

Rod Girle

Modal Logics and Philosophy

2.4.1

e. $(p \rightarrow q) \rightarrow \sim \diamond(p \& \sim q)$

1. $\sim((p \rightarrow q) \rightarrow \sim \diamond(p \& \sim q))$	n	NTF
2. $p \rightarrow q$	n	1, sand ant.
3. $\sim \sim \diamond(p \& \sim q)$	n	1, sand kons.
4. $\diamond(p \& \sim q)$	n	3, DN
5. nRk	-	-
6. $p \& \sim q$	k	4,5 $\diamond S5$
7. p	k	6, &-opløsning
8. $\sim q$	k	6, &-opløsning
↑		

f. $(\sim \diamond p \rightarrow \Box(p \rightarrow q))$ – (strict implication)

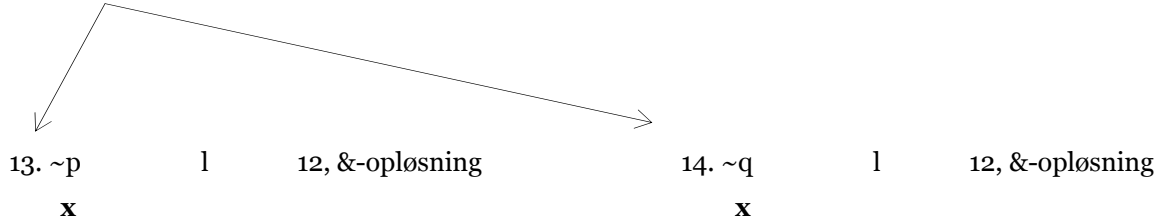
1. $\sim(\sim \diamond p \rightarrow \Box(p \rightarrow q))$	n	NTF
2. $\sim \diamond p$	n	1, sand ant.
3. $\sim \Box(p \rightarrow q)$	n	1, falsk kons.
4. $\Box \sim p$	n	2, mn
5. $\diamond \sim(p \rightarrow q)$	n	3, mn
6. nRk	-	-
7. $\sim(p \rightarrow q)$	k	5,6, $\diamond S5$
8. p	k	7, sand ant.
9. $\sim q$	k	7, falsk kons.
10. $\sim p$	k	4, 6, $\Box S5$
x		

g. $(\sim\Diamond p \rightarrow \sim\Diamond(p \& q))$

1. $\sim(\sim\Diamond p \rightarrow \sim\Diamond(p \& q))$	n	NTF
2. $\sim\Diamond p$	n	1, sand ant.
3. $\sim\sim\Diamond(p \& q)$	n	1, falsk kons.
4. $\Diamond(p \& q)$	n	3, DN
5. nRk	-	-
6. $p \& q$	k	4,5, $\Diamond S5$
7. p	k	6, $\&$ -opløsning
8. q	k	6, $\&$ -opløsning
9. $\Box\sim p$	n	2, mn
10. $\sim p$	k	5, 9, $\Box S5$
x		

h. $\sim\Diamond(\Diamond(p \& q) \& \sim\Diamond(p \& q))$

1. $\sim\sim\Diamond(\Diamond(p \& q) \& \sim\Diamond(p \& q))$	n	NTF
2. $\Diamond(\Diamond(p \& q) \& \sim\Diamond(p \& q))$	n	1, DN
3. nRk	-	-
4. $\Diamond(p \& q) \& \sim\Diamond(p \& q)$	k	2,3, $\Diamond S5$
5. $\Diamond(p \& q)$	k	4, $\&$ -opløsning
6. $\sim\Diamond(p \& q)$	k	4, $\&$ -opløsning
7. kRl	-	-
8. $p \& q$	l	5, 7, $\Diamond S5$
9. p	l	8, $\&$ -opløsning
10. q	l	8, $\&$ -opløsning
11. $\Box\sim(p \& q)$	k	6, mn
12. $\sim(p \& q)$	l	7, 11, $\Box S5$



i. $\nabla p \equiv \nabla \sim p$

$(\nabla p \equiv \nabla \sim p) \equiv ((\diamond p \ \& \ \diamond \sim p) \equiv (\diamond \sim p \ \& \ \diamond \sim \sim p))$

1. $\sim((\diamond p \ \& \ \diamond \sim p) \equiv (\diamond \sim p \ \& \ \diamond \sim \sim p))n$ NTF

- | | | |
|--|---|----------------|
| 2. $\diamond p \ \& \ \diamond \sim p$ | n | 1, sand ant. |
| 3. $\sim(\diamond \sim p \ \& \ \diamond \sim \sim p)$ | n | 1, falsk kons. |
| 4. $\diamond p$ | n | 2, &-opløsning |
| 5. $\diamond \sim p$ | n | 2, &-opløsning |

- | | | |
|---------------------------|---|-------------------|
| 6. $\sim \diamond \sim p$ | n | 3, &-opløsning |
| 7. $\square \sim \sim p$ | n | 6, mn |
| 8. nRk | - | - |
| 9. $\sim p$ | k | 5, 8 |
| 10. $\sim \sim p$ | k | 7, 6 $\square S5$ |
| x | | |

- | | | |
|---------------------------------|---|----------------------|
| 11. $\sim \diamond \sim \sim p$ | n | 3, opløsning |
| 12. $\square \sim \sim \sim p$ | n | 11, mn |
| 13. nRl | - | - |
| 14. p | l | 5, 13, $\diamond S5$ |
| 15. $\sim \sim \sim p$ | l | 12, 13, $\square S5$ |
| 16. $\sim p$ | l | 15, DN |
| x | | |

- | | | |
|---|-----|-----------------|
| 17. $\sim(\diamond p \ \& \ \diamond \sim p)$ | (n) | 1, falsk ant. |
| 18. $(\diamond \sim p \ \& \ \diamond \sim \sim p)$ | (n) | 1, sand kons. |
| 19. $\diamond \sim p$ | (n) | 18, &-opløsning |
| 20. $\diamond \sim \sim p$ | (n) | 18, &-opløsning |

- | | | |
|-----------------------|-----|-----------------------|
| 21. $\sim \diamond p$ | (n) | 17, &-opløsning |
| 22. $\square \sim p$ | (n) | 21, mn |
| 23. nRm | - | - |
| 24. $\sim \sim p$ | (m) | 20, 23, $\diamond S5$ |
| 25. $\sim p$ | (m) | 22, 23 |
| x | | |

- | | | |
|----------------------------|---|-----------------|
| 26. $\sim \diamond \sim p$ | n | 17, &-opløsning |
| 27. $\square \sim \sim p$ | n | 26, mn |
| 28. nRo | - | - |
| 29. $\sim p$ | o | 19, 28 |
| 30. $\sim \sim p$ | o | 27, 28 |
| x | | |

j. $\Delta p \equiv \sim \Delta \sim p$

$$(\Delta p \equiv \sim \Delta \sim p) \equiv ((\Box p \vee \Box \sim p) \equiv (\sim(\Box \sim p \vee \Box \sim \sim p)))$$

1. $((\Box p \vee \Box \sim p) \equiv (\sim(\Box \sim p \vee \Box \sim \sim p)))$

n

NTF



2. $(\Box p \vee \Box \sim p)$

n

1, sand ant.

3. $\sim \sim (\Box \sim p \vee \Box \sim \sim p)$

n

1, falsk kons.

4. $(\Box \sim p \vee \Box \sim \sim p)$

n

3, DN



5. $\Box \sim p$

n

4, v-opløsning



6. $\Box p$

n

2, v-opløsning

10. $\Box \sim p$

n

2, v-opløsning

7. nRn

-

-

11. nRn

-

-

8. p

n

6, 7, $\Box S5$

12. $\sim p$

n

10, 11, $\Box S5$

9. $\sim p$

n

5, 7, $\Box S5$

13. $\sim p$

n

5, 11, $\Box S5$

x

↑