually favour the skeptic.

6.2 Solving the Skeptical Problem

In order to solve this seeming problem of knowledge, Dretske returns to his account of relevant alternatives. As we saw in section 4.2 knowledge is to some degree closed under deductive inference. However, as we also saw in sections 5.1 and 5.2 there are certain complications with knowledge as a fully-penetrating operator. These problems were further illuminated in sections 5.3 and 6.

The solution of the problem, according to Dretske, is to deny unrestricted closure. In other words, to treat the *K-operator* as a semi-penetrating operator. The *K-operator* does penetrate to some logical consequences of a proposition, but not to all. It does not penetrate to logical consequences which can be considered irrelevant in the particular context. In the context where I claim to have two hands, the logical consequences of that proposition are not that I thereby claim to know that I’m not a brain in a vat. So, as for the skeptic argument that,

1.	K_{Sp}	premise	
2.	$K_{Sp} \rightarrow K_S \neg q$	premise	
3.	$\neg K_S \neg q$	premise	
\therefore	$\neg K_{Sp}$	conclusion	<i>modus tollens, 2,3</i>

... we should simply deny the second premise.

2.	$K_{Sp} \rightarrow K_S \neg q$	premise	<i>false</i>
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Closure does not hold in this case, because being a brain in a vat is not a relevant alternative.

7 Problems and Objections

One major question remains unanswered throughout Dretske's article: Why should the skeptic accept that being a brain in a vat is not a relevant alternative? If a fully convinced skeptic actually poses this question to someone who claims to know that he has two hands, why is this not a relevant alternative?

In other words, what is missing from Dretske's article is a reasonable account of how we partition relevant from irrelevant alternatives.

“The explanation for why the modal relationship between R and P ($R \rightarrow P$) fails to carry over (penetrate) to the logical consequences of P (i.e., $R \rightarrow Q$ where Q is a logical consequence of P) is to be found in the set of circumstances that are taken as *given* or *fixed*, in subjunctive conditionals”. (DRETSKE: 1970)

An account of subjunctive conditionals, that could probably yield the proper framework for Dretske's theory would be David Lewis' or Robert Stalnaker's accounts. An approach quite similar to Dretske's can be found in Nozick's "Philosophical Explanations". Nozick also denies closure.

In conclusion, what Dretske needs, in order to make his suggestion work, is an adequate framework for partitioning the relevant alternatives from the irrelevant alternatives.

But even if Dretske's suggestion can be captured in Lewis' or Stalnaker's framework, I see no reason for the skeptic not to reply that Dretske is begging the question. Concluding that the Brain-in-Vat-Alternative is irrelevant, because it is a far away world, or a remote possibility etc. is definitely making an assumption that the skeptic will not accept.

Anders J. Schoubye
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