

Moore, Bradley and Indicative Conditionals

Richard Bradley (2000: 220) endorses the following constraint on conditionals.

Preservation Condition

If $Pr(A) > 0$ but $Pr(B) = 0$, then $Pr(A \rightarrow B) = 0$

The argument for this condition is the following passage.

[A]ll it says is that one cannot be certain that B is not the case if one thinks that it is possible that if A then B , unless one rules out the possibility of A as well. You cannot, for instance, hold that we might go to the beach, but that we certainly won't go swimming and at the same time consider it possible that if we go to the beach we will go swimming! To do so would reveal a misunderstanding of the indicative conditional (or just plain inconsistency). (2000: 220)

The importance of this claim comes in the following sentences.

[T]here is in fact no Boolean semantic theory for indicative conditionals that can ensure that it is satisfied, on the reasonable assumption that neither indicative conditionals nor their antecedents generally imply their consequents. (2000: 200)

This seems to be a mistake, at least on a generous interpretation of what the Preservation Condition states. In this note I want to do three things. First, sketch a Boolean semantic theory of indicative conditionals that does satisfy a weak reading of the Preservation Condition. The theory will not be very plausible, but it is I think plausible that this theory plus some bells and whistles is right. Second, argue that this weak reading of the Preservation Condition is all that the data justifies. Third, note where I think Bradley's argument against the possibility of such a theory goes wrong.

First the simple theory. Let $\diamond p$ mean *It might be the case that p* , where this *might* is interpreted as an epistemic modal. I won't assume much about the semantics of epistemic modals, except that (1) is both logically consistent and not the kind of thing one could ever assent to.

(1) $\diamond p \wedge \neg p$.

That there is a big gap between what can be true and what can be asserted is just a consequence of the fact that once we invoke epistemic modals, versions of Moore's paradox start to crop up. Note that (1) has these features both on the contextualist semantics of Keith DeRose (1991, 1998) and the relativist semantics of Andy Egan, John Hawthorne and Brian Weatherson (forthcoming) and John MacFarlane (unpublished). For present purposes it won't matter just what the right semantics for *might* is, just that it is intimately connected to the indicative conditional.

I'll also use $\Box p$ simply as shorthand for $\neg\Diamond\neg p$.

Now here's the theory.

$$\text{If } p \text{ then } q = (p \wedge q) \vee \Box(p \supset q)$$

That is, *If p then q* is true iff it must be that the material conditional is true. Now assume that is what the indicative conditional means, and imagine someone who wants to deny Bradley's Preservation Condition. I interpret $Pr(p) > 0$ as $\Diamond p$ here, following the way Bradley interprets it in the paragraph immediately after the condition is stated. Such a person endorses $\Diamond A$. And they endorse $\neg\Diamond B$. So there are epistemic possibilities in which A is true. None of these are possibilities in which B is true. So these are epistemic possibilities in which $A \wedge \neg B$ is true. Now they also endorse $\Diamond(A \rightarrow B)$. That can't be because there is an epistemic possibility in which $A \wedge B$ is true, because we have $\neg\Diamond B$. So it must be because it is possible that $\Box(A \supset B)$ is true.

As a general rule, the principle $\Diamond q \rightarrow \Box\Diamond q$ is wildly implausible when \Diamond is epistemic *might*. Nonetheless, denying a particular instance of it sounds bad. That's not surprising, since its negation is $\Diamond q \wedge \Diamond\neg\Diamond q$, which is just an instance of (1). That is, it's denial is Moore-paradoxical. So we would not expect anyone to comfortably deny it. But someone who denies Bradley's Preservation Condition must do just this. For as we said, they must say that there are epistemic possibilities in which $A \wedge \neg B$, but also that there might not have been. For we showed they are committed to $\Diamond\Box(A \supset B)$, which just is the claim that there might not be any epistemic possibilities in which $A \wedge \neg B$. So any specific denial of Bradley's Preservation Condition is Moore-Paradoxical.

That is to say, our toy semantic theory does validate a weak version of the Preservation Condition. It validates the version that says you can never felicitously deny a particular instance of it. And note that the

argument in favour of the Preservation Condition really only supports this weak version. The only argument for the Preservation Condition notes the absurdity of denying a particular instance. And I agree, this does sound absurd, just like uttering any Moore-paradoxical sentence sounds absurd. But just as we shouldn't conclude from the absurdity of (1) that it couldn't be true, we shouldn't conclude from the denial of all instances of the Preservation Condition that one of them couldn't be true.

In a nutshell, the point is that if the semantics of indicative conditionals refers to epistemic possibilities, then it is possible that any denial of the Preservation Condition is Moore-paradoxical, rather than false. Whether this is true or not will depend on working out the complete theory of indicative conditionals, but if the theory sketched here is plausible (and I think it is) there is a reason to think the Preservation Condition fails.

In conclusion, note that even if the Preservation Condition is true, having modal operators in the truth conditions for indicative conditionals could still block Bradley's argument. For the next step of his argument relies on the following premise.

If X doesn't entail Z and Y doesn't entail Z then there is a probability function Pr such that $Pr(X) > 0$ and $Pr(Y) > 0$ and $Pr(Z) = 0$.

Now that's true when X , Y and Z are not themselves modal claims. But it isn't true in general. Here's a simple case where that isn't true. Assume we're working in a probability/modal logic where the probability function is governed by the standard probability calculus, the modal logic is S4, and we add the following (fairly intuitive) axiom linking the two.

$$Pr(p) > 0 \leftrightarrow \diamond p$$

(Again, it's worth noting that Bradley freely moves back and forth between $Pr(p) > 0$ and $\diamond p$, so we aren't changing the subject unduly by positing this as an axiom.) Now let X be $\diamond q$ and Z be q . Obviously X doesn't entail Z . But we can't have $Pr(X) > 0$ and $Pr(Z) = 0$ because in this logic the following chain of biconditionals holds.

$$Pr(X) > 0 \leftrightarrow \diamond X \leftrightarrow \diamond \diamond q \leftrightarrow \diamond q \leftrightarrow \diamond Z \leftrightarrow Pr(Z) > 0$$

More complicated cases are possible as well. Strengthen the modal logic to S5 while keeping the axiom $Pr(p) > 0 \leftrightarrow \diamond p$, let X and Z be atomic sentences, and let Y be $(X \wedge Z) \vee \square(X \supset Z)$. (Or, in other words, $X \rightarrow Z$ on the above definition.) Then even though neither X nor Y entails Z , it is impossible to have both X and Y with positive probability while Z 's probability is zero. The point is that sentences about possibilities and probability are subject to different modal constraints than non-modal sentences, and it is possible that even if the Preservation Condition is true, these constraints rather than the non-Boolean nature of conditionals explain its truth. So if the truth-conditions of indicative conditionals should be spelled out using modal locutions, especially epistemic modals, we have reason to believe both that the Preservation Condition is false, and that it does not entail that "the logic of conditionals is non-classical" (Bradley 2000: 221).

References

- Bradley, Richard (2000) "A Preservation Condition for Conditionals," *Analysis* 60: 219-22.
- DeRose, Keith (1991) "Epistemic Possibilities," *Philosophical Review* 100: 581-605.
- (1998) "Simple *Might*'s, Indicative Possibilities, and the Open Future," *The Philosophical Quarterly* 48: 67-82.
- Egan, Andy, John Hawthorne and Brian Weatherson (forthcoming) "Epistemic Modals in Context" in *Contextualism in Philosophy*, edited by Gerhard Preyer and Georg Peter, Oxford University Press.
- MacFarlane, John (unpublished) "Epistemic Modalities and Relative Truth", available at <http://philosophy.berkeley.edu/macfarlane/epistmod.pdf>.