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# Duplication, Growth and 'Total Return Economics'*

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**Synopsis:** Maintenance consumption is an expense recovered in product prices, yet also a source of taste satisfaction which must be exhausted, rather than reinvested, from the capital affording it. This riddle is solved in the 'duplication rules': the cost of maintenance consumption is recovered in pay and prices, but an equal flow is exhausted from the human capital of the worker earning the pay. The rules impact tradition in several ways. If output is defined in principle as value added, then it cannot also be described as consumption plus net investment without double-counting the maintenance consumption recovered in prices. Also rate of return in the stationary state is not zero, but is the rate sufficient to offset the exhaustion of individual human capital. The rules lead to new insights into economic return, and support an argument that all growth at the scale of closure is due to productivity gain rather than to thrift.

**Key words:** capital, human capital, yield, output, return, consumption, waste, gift, growth, pay

**JEL classification:** B4, O47

## 1. Basics and the duplication rules

This paper condenses and revises arguments which I collectively call 'total return economics', TRE (see Getty 2001). Both mean to reconceptualize economics, from a blank slate, toward the practical aim of explaining and predicting behavior. We will adapt old terms where we can, and invent new ones where we must.

The prime dimensions of economics are value and time. TRE measures value in disinflated dollars, without a physical numeraire and without specifying the means of disinflation. Capital stocks are measured in dollars, and flows such as output are measured in dollars per unit time. 'Rates', in TRE, are ratios of flows to assets generating them, and are therefore pure numbers over time. Stocks will have the usual notation in upper-case letters, and flows and rates in lower-case ones, but otherwise we will follow traditional notation where a tradition can be found. Flows and rates will generally be shown as instantaneous rather than discrete-period. All values are time-variable unless otherwise stated. Since values are as of the current instant, as a default assumption, time-denoting subscripts will be shown only for the exceptions, e.g. (1) and (5) below.

TRE pictures a reproductive population of 'owners', meaning individuals of all ages and endowments, competing in markets for the means of satisfying current and future tastes. This means is capital. The outflow of value from capital, called yield, is

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the means of satisfying current tastes. Capital is productive, generating more capital in output. It is priced at the integral of expected future yields discounted to present value at the rate of return, or ratio of output to capital value, at which it is expected to generate them. That is the unique rate such that a rational actor should be indifferent between buying the stream of yield later or the capital now. This well-known ‘present value’ rule is

$$J = \int_0^\infty \eta_t \mathrm{e}^{-\bar{r}_t t}\,\mathrm{d}t, \tag{1}$$

where $\bar{r}_t = (1/t)\int_0^t r_z\,\mathrm{d}z$, and where $J$ is capital value, $\eta$ (*eta*) is its yield, $r$ is rate of return, $t$ is time and $z$ is a variable of integration.

Yield means net yield after plowback in the asset generating it. ‘Pos’ and ‘neg’ mean positive and negative. Yield is pos yield $\eta_+$ less neg yield $\eta_-$. Neg yield is investment received from outside. Yield is either ‘transfer yield’ $\tau$ (*tau*) satisfying ‘transfer tastes’ for reinvestment in a different asset, or is ‘final yield’ $\phi$ (*phi*) exhausted from the economy in satisfying ‘final tastes’. The litmus test for telling transfer yield and tastes from final ones is that owners must annihilate capital, rather than shift it to another asset or another owner, in order to satisfy final tastes. Final yield is yield without any compensation, either to the owner or his donees, except in taste satisfaction. Transfer yield moves value from one pocket to another, whether or not the source and destination pockets belong to the same trousers, while final yield exhausts value from the economy. Although TRE agrees with tradition that all final yield is consumption, by the way, we will soon see that the converse is not true. Meanwhile the ‘yield rule’ is

$$\eta = \eta_+ - \eta_- = \tau + \phi. \tag{2}$$

Transfer yield is likewise the difference between positive and negative components. Neg transfer yield is the same as neg yield. All final yield is pos yield, so that pos yield is pos transfer yield plus final yield. Then

$$\tau = \tau_+ - \tau_-, \quad \tau_- = \eta_- \quad \text{and} \quad \eta_+ = \tau_+ + \phi. \tag{3}$$

Also define ‘yield rate’ $\varepsilon$ (*epsilon*) and ‘final yield rate’ $\phi_{\text{rate}}$ by

$$\varepsilon = \frac{\eta}{J} \quad \text{and} \quad \phi_{\text{rate}} = \frac{\phi}{J}. \tag{4}$$

Capital is of two factors. These are ‘self-value’ and ‘property,’ often shortened to ‘self’ and ‘prop’. They correspond to the traditional terms human capital and physical capital. I coin the new words in order to shed old baggage and to keep key words short. For easier reading, however, I keep the old notations $H$ for self and $K$ for prop. Then $J = H + K$. A distinguishing feature of TRE is its insistence that both factors, and not prop alone, are priced by the present value rule (1). This conclusion is forced by TRE’s definition of capital in principle as the means of current and expected taste satisfactions. Consequently (1) implies

$$H = \int_0^\infty \eta_{H,t}\, \mathrm{e}^{-\bar{r}_t t}\, \mathrm{d}t, \tag{5}$$

where $\eta_H$ is ‘work yield’, to be clarified later. If $H$ here means the self of an individual, as distinct from a population, we may prefer to change the upper limit of integration from $\infty$ to $\lambda - x$, where $x$ is current age and $\lambda$ (*lambda*) is remaining life expectancy.

Each asset of either factor is reduced in value by current pos yield, and conversely replenished by any neg yield and by retained output. Thus any pos yield must deplete capital, other things equal, and final yield must deplete it without reinvestment in any other asset. ‘Free goods’, defined traditionally as goods available without sacrifice, such as air or sunlight, are clarified in TRE as any final taste satisfactions in excess of current final yield from assets of the owner satisfied. TRE, like other economic systems, treats free goods as fortuitous events which cannot influence behavior in advance because they occur at random. Since the aim is prediction of behavior, economists generally neglect them. Therefore free goods are excluded from final yield. Note on the other hand that final yield, by definition, can never be greater than the final taste satisfaction it provides. That is why there are no negative free goods, or ‘free bads’. Any exhaust of capital without equal taste satisfaction is implicitly not yield, but is rather deadweight loss. The truism that all final yield requires equal exhaust is crucial to TRE, and can be called the ‘exhaust principle’.

Are there any exceptions? What about a diamond ring, which confers final yield in the form of pleasure in wearing it, and yet scarcely depreciates over the centuries? The answer is that value is exhausted in yield, but simultaneously replenished by the output of the ring. That process is modeled in (1) where yield is assumed constant to infinity. Capital is depleted by current yield, but replenished by the rise in value of all future yields as they lessen their time discounts by moving nearer. This demonstrates the sense in which all capital is productive, however inert, and confirms that there are no exceptions to the exhaust principle.

TRE is both microeconomic and macroeconomic, since it is meant to describe reality at every scale. All of its equations, unless otherwise noted, hold at the scale of the individual asset as well as at all larger scales. Thus transfer yield, in general, may be reinvested in other assets of either factor and of the same or another owner. At the scale of the individual owner, net transfer yield from all his capital of both factors is ‘gift’ $\gamma$ (*gamma*). At the scale of the closed economy, where all transfers and gifts cancel, yield is final yield alone. Transfers are zero-sum. TRE highlights ‘parental gift’, meaning support of the young by adults regardless of relatedness. If TRE accounted at the scale of the household rather than of the individual, most parental gift would escape view.

Final tastes are satisfied in consumption c alone, but not in all consumption. TRE distinguishes a part of that flow as ‘insumption’ $\iota$ (*iota*) which is invested in self. Insumption is transfer yield, not final yield, and generally does not satisfy final tastes of owners paying for it. It is largely nurture and education, and much of it is parental gift from adults to the young. Education is seldom a taste-satisfying end in itself, whether or not it ought to be, and taste satisfactions from nurture afforded by donors are incidental free goods to the donees, since the donees did not pay for them. Therefore TRE excludes insumption from the kinds of consumption satisfying final tastes in final yield. That is why consumption and final yield are not always the same. The litmus test for insumption is that a penny less would mean a penny less gain, other things equal, in present value of future work yield capitalized in (5).

The rest of consumption is maintenance consumption or 'maint' $\mu$ (*mu*), defined as an expense recovered in pay and then in product value, and any residual waste consumption or simply 'waste' $\upsilon$ (*upsilon*) recovered neither in self nor in pay. Thus

$$c = \iota + \mu + \upsilon. \tag{6}$$

To help clarify maint, first think of 'vital consumption' as the consumption any owner needs to survive and work, but not to learn new skills or to raise descendants. Then vital consumption is a part of insumption for the young, along with learning, and is typically equal to maint among adults. Since maint is defined as an expense recovered in pay, however, maint cannot exceed pay. Therefore maint must be less than vital consumption if pay is less, and pay may be less for adult cohorts whose vital consumption is subsidized from their own prop yield or from gift received from other cohorts. It seems that youngest and oldest adult cohorts of Americans today are indeed subsidized, by continuing parental gift in the first case and by social security in the second. With that qualifier, maint may be taken as vital consumption of adults.

Now return to insumption. It seems likely that some of this flow, though not much, gives rise to final taste satisfactions and is paid for by the owner satisfied. A collegiate working his way through college may enjoy the steak that builds his soma, and even some pages of the textbooks that build his knowledge. Should TRE therefore relent on its claim that all insumption is transfer yield, and none final yield? It should not. Whatever he paid for insumption, by definition, bought that much growth in self-value. If he paid nothing additional for the taste satisfactions, they are fortuitous free goods. If he did pay something more for them, say by having chosen a pricey country club school whose tuition bought enticements beyond education, the cost difference is waste recovered neither in self nor in pay. Insumption is always transfer yield, and never final yield, even though it may bring concomitant free goods or waste.

Although all economists realize that some consumption is wasted and some constructive, the dichotomy between maint and insumption is probably unique to TRE. That is why we had to coin the words. In property terms, this is the dichotomy between expense and investment. Expense is a cost recovered in current sales, in this case the worker's literal or imputed pay, while investment (insumption) is a cost recovered in capital now and in sales later. As a rough rule of thumb, nurture of the young is insumption while nurture of adults is maint. All education, at every age, can be taken as insumption. The litmus test, again, is effect of the consumption on current pay versus expected future pay. Recovery in the first means maint, recovery in the second means insumption, and recovery in neither means waste. Thus the worker's meal is typically maint, while his children's meals are typically insumption.

Although TRE tends to describe the two factors in equations of the same form, and to put them on equal footing in many ways, there is a root difference. Self is the factor whose maintenance satisfies the owner's final tastes, so that the cost of maint must be exhausted from capital according to the exhaust principle. That is sometimes true of prop, but always true of self. Even though TRE recognizes some worker expense which is recovered in pay but does not satisfy final tastes, such as bus fare to work, maint is the specific expense applied to minimize capital deterioration. And all maint satisfies final tastes to keep body and soul together. In this case, the litmus test is motivation independent of effect on pay. Any outlay which enables pay, but which the owner would have spent even if it did not, is maint. Even trips to the dentist meet this test, or payment of taxes on earnings, because such costs avoid worse ones whether they are recovered in pay or not.

These reflections allow

$$\phi = \mu + \upsilon \tag{7}$$

meaning that final yield equals maint plus waste. But this brings an apparent contradiction. Maint was defined as an expense recovered in pay and product value. Final yield, by the exhaust principle, must be exhausted from capital and reinvested in nothing. How can these facts be reconciled?

We find the answer by imagining parallel cases in property accounting. Suppose that most owners of classic cars love to lube them on weekends, and would lube them whether that kept up car values or not. The result is that the price of classic cars would include a "lubing premium" equal to the present value of that expected taste satisfaction. Lube oil would not, since most of it is used on recent cars whose maintenance gives no pleasure, and this majority use determines the price. An accountant might reason that the maintenance value of the lube job is an expense recovered in the value of the current performance of the car, meaning its imputed sales, while taste satisfaction value is amortized from the lubing premium.

Although the taste satisfaction value of lubing classic cars might be different from the expense cost, the expense and final yield values of maint are the same, since any work expense not recovered in final yield was defined as non-yield expense. This tautology allows the 'first duplication rule'

$$\mu = \mu_{\text{yield}} = \mu_{\text{expense}}, \tag{8}$$

where the second and third sides are respectively the final yield value and expense value of maint.

Note that if only one owner in the world enjoyed lubing his classic car, while all others begrudged the work or paid the garage to do it, then classic cars would not add the price premium shown in the first example. In that case, the one owner's enjoyment would be a free good. That distinction shows why it is practical to treat the taste satisfaction value of maint as final yield exhausted from a capital premium rather than as a free good paid from nothing. Since all owners gain this satisfaction, rather than only a few, it is realistic to add a capital premium to self just as we would to classic cars.

It is clear that the premium must be added to self, rather than to the prop assets, say food or clothing, which are consumed in maint. These prop assets are wholly converted into products, through the medium of the worker's services, so that no part of their value is exhausted. Final yield must be exhausted from capital of the asset enabling the yield, which is uniquely the self of the worker who earns it. All waste, on the other hand, is exhausted from prop assets (the consumption goods) of the owner enjoying the waste. It follows that all of the final yield value of maint is exhausted from self of the owner realizing that yield, while all his waste is exhausted from his prop. If we therefore define

$$\phi = \phi_H + \phi_K, \tag{9}$$

where $\phi_H$ and $\phi_K$ are respectively exhausts from self and prop, the arguments just given lead to the 'second duplication rule'

$$\phi_H = \mu \quad \text{and} \quad \phi_K = \upsilon. \tag{10}$$

(8) and (10) are the first two 'duplication rules'. Two more will be added soon.

The duplication rules are the most remarkable feature of TRE. There is nothing like them in tradition. At first glance, they seem absurd. They seem to say that maint, the form of consumption which keeps self intact and productive, actually depletes self. Not at all. We cannot live longer by deferring maint, and will live shorter if we do. Maint is not the cause of the exhaust of self with age, but its compensation in final yield. Maint no more explains that loss than the recovery of the bakery's depreciation in the value of bread explains the depreciation of the bakery. The explanation is 'senescence', meaning the erosion of self as the pool of future work yields on which it is capitalized in (5) is drained by age.

We will say more about senescence later. Meanwhile the shock of the duplication rules is that the erosion of self with age, like that of the bakery, is compensated by yield. Their logic lies in the biological imperative. Maint, defined for economic purposes as the flow of consumption enabling work services and hence pay, amounts biologically to the flow sustaining productive life. To say that maint is both expense and final yield is to say that the means of life is an aim of life, along with transfer yield available to bring the next generation into place. Self, as the sole means of that final yield, capitalizes expected maint into its value. Thus economics is biology, as seen by Alfred Marshall over a century ago, with waste to relieve and humanize the Darwinian rigor.

The concept of 'maint lag' helps clarify the process. In TRE, flows are measured as of the current instant, rather than accumulated over a period. Maint is defined as conversion of consumption into worker productiveness and hence pay. When does this conversion happen? Clearly the steak is not yet converted into pay when bought at the checkout counter, or even when served on the plate, or even as each bite is ingested. Food in the stomach is not yet converted, in the TRE sense, and therefore is still prop. Only as it is digested into the blood stream does it revitalize the worker and cross the threshold into maint. As it is digested, hunger and lassitude return. This helps make intuitive sense of the duplication rules. Each bit of maint is drained in usage, and the drain measures the exhaust of self.

## 2. Output

Output $y$ may be defined in principle as value-added, or creation of economic value as distinct from conversion. A textbook example of this distinction is the value-added chain, as from wheat to flour to bread. Each link adds a little to what was already produced, so that the sum of outputs is the sum of increments or flow of final products alone. Creation of economic value, at first glance, might seem to mean direct creation of final yield as well as of capital. But the exhaust principle reminds us that final yield must be exhausted from capital already in place, so that creation of final yield is impossible. Therefore all output is implicitly creation of capital, meaning capital of both factors. This can be written as

$$y = y_H + y_K \tag{11}$$

where the right side terms mean creation of self and of prop, respectively.

Value-added is identical to total return, or the sum of capital growth and yield, since internally created capital has no other dispositions and the sum of growth and yield has no other source. This tautology is the 'equivalence rule'. Then the 'total return rule' is

$$y = \dot{J} + \eta, \tag{12}$$

and for the factors,

$$w = \dot{H} + \eta_H \quad \text{and} \quad p = \dot{K} + \eta_K, \tag{13}$$

where work $w$ and profit $p$ are the outputs of self and prop, and $\eta_K$ is 'prop yield'. Also $y = w + p$. Keep in mind that $y_H$ and $y_K$ mean creation *of* these factors while $w$ and $p$ mean creation *by* them, so that the four flows are distinct.

For prop assets, the term 'net investment' means all investment from outside (neg prop yield) plus all retained output (profit). Since retained profit is all profit less pos prop yield, we see that net investment is profit less prop yield, or simply prop growth $\dot{K}$ (by (13)). Thus the growth flow of prop will be called either 'net investment' or 'prop growth'. The flow $\dot{H}$ is 'self-growth'.

While prop yield is well understood, work yield $\eta_H$ needs explanation. Like prop yield, it is the difference between positive and negative components. This is notated $\eta_H = \eta_{H+} + \eta_{H-}$, where the right-hand terms are 'pos work yield' and 'neg work yield'. Neg work yield is insumption afforded from outside, from parental or other gift and possibly from the owner's own prop yield. In either case, it is identical to the negative component in 'work transfer yield' $\tau_H$. That is, $\eta_H = \tau_{H-}$. Pos work yield is final yield $\phi_H$, which equals $\mu$ by (10), plus pos transfer work yield $\tau_{H+}$ invested in prop or given away. Then the forms of and applicable to self are

$$\eta_H = \tau_H + \mu, \quad \eta_{H-} = \tau_{H-} \quad \text{and} \quad \eta_{H+} = \tau_{H+} + \mu, \tag{14}$$

where $\tau_H = \tau_{H+} - \tau_{H-}$.

These definitions allow a restatement of the economics of the individual. Each owner owns one asset of self and many of prop. We may deem that even the young are beneficial owners of some prop, including the food in their mouths at a minimum. The owner may transfer prop yield from one prop asset to another, or to self through insumption, and he may invest pos work yield in his prop assets. At the scale of his total capital, as we saw, his transfer yield is gift alone. Gift is pos gift $\gamma_+$ passed to others, less neg gift $\gamma_-$ received from others. Thus the owner's total pos yield is his pos gift plus his final yield, while his total neg yield is his neg gift. That is,

$$\eta_+ = \gamma_+ + \phi_+ \quad \text{and} \quad \eta_- = \gamma_-, \quad \text{where } \gamma = \gamma_+ - \gamma_-, \tag{15}$$

at the scale of the owner. Gift includes social transfer payments, mediated through taxes or charities, and parental gift from adults to the young. (15) cautions us against supposing that only final yield imposes real costs to the individual, since pos gift is also net outflow, and therefore that final tastes alone drive economic behavior. All of total pos yield, meaning pos gift as much as final yield, costs him equally, and may be supposed to be spent on end objectives of equal priority. Final tastes are final only in that their satisfaction depletes the universe of wealth as well as his personal capital.

## 3. The growth dichotomy

Capital growth, which is the rest of total return after yield, is one of the most interesting topics in economics. We all want to know more about the roles of thrift and productivity in this flow. But what exactly does that mean? Productivity is rate of return $r$, and thrift must have something to do with restraint of yield rate $\varepsilon$. Now define 'growth rate' $g$ by $g = \dot{J}/J$, and divide (12) by $J$ to get

$$r = g + \varepsilon \quad \text{or} \quad g = r - \varepsilon \tag{16}$$

as another form of the total return rule. The share of productivity in growth would thus mean *r*/*g*. Since yield rate $\varepsilon$ is usually positive, however, *r*/*g* is usually greater than unity. This cannot be the ratio we are looking for.

What we want is the extent to which *increase* in productivity accounts for growth. Therefore TRE's approach is therefore to compare changes in $r$, $g$ and $\varepsilon$ over chosen periods. Define 'free growth rate' $\psi$ (*psi*) as increase in $r$ since a chosen past moment of zero growth, and define 'thrift growth rate' $\theta$ (*theta*) as decrease in yield rate $\varepsilon$ since that same moment. That is,

$$\psi = r - r_0 \quad \text{and} \quad \theta = \varepsilon_0 - \varepsilon \tag{17}$$

where subscript 0 denotes values at the past moment of 'origin' in stasis. Note $g = g - g_0$ by definition. Now define 'free growth' $\dot{J}_\psi$ and 'thrift growth' $\dot{J}_\theta$ by

$$\dot{J}_\psi = \psi J \quad \text{and} \quad \dot{J}_\theta = \theta J. \tag{18}$$

Free growth is free in the sense that it costs no restraint in yield rate. These definitions apply at all scales, and to both factors. The origin in stasis, by the way, need not be so far back in time as to make information outdated. Recessions seem to recur every decade or so, and stock market volatility suggests many moments of negative growth every day. Growth in reproducible assets, including self, can also be dated from the moment of first investment. Now verify

$$\frac{\psi}{g} + \frac{\theta}{g} = 1 \quad \text{and} \quad \dot{J}_\psi + \dot{J}_\theta = \dot{J}, \tag{19}$$

and define $\psi/g$ as the 'productivity index', or 'prod index', and $\theta/g$ as the 'thrift index'.

The practical value of the 'growth dichotomy,' meaning the distinction between free growth and thrift growth, comes from the fact that the two are influenced in different ways. Thrift is an option within our power, at least sometimes, while productivity gain seems harder to elicit and manipulate. Some see it as 'exogenous', while others favor Paul Romer's (1986) view that 'meta-ideas' such as free markets and free information help motivate and sustain it. The equations of TRE treat thrift growth as a quantity determined by inflows and outflows, and free growth as a given not fully explained. For these practical reasons, TRE distinguishes 'thrift output' $y_\theta$ as output less free growth. That is,

$$y_\theta = y - \dot{J}_\psi = \dot{J}_\theta + \eta = r_0 J. \tag{20}$$

Likewise 'thrift return' $r_\theta$ is defined as

$$r_\theta = r - \psi = r_0 = \frac{y_\theta}{J} = \theta + \varepsilon = \varepsilon_0. \tag{21}$$

Now let us consider free growth of the factors. Define

$$r_H = \frac{w}{H}, \quad r_K = \frac{p}{K}, \quad \varepsilon_H = \frac{\eta_H}{H}, \quad \varepsilon_K = \frac{\eta_K}{K}, \quad g_H = \frac{\dot{H}}{H} \quad \text{and} \quad g_K = \frac{\dot{K}}{K}, \tag{22}$$

and divide the equations of (13) of by $H$ and $K$ respectively for

$$r_H = g_H + \varepsilon_H \quad \text{and} \quad r_K = g_K + \varepsilon_K. \tag{23}$$

Free and thrift growth rate, and growth flows, of the factors are

$$\begin{gathered}\psi_H = \Delta r_H, \quad \theta_H = -\Delta\varepsilon_H, \quad \psi_K = \Delta r_K, \quad \theta_K = -\Delta\varepsilon_K, \\ \dot{H}_\psi = \psi_H H, \quad \dot{H}_\theta = \theta_H H, \quad \dot{K}_\psi = \psi_K K \quad \text{and} \quad \dot{K}_\theta = \theta_K K\end{gathered} \tag{24}$$

where $\Delta$ means change since some moment of stasis. Also verify

$$\dot{H} = \dot{H}_\psi + \dot{H}_\theta \quad \text{and} \quad \dot{K} = K_\psi + K_\theta. \tag{25}$$

Note that one factor might be growing at the expense of the other in a moment of overall stasis, so that origin moments would disagree. Therefore we cannot assume $\dot{H}_\psi + \dot{K}_\psi = \dot{J}_\psi$ or $\dot{H}_\theta + \dot{K}_\theta = \dot{J}_\theta$, although we may sometimes suppose those conditions for simplicity. (24) and (25) allow definition of 'thrift work' and 'thrift profit' as

$$w_\theta = w - \dot{H}_\psi = \dot{H}_\theta + \eta_H \quad \text{and} \quad p_\theta = p - \dot{K}_\psi = \dot{K}_\theta + \eta_K. \tag{26}$$

Divide by $H$ and $K$ to find 'thrift work rate' and 'thrift profit rate'

$$r_{\theta,H} = \theta_H + \varepsilon_H \quad \text{and} \quad r_{\theta,K} = \theta_K + \varepsilon_K. \tag{27}$$

It is useful to decompose thrift growth into positive and negative components. Pos thrift growth is neg yield (investment from inside) plus thrift output, and neg thrift is pos yield. That is,

$$\dot{J}_{\theta+} = y_\theta + \eta_- \quad \text{and} \quad \dot{J}_{\theta-} = \eta_+, \tag{28}$$

so that $\dot{J}_{\theta+} - \dot{J}_{\theta-} = \dot{J}_\theta$ as required.

TRE is particularly interested in self, partly because it is the factor about which most needs to be learned. While prop prices are revealed in asset sales, the value of self must be inferred indirectly. Let us therefore try to clarify the thrift growth of self. Since insumption was defined as a zero-sum conversion of consumption goods and services into self, and since we may take it as axiomatic that there is no other medium of external input into self, we conclude that externally supplied insumption is the whole of neg work yield. But each owner also invests work in himself, particularly during childhood and youth, through such means as exercise and learning. Since work is defined as output, which includes free growth, any free growth in self, say when discovery of a nearby gold field adds fortuitously to the market value of the skills of a land lawyer, is also technically part of 'self-invested work'. The litmus test, for self and prop alike, is that free growth rate is change in rate of return since an origin in stasis. Let us term the thrift portion of self-invested work 'insumed work' $\iota_w$, and include it as part of insumption. Then insumed work is self-invested work less free self-growth.

Externally supplied insumption, meaning insumed parental or other gift plus any insumption from the owner's prop yield, equals neg work yield. We saw that it equals the negative component in the transfer yield of self-value, or $\tau_{H-}$. Conversely pos work transfer yield $\tau_{H+}$ is the flow from pay spent on prop investment or gift. Insumed work means all thrift work less any simultaneously transferred out in these forms. Then

$$\iota_w = w_\theta - \tau_{H+} \quad \text{and} \quad \iota = \tau_{H-} + \iota_w = \tau_{H-} + w_\theta - \tau_{H+} = w_\theta - \tau_H. \tag{29}$$

Now (26), (14) and (29) combine for

$$\dot{H}_\theta = w_\theta - \tau_H - \mu = \iota - \mu, \tag{30}$$

showing that maint is the negative component in thrift self-growth.

Self, like a building, originates from outside investment and depreciates to zero as life ends. Tritely, any such asset can be priced as the undiscounted integral either of all past growth or of all future decline in value. That is,

$$H(x) = \int_{x_0}^{x} \dot{H}_t \,\mathrm{d}t = \int_{x}^{\lambda} -\dot{H}_t \,\mathrm{d}t, \tag{31}$$

where $x$ is present age, $x_0$ is age at first received investment (ovulation), and $\lambda$ (*lambda*) is economic lifespan of the worker. Since $\dot{H} = \dot{H}_\psi + \dot{H}_\theta = \dot{H}_\psi + \iota - \mu$, by (30), the 'third duplication rule' becomes

$$H(x) = \int_{x_0}^{x} \iota_t - \mu_t + \dot{H}_{\psi,t} \,\mathrm{d}t = \int_{x}^{\lambda} -\iota_t + \mu_t - \dot{H}_{\psi,t} \,\mathrm{d}t. \tag{32}$$

This rule is useful in evaluating young cohorts assumed to show negligible maint, and older cohorts which may show negligible insumption.

## 4. Return as the absolute maximand

Now let us take a closer look at output and rate of return $r$. Tradition defines this rate as the ratio $p/K$, or equivalently $r_K$, while TRE prefers to generalize it as $y/J$. We may agree with tradition that for prop assets at least, the equilibrium rate of return is risk-dependent. Owners are generally risk-averse, in varying degrees, and tend to bid less for assets whose return is expected to prove more volatile. This effect makes price vary inversely, and therefore rate of return directly, as asset risk in the sense of volatility. Its probable explanation is the narrow margin for income reduction in competition in saturated niches. This being so, one would expect a direct correlation between risk and return in investments in self as well.

Capital theory is largely an effort to make sense of interest and rate of return. On that ground and others, TRE is probably capital theory. But TRE approaches the question differently. Tradition tends to explain a riskless interest rate first, and then to add increments for risk. TRE says nothing about riskless assets, and makes no attempt to quantify risk discounts. Rather it seeks an explanation for the 'social' or economy-wide rate of return, which is implicitly the return on assets of average risk.

While other approaches work from the bottom up, that is, TRE begins and ends in the middle.

TRE defines the social rate of return as the ratio of all output to all assets of both factors, but cannot find data to measure it except as the ratio of profit to property in the business sector. Is this sample typical? To find out, let us try to identify the 'absolute maximand' which drives the behavior of owners. Might the maximand be final yield? Might it be yield in general? Clearly neither, since owners defer yield by saving. Then is capital the maximand? Again, clearly not, since owners divest capital in gift and waste. Then what about output, which is the sum of capital growth plus yield? Now we are much warmer. Output is broadly the right answer, but a slightly more informative way of saying the same thing is that each owner maximizes the risk-adjusted rate of return to the capital he has, according to his degree of risk aversion. This maximand holds for all personalities and circumstances. It allows most waste to Reggie Van Gleason, most thrift to the Practical Pig, most benefaction to Mother Theresa, and most malefaction to Bin Laden or Doctor No. Since capital is defined as present value of expected satisfactions time-discounted at the rate owners actually apply to them, and since behavior is 'revealed preference' for these ultimate satisfactions, the choice of risk-discounted rate of return as the absolute maximand seems to rise to the level of certitude or tautology.

Each owner may therefore be trusted to arbitrage among his assets of both factors, as far as practical, to maximize risk-adjusted rate of return to all together. He will divest low-return assets, after allowance for risk, and invest in higher-return ones. This market action leads to equilibrium in risk-adjusted rate of return across capital of all sectors and both factors, with the qualification that inefficient markets in self should mean a slower response in that factor when equilibria are disturbed.

We thus expect that return to both factors and all sectors is uniform but for differences in risk, meaning volatility in rate of return. If houses or self are perceived as less risky than businesses, on average, then they are expected to carry lower return. Absent an argument why either should be so perceived, I prefer the neutral assumption that the social rate equals the rate to the business sector, meaning the cap-weighted average of returns to equity and debt claims on that sector.

## 5. The duplication rules and tradition

The gist of the argument leading to the duplication rules is:

1. Maint satisfies final tastes of all owners, and must therefore be exhausted in final yield from the capital affording the satisfaction.
2. This capital is uniquely self.
3. The exhaust of self provides no other final yield.

The duplication rules follow. Thrasymachus would object, and rightly, that they describe nothing but creatures of the mind. Each owner's self is self-appraised, and would have much less value to a slaver, since the slave's maint is expense only, and not yield, from the slaver's perspective. Then how can purely subjective things support useful analysis? The answer is that all value of both factors is subjective, and economics is the study of value. Subjectivity is all. It can be studied usefully because owners belong to a common species with common traits.

There is no question that TRE makes surprising and testable predictions. It will predict that the real social rate of return exceeds the sum of the growth and waste rates by several percent per year. Insofar as tradition addresses this question, it seems to equate the social rate of return with the population growth rate (per overlapping generations theory) or the technological growth rate (per the golden rule of accumulations). Thus TRE's prediction exceeds these traditional ones by several percent per year. TRE's version saves the appearances insofar as good data can be found, for example in the business sector (*ibid.*). Models of return in the residential sector are unreliable (*ibid.*), and can be set aside until more is known. Why extrapolate from skimpy and cryptic data on house rentals to find low returns, when robust data from the apartment rental business show higher ones?

The duplication rules, which seem absurd at first glance, resolve some of the paradoxes of tradition. Since prop value is replenished indefinitely through the depreciation component in cash flow, some have concluded that 'rate of return in the stationary state is zero'. This doctrine more or less fits the two traditions just cited. But people would not own prop in the stationary state if its output were zero, and archaeological evidence shows that people owned hand axes and other prop over millennia of stasis. Nor do people throw prop away during episodes of zero or negative growth today. Tradition seems to miss the fact that people are not replenished from their depreciation, as prop is, and must be replenished from output. The social rate of return—setting waste aside—is neither the population growth rate nor the technological growth rate, but the technology-adjusted population replenishment rate.

To see how, write

$$\begin{aligned} y = \dot{J} + \eta = \dot{H} + \dot{K} + \tau + \phi = \iota - \mu + \dot{H}_\psi + \dot{K} + \tau + \mu + \upsilon \\ = \iota + \dot{H}_\psi + \dot{K} + \tau + \upsilon, \end{aligned} \tag{33}$$

by (12), (2), (7), (25) and (30). Transfer yield $\tau$ is always zero at closure, and presumably the stationary state rules out the growth terms $\dot{H}_\psi$ and $\dot{K}$. Thus simplifies to $y = \iota + \upsilon$ in the stationary state at closure. Output in stasis is waste plus population replenishment through insumption, and the social rate of return in stasis is the ratio of this flow to total capital.

Tradition also holds that output is consumption plus gross or net investment, and that income is equivalently pay plus gross or net profit (in TRE's terms, rather than tradition's). Let us check this at the scale of the individual owner. His total output from both factors is

$$y = \dot{J} + \eta = \dot{H} + \dot{K} + \tau + \phi. \tag{34}$$

Transfer yield $\tau$ at the scale of the owner is gift $\gamma$, and final yield is maint $\mu$ plus waste $\upsilon$. Then

$$y = \dot{H} + \dot{K} + \gamma + \mu + \upsilon \tag{35}$$

at that scale. $\dot{H}$ is free self-growth $\dot{H}_\psi$ plus thrift self-growth, and the latter, by (30), is insumption $\iota$ less maint. Also (6) shows that consumption is insumption plus maint plus waste. That is,

$$\dot{H} = \dot{H}_\psi + \iota - \mu \quad \text{and} \quad c = \iota + \mu + \upsilon. \tag{36}$$

Substitute these equations in the last, and cancel terms, to get the 'net product rule'

$$y = c + \dot{K} + \dot{H}_\psi + \gamma - \mu, \tag{37}$$

reading 'output equals consumption plus net investment plus free self-growth plus gift less maint'. The gift term cancels as we increase the scale to economic closure.

Then the belief that output is consumption plus net investment misses free self-growth, but double-counts maint. Since maint is an expense recovered in prices of other goods, it should not be included in final goods separately. And the double-counting is unmistakable. The fact that pay depends on maint, as bread on flour, is decisive. The duplication rules recognize the dual nature of maint as recovered expense and unrecovered final yield, and resolve the double-counting problem by exhausting its final yield value from self.

Tradition is therefore mistaken in supposing that all of a worker's pay, or anyhow all but the part such as bus fare to work which he would not have spent anyhow, given the means, compensates his value added. Once again, the traditional view brings a paradox. Assuming that output is more or less proportional to capital, the worker's output (work) should approach zero at the end of productive life as his work-producing capital (self) must. TRE and common sense predict however that pay, unless subsidized, cannot fall below the maint level at any age. And that is what the data show. Although the worker's value added approaches zero, his services are still worth the maint expense converted into them.

It appears therefore that the National Income and Products Accounts (NIPA), which measure Net National (or Domestic) Product and Income as consumption plus net investment and as pay plus profit respectively, both double-count maint while missing free self-growth. Thus NIPA includes no good measure of output in the sense of value added. I argue (*ibid.*) however that GDP or GNP are good rough measures of 'total cash flow' from both factors, meaning gross yield before plowback, which is the flow available for national budgeting. This is probably the purpose meant by Keynes (1936) and Kuznets (1934).

Note that with no duplication rule, and no exhaust of the final yield value of maint, exhaust from the economy would be limited to waste. Then output and rate of return in the stationary and wasteless state would indeed be zero, since output would have no outlet but growth and waste. No wonder that Schumpeter (1954) and others came to the conclusions noted. Economics without the duplication rules is paradox and unreality.

## 6. Measuring the productivity (prod) index

Although TRE contributes nothing toward an explanation of the causes of free growth, it adds a method for distinguishing and measuring its effects. This can matter to policy makers, since thrift growth and free growth have different prescriptions. The prod index was clarified in (19) as the ratio $\Delta r/g$, when $\Delta r$ is increase in real rate of return since a chosen past moment of zero growth. Free growth is therefore retrospective in nature, but reasonably current if the moment of stasis chosen was recent. The prod index is also convenient to measure in commercial markets, since rates of growth and return in these markets tend to be well studied and tabulated. Ibbotson Associates (2001), for example, records such data from 1926 through 2000 in its *Stocks*, *Bonds*, *Bills* and Inflation 2001 (SBBI 2001). Table 1 calculates yearly prod indices for large-cap stocks from that source. Since SBBI 2001 disinflates some tables and not others, but includes inflation tables to allow conversion, and since traditions vary as to disinflation methods, those applied here are clarified in an appendix to this study.

Calculation of the prod index requires identification of a past moment of zero growth. In general, the record does not isolate such moments. We find pos growth in some periods, and neg growth in others, but no periods or explicit moments of zero growth. Table 1 solves that problem by interpolation. A 'slump-end year' is defined as any neg growth year followed immediately by a pos growth year. Its values of $r$ and $g$ are notated $r_a$ and $g_a$, while those of the following year are $r_b$ and $g_b$. Thus $g_a$ is always negative, and $g_b$ is always positive. We want to find the value of $r$ at the moment when $g$ passed upward through zero. Linear interpolation (see the appendix) requires

$$r_0 = \frac{(\text{abs})g_a}{g_b + (\text{abs})g_a}(r_b - r_a) + r_a, \tag{38}$$

recalling that $r_a$ may be positive or negative. A slump-end year may be preceded by a pos-growth year, so that the slump lasted one year only, or by any number of consecutive neg-growth years.

The arithmetical mean of all shown values of the prod index is 0.95. Thus, Table 1 makes a good case for the predominance of free growth over thrift growth in explaining movements in the stock markets. Note that values greater than unity are not necessarily in error, since neg thrift growth is possible even when overall growth is positive.

Table 1 is surprising only as to degree. Tradition recognizes the predominance of technology over thrift in explaining macroeconomic growth. But tradition would not

*Table 1.* The prod index for large cap stocks[a]

| Year | $g^{b}$ | $r^{c}$ | $r_0{}^{d}$ | Prod index[e] | Year | $g$ | $r$ | $r_0$ | Prod index |
|---|---|---|---|---|---|---|---|---|---|
| 2000 | −0.1353 | −0.1208 | N/A | | 1966 | −0.1644 | −0.1298 | 0.0337 | |
| 1999 | 0.1685 | 0.1788 | 0.0283 | 0.89 | 1965 | 0.0714 | 0.1033 | 0.0304 | 0.97 |
| 1998 | 0.2506 | 0.2654 | 0.0283 | 0.95 | 1964 | 0.1168 | 0.1511 | 0.0304 | 0.99 |
| 1997 | 0.2391 | 0.3113 | 0.0283 | 0.97 | 1963 | 0.1724 | 0.2081 | 0.0304 | 1.01 |
| 1996 | 0.1694 | 0.1912 | 0.0283 | 0.96 | 1962 | −0.1303 | −0.0983 | 0.0304 | |
| 1995 | 0.3157 | 0.3403 | 0.0283 | 0.99 | 1961 | 0.2246 | 0.2604 | 0.0348 | 1.00 |
| 1994 | −0.0421 | −0.0133 | 0.0283 | | 1960 | −0.0445 | −0.0099 | 0.0348 | |
| 1993 | 0.0431 | 0.0705 | 0.0375 | 0.77 | 1959 | 0.0698 | 0.1030 | 0.0413 | 0.88 |
| 1992 | 0.0156 | 0.0464 | 0.0375 | 0.57 | 1958 | 0.3630 | 0.4088 | 0.0413 | 1.01 |
| 1991 | 0.2325 | 0.2667 | 0.0375 | 0.99 | 1957[f] | −0.1733 | −0.1340 | 0.0413 | |
| 1990 | −0.1267 | −0.0874 | 0.0375 | | 1955 | 0.2603 | 0.3107 | 0.0595 | 0.97 |
| 1989 | 0.2261 | 0.2565 | 0.0469 | 0.93 | 1954 | 0.4552 | 0.5339 | 0.0595 | 1.04 |
| 1988 | 0.0799 | 0.1187 | 0.0469 | 0.90 | 1953 | −0.0724 | −0.0160 | 0.0595 | |
| 1987 | −0.0238 | 0.0079 | 0.0469 | | 1952 | 0.1090 | 0.1733 | 0.0670 | 0.98 |
| 1986 | 0.1350 | 0.1715 | 0.0478 | 0.92 | 1951 | 0.1123 | 0.1714 | 0.0670 | 0.93 |
| 1985 | 0.2257 | 0.2736 | 0.0478 | 1.00 | 1950 | 0.1599 | 0.2450 | 0.0670 | 1.11 |
| 1984 | −0.0256 | 0.0222 | 0.0478 | | 1949 | 0.1206 | 0.2097 | 0.0670 | 1.18 |
| 1983 | 0.1347 | 0.1803 | 0.0598 | 0.89 | 1948[f] | −0.0336 | 0.0272 | 0.0670 | |
| 1982 | 0.1089 | 0.1688 | 0.0598 | 1.00 | 1945 | 0.2847 | 0.3343 | 0.0709 | 0.93 |
| 1981 | −0.1866 | −0.1281 | 0.0598 | | 1944 | 0.1169 | 0.1728 | 0.0709 | 0.87 |
| 1980 | 0.1337 | 0.1781 | 0.0554 | 0.92 | 1943 | 0.1629 | 0.2204 | 0.0709 | 0.92 |
| 1979[f] | −0.0100 | 0.0453 | 0.0554 | | 1942 | 0.0314 | 0.1011 | 0.0709 | 0.96 |
| 1976 | 0.1434 | 0.1816 | 0.0507 | 0.91 | 1941[f] | −0.2758 | −0.1942 | 0.0709 | |
| 1975 | 0.2454 | 0.2821 | 0.0507 | 0.94 | 1938 | 0.2799 | 0.3847 | 0.0816 | 1.08 |
| 1974[f] | −0.4192 | −0.3446 | 0.0507 | | 1937 | −0.4169 | −0.3698 | 0.0816 | |
| 1972 | 0.1222 | 0.1505 | 0.0558 | 0.77 | 1936 | 0.2671 | 0.3232 | 0.0465 | 1.04 |
| 1971 | 0.0743 | 0.1060 | 0.0558 | 0.68 | 1935 | 0.3838 | 0.4339 | 0.0465 | 1.01 |
| 1970[f] | −0.0533 | −0.0141 | 0.0558 | | 1934 | −0.0797 | −0.0340 | 0.0465 | |
| 1968 | 0.0294 | 0.0605 | 0.0337 | 0.91 | 1933 | 0.4608 | 0.5321 | 0.0249 | 1.10 |
| 1967 | 0.1705 | 0.2032 | 0.0337 | 0.99 | 1932[g] | −0.0485 | 0.0235 | 0.0249 | |

[a] From SBBI 2001.

[b] From Table A-3 of SBBI 2001, disinflated per Table A-15.

[c] From Table A-20. Values shown are disinflated by Ibbotson Associates.

[d] Interpolated by means of equation (38) from the last previous slump-end year.

[e] $(r-r_0)/g$.

[f] Preceded by one or more neg growth years omitted in this table.

[g] No slump-end years appear before 1932.

have expected a predominance of 19 to one. I will argue, even so, that the ratio at the scale of closure should probably be still higher.

## 7. The growth principles

Thrift growth is reduction in yield rate compared with an origin moment in stasis. Yield at the scale of closure is final yield, or maint plus waste. Since growth cannot come from maint reduction, thrift growth at closure means reduction in waste rate relative to the origin. That would require that waste rate was higher in stasis than in growth, which is not an intuitively likely hypothesis. Although the denominator (capital) was less in stasis, intuition suggests that waste flow was less too. Recessions and setbacks are typically periods of scrimping rather than splurging.

There are further problems. Less waste brings growth only insofar as 'waste conversion' keeps capital intact and no less productive. That can be a tall order in a competitive world where niches are already more or less saturated at the current level of technology. A man cannot dig with two shovels. Only invention widens the niches, and invention is free growth. This is the truth behind the 'golden rule of accumulations', which equates overall growth rate with the technology growth rate. What the golden rule forgets is that the workforce itself must be replenished over the generations by output converted into insumption, and hence by a positive rate of return.

Thrift without waste conversion lowers productivity. Suppose that cigarettes are waste goods, for the sake of political correctness, and that we can manage to stop producing them. Unless we can replace this loss with production of non-waste goods, the social rate of return has lost as much as the waste rate. This shows that the prod and thrift indices are not independent. Ideally, we would have defined their marginal forms as $\partial r/\partial g$ and $-\partial \varepsilon/\partial g$ to allow for effects of one on the other. But in practice, such cross-effects are probably mild. People are unlikely to give up their vices and indulgences unless they get value in return, and the only value aside from waste flows is productive flows such as maint or insumption or investment.

Since the indices in their total-derivative form describe growth already achieved, they are probably adequate for their purposes. *Ex post*, less cigarettes probably means more growth. *Ex ante*, it well may not. Let us therefore define 'thrift effect' $\theta_{\text{effect}}$ as the share of growth rate due to thrift after allowing for impact of thrift on productivity. This is

$$\theta_{\text{effect}} = \theta E \tag{39}$$

where 'conversion efficiency' $E$, a pure number, is the ratio of productive output gained over waste output lost. I will not attempt a more rigorous definition. Since $\theta$ is reduction in waste rate $\upsilon/J$ since an origin in stasis, we can also write

$$\theta = \frac{\upsilon_0}{J_0} R, \quad \text{implying} \quad \theta_{\text{effect}} = \frac{\upsilon_0}{J_0} RE, \tag{40}$$

where 'waste reduction factor' $R$ is defined by

$$R = \frac{-\Delta\upsilon_{\text{rate}}}{\upsilon_{\text{rate},0}}. \tag{41}$$

Meanwhile note

$$\frac{\upsilon_0}{J_0} = \frac{\upsilon_0}{\mu_0 + \upsilon_0} \frac{\mu_0 + \upsilon_0}{J_0}. \tag{42}$$

$\mu_0 + \upsilon_0$ is final yield at the origin moment in stasis. (2) and (12) show that in stasis, at the scale of economic closure, final yield equals output. Therefore $(\mu_0 + \upsilon_0)/J_0 = y_0/J_0 = r_0$, noting $r_0 = r_\theta$ by. Then

$$\frac{\upsilon_0}{J_0} = \frac{\upsilon_0}{\mu_0 + \upsilon_0} r_0. \tag{43}$$

Further define the 'waste index' $W$ as the ratio of waste to final yield, so that $W_0$ gives this ratio at the moment of the origin in stasis. That is,

$$W_0 = \frac{\upsilon_0}{\mu_0 + \upsilon_0}, \quad \text{implying} \quad \frac{\upsilon_0}{J_0} = W_0 r_0. \tag{44}$$

from (43). By (40), this gives the 'thrift effect rule'

$$\theta_{\text{effect}} = W_0 r_0 RE. \tag{45}$$

(45) is atypical of the rules of TRE in several ways. Its arguments, aside from $r_0$, appear in no other rules, and are probably impractical to measure. Rather they are meant to be weighed intuitively, and applied to what we know of $r_0$. Then let us evaluate $r_0$ first. The unweighted average of all values of $r_0$ interpolated in Table 1 comes to 5.16%. This however gives $r_0$ for large-cap stocks, which are generally leveraged and therefore probably riskier than average assets. $r_0$ at closure means average-risk rate of return, in stasis, and is probably less. Later we will model it at 3%–3.5% per year.

The waste index $W$ cannot exceed unity, since it gives the ratio of waste to the sum of waste and maint, and since maint and waste are both always positive. Within that limit, what might the upper limit of $W$ be? How much can workers afford to waste, and what share of the world's final yield is spent by workers?

Then is $W_0$, the waste index in stasis, less or more than in growth? This brings us to the waste reduction factor $R$. While $W_0$ is limited to the range between zero and unity, we saw that $R$ is negative unless waste rate is less in growth than in stasis. Finally we come to conversion efficiency $E$. What does it cost to convert tobacco fields and cigarette factories into cotton fields and shirt factories, and to convert customer preferences accordingly, when the market for shirts is presumably already saturated?

Suppose for illustration that $r_0$ is 4% per year, that $W_0$ is 0.5, and that $R$ and $E$ are 0.25 and 0.75, respectively. Then thrift effect would come to 0.375% per year, or about one sixth of the growth witnessed over the past century. If we cut each of the estimates for $W_0$, $R$ and $E$ by half, which strikes me as more plausible, thrift effect comes to about one hundredth of growth actually seen over that span. And if $R$ is negative, which seems likely, then (45) shows that thrift effect is also negative.

It is for such reasons that I think the prod index at closure is even higher than the 0.95 found for large-caps. The corporate subsector grows partly by net transfer from other areas of the U.S. and world economy. My rule of thumb is that growth is essentially free growth at closure, so that the prod index is roughly unity and is as likely to be a little more as a little less. Then the ‘growth principles’ are

$$\overset{\bullet}{J}_\theta > 0 \quad \text{and} \quad \overset{\bullet}{J}_\theta > \overset{\bullet}{J}_\psi \tag{46}$$

at closure.

It does not necessarily follow that thrift growth of each factor equilibrates to zero. Each may thrift-grow at the expense of the other, since thrift growth in that case would not require a reduction in waste rate compared to the origin in stasis.

Let me repeat that (45) is not claimed to prove the growth principles. Rather it frames the debate, and sets hurdles for counterarguments to clear. If the growth principles are wrong, which of the factors $W_0$, $R$ and $E$ have I underestimated? Meanwhile measurements of the prod index in Table 1 support the principles, and make refutation harder.

## 8. Age growth and time growth

It was R.A. Fisher (1930), and other founders of population biology, who showed how changes in members of a population are effects of age and time separately. The measure of the effect of time, for Fisher (1930), was the ‘Malthusian parameter’ or population growth rate. Individuals meanwhile varied in ‘reproductive value’ as a function of age. Reproductive value meant number of expected future offspring, as a function of current age, net of effects of the Malthusian parameter, after factoring out risks of mortality at successive later ages. This number rose steadily from birth to sexual maturity, since each day that passed meant greater chances of maturing, and then declined as the period of remaining fertility was used up. Although TRE focuses on self rather than fertility, measured in dollars rather than reproductive value, it will draw a value/age curve somewhat like Fisher’s (1930).

Age growth is differences among cohorts of different ages at the same moment, as in a family photograph. Time-growth is differences among cohorts of a single age at different times, like a montage of baby pictures of all members of the family. Overall growth, or the combined effects of the two, is like a series of class reunion photos of the same old gang over the decades. The notation is $D_x\, H(x,t)$ for age-growth and

$D_t\ H(x,t)$ for time-growth, where $x$ is age and $t$ is time. Then overall growth is

$$\frac{\mathrm{d}}{\mathrm{d}t}\,H(x,\ t) = D_x H(x,t)\frac{\partial x}{\partial t} + D_t H(x,t) = D_x H(x,t) + D_t H(x,t), \tag{47}$$

since $\partial x/\partial t = 1$.

It is natural to define insumption as the positive component in age-growth of self, just as it is in thrift growth. Then define senescence $\delta$ (*delta*) as the negative component. That is,

$$D_x H(x,t) = \imath - \delta \quad \text{or} \quad \delta = \imath - D_x H(x,t), \tag{48}$$

which combines with (47) to give

$$\frac{\mathrm{d}}{\mathrm{d}t} H(x,t) = D_t H(x,t) + \imath - \delta. \tag{49}$$

Also

$$\frac{\mathrm{d}}{\mathrm{d}t} H(x,t) = \dot{H} = \dot{H}_\psi + \dot{H}_\theta = \dot{H}_\psi + \imath - \mu, \tag{50}$$

by (25) and (30).

We reasoned in Section 7 that thrift growth at the scale of closure is likely to be negligible. The same is true of age-growth in Fisher's (1930) model, where the Malthusian parameter is a function of time but not of age. (Fisher had assumed for simplicity that the age distribution is constant.) Economics adds the vector of rising individual productivity through innovation, but we may be wisest to suppose that free growth (rising productivity) is a tide which lifts all boats and cohorts at the same rate at any given moment. Insofar as that is so, free growth is time growth. Age growth, meanwhile, is essentially thrift growth. The young age-grow from parental gift, which they receive as neg transfer yield, and adults lose value in maint, which is final yield. Growth and decline by neg and pos yield are components of thrift growth. Growth other than through yield is self-invested work, which includes no free growth if we are comparing cohorts at a single moment and if free growth is a function of time alone. If the 'rising tide' metaphor is reasonable, so that most thrift growth is time growth and conversely, then it is possible to study data such as age-earnings profiles, and family photographs, with the zero-sum logic of the thrift output rules.

If both age-growth and thrift growth of self were ruled out at closure, then (49) and (50) would allow $\dot{H}_\psi = D_t H(x,\ t)$ and $\delta = \mu$ at that scale. But we saw that self is allowed to grow at the expense of prop, or conversely. Absent theory or evidence as to which should thrift-grow at the expense of the other, we will sometimes assume $\mu \approx \delta$ for simplicity. In general, we will sometimes assume the 'growth simplifications'

$$\dot{H}_\psi \approx D_t H(x,t), \quad \text{implying} \quad \dot{H}_\theta \approx D_t H(x,t), \quad \text{and} \quad \mu \approx \delta \tag{51}$$

for modelling. The relation $\mu \approx \delta$ will be called the 'fourth duplication rule'.

The growth principles tend to advise policy makers against inducements to save other than by waste reduction, and to count on little growth even from that. Sin taxes and luxury taxes may have a role in good government on other grounds, and sometimes help growth too. But it is time to critique laws which tax all consumption, or tax corporate dividends twice and plowback once, or tax income more than capital gains, or tax realized capital gains and not unrealized ones. (There are practical ways to tax unrealized gains without forcing sales.) Although such laws inhibit waste consumption, along with all other consumption, they probably do more harm by skewing decisions away from optimality. Since either factor might thrift-grow at the other's expense, I suppose that such laws might induce more shovels and fewer diggers (because of less maint and insumption) than in ratios which would otherwise maximize risk-adjusted return. But the larger cost is in 'capital constipation', or reluctance to sell, and incur tax events, when assets might be worth more in other hands.

## 9. Pay, sold work and maintenance (maint) preponderance

Economics as an empirical science needs comparison of theory to evidence, and market evidence is recorded in sales. Then it is useful to study how sales data can reveal the more fundamental concepts of capital and output and yield. Sales analysis must first solve the problem of imputation. Many goods and services are exchanged for considerations other than literal dollars. The unpaid work of housewives is a favorite textbook example. For uniformity of treatment, then, let us impute that all economic goods and services are sold for real or imaginary dollars except the self-invested work of learning and exercise. Then we will impute that a man pays himself a valet's salary to dress himself and brush his teeth, if these services count as maint or waste, and a handyman's salary to mow his lawn, but pays himself nothing for the work of studying French or calculus if this effort is insumption meant to pay off in higher literal or psychic earnings later, rather than to sustain pay now. We will soon see the reason for this exception.

What is sold, less the part created earlier or by others or both, is the part created currently by the seller. This tautology is the 'realized output rule'. It is one of the foundations of accounting. The cost of the part provided currently by others is 'cash expense,' regardless who created it or when, and the part liquidated from assets already on hand, including depreciation, is 'liquidated expense'. The two kinds together are 'expense,' and the residue produced by the seller is his 'realized output'. Then the realized output rule is

$$\text{realized output} = \text{sales} - \text{expense}. \qquad (52)$$

TRE applies this tautology to both factors, and defines literal or imputed 'pay' as the sales of worker services. Expense is defined in principle as cost recovered in sales, and we saw that the cost recovered in pay is maint plus any 'non-yield expense,' such as bus fare to work, which the owner would not have borne willingly except to enable the pay. TRE prefers to define 'pure pay' $\sigma_H$ ($\sigma$ is *sigma*) as pay less any non-yield expense, and to write the realized output rule for self as the 'sold work rule'

$$w_\sigma = \sigma_H - \mu, \tag{53}$$

where $w_\sigma$ is 'sold work'. We say 'sold' rather than 'realized' in this case where the longer word is not established by tradition.

One practical reason for excluding insumed work from services earning imputed pay is to clarify the limits of maint. By the sold work rule, maint is limited by pay and deducted from pay. The cost of insumption cannot be spent partly on maint, just as a dollar of investment cannot include a share spent on expense. Consumption is either or neither, but not both. The exclusion is justified, and will lead to enrichments of theory in such forms as the maint preponderance rule (*infra*).

But no other work services are excluded. Note carefully that sold work, despite its name, includes the unpaid work of all owners during the hours when they attend to their needs, and one another's, without cash being spent. Some of these services count as maint for themselves or for others, some as prop investment, and some such as parenting count as insumption for others. Sold work, in the TRE sense, really means all 'external work', or work not directly and simultaneously insumed. Note however that it cannot include waste, which must be exhausted from prop according to (10).

All work including sold work reaches zero when self does at the end of life. If the curve of self as a function of age is continuous to final age $\lambda$, then, the 'last day rules' are

$$H(\lambda) = w(\lambda) = w_\sigma(\lambda) = 0 \quad \text{and} \quad \sigma_H(\lambda) = \mu(\lambda), \tag{54}$$

by the sold work rule. Thus pure pay converges to maint at the end. Although this inference is intuitively obvious, and is reflected in age-earnings profiles, we saw that it contradicts the traditional doctrine that all pay is recompense for sold work. We will soon go further, and show that less than half is such on average.

Realized output is generally not the whole of output, since some, including insumed work, may be held back as growth in self or inventories or fixed capital. Growth in general is unrealized output plus plowback from sales plus neg yield less pos yield. Thus some realized output contributes to growth through plowback. This is also true of self, in a sense, since a worker might spend part of his pay on tuition. But it seems most natural to account for the plowback as reciprocal transfer yields; the worker buys the tuition from his pos work yield, and simultaneously converts its cost into self through an equal flow of neg work yield. The point here is that his pay was property, and that it bought other property in the form of the right to attend class. Thus there were transfers from sold work to prop investment and back to self. This accounting interpretation simplifies the equations of TRE by obviating a separate symbol for plowback insumption. The same interpretation would be compelled if the worker had bought a textbook rather than tuition, since the textbook is clearly property whose value is converted into self over time. Let us therefore write the ‘insumption rule’ as

$$\iota = \iota_w + \tau_{H-} = \iota_w + \eta_{H-}, \tag{55}$$

meaning that each owner’s insumption is the sum of his work insumption and his neg work yield.

By the same reasoning, sold work equals the worker’s pos transfer yield. His sold work is his pure pay less his maint. This residue (sold work) is the sole source of his pos work transfer yield in prop investment or gift, and has no other possible disposition. Consequently,

$$w_\sigma = \tau_{H+} \tag{56}$$

for each owner. Sold work or pos work transfer yield may also be called ‘discretionary pay,’ to save words and to highlight the fact that pay depends on the maint remainder. Now by (29) and (55) and (56), and by the fact $\tau_H = \tau_{H+} - \tau_{H-}$, note

$$w_\theta = \iota + \tau_H = \iota_w + \tau_{H-} + \tau_H = \iota_w + \tau_{H+} = \iota_w + w_\sigma. \tag{57}$$

This may be read as ‘thrift work equals insumed work plus sold work’. (57) is the logical outcome of our decision to exclude the former from the latter.

We are ready to consider the general proportions of maint and sold work as components of pure pay. We reasoned in the growth principles that thrift growth is likely to be negligible at closure. By (20), then, and by the fact that all yield is final yield at closure, we see $y_\theta \approx \mu + \upsilon$ at closure. Thus the fact $y_\theta = w_\theta + p_\theta$ allows

$$\mu + \upsilon \approx \mathrm{w}_\theta + \mathrm{p}_\theta, \quad \text{or} \quad \mu \approx \mathrm{w}_\theta + \mathrm{p}_\theta - \upsilon, \tag{58}$$

at closure. Substitute (57) in this to get the ‘maint preponderance rule’

$$\mu \approx \mathrm{w}_\sigma + \iota_w + \mathrm{p}_\theta - \upsilon, \tag{59}$$

showing that maint, at the scale of closure, tends to exceed sold work by the whole of insumed work and unwasted thrift profit combined.

By (53), (55) and (14), verify

$$\sigma_H = \mu + w_\sigma = \mu + \tau_{H+} = \eta_{H+}, \quad \text{or} \quad \eta_H = \sigma_H - \eta_{H-} \tag{60}$$

showing that work yield, for any worker, equals his pure pay less his neg work yield. (60) is useful in modelling the value of self as a function of age. Neg work yield, by (55), is insumption other than insumed work. It is insumed gift from others, typically parental gift received by the young, plus any insumption from the owner’s prop yield. Adults collectively receive no gift, at the scale of closure, and I will argue that adults insume little from prop yield. Adult insumption is substantially the insumed work of on-the-job training and experience. For adults, then, $\eta_{H-}$ should prove negligible, so that work yield should approximate pure pay. Meanwhile there are abundant data for pay as a function of age. If pure pay can be estimated from data, then, it is possible to model self as a function of age from (5) and (60). We will do so soon.

## 10. Pay equilibrium

David Ricardo (1817) may have been the first economist to see that workers are effectively commodities whose price (pay) adapts supply to meet demand. Oversupply means that pay is bid down toward the maint level, which is the threshold sufficient for job performance but not for reproduction, while undersupply drives pay upward toward the employer's alternative cost of sustaining idle plant and machinery. Several points emerge from this powerful insight. It is clear that self and prop compete in the marketplace. Owners, who can be trusted to maximize risk-adjusted rate of return, will bid up self when it performs better by that standard than prop does, and conversely. This dynamic should tend to equalize risk-adjusted rate of return between the factors, as required from the reasoning in Section 4.

Ricardo's (1817) insight can also help explain the typical skimpiness of inheritance. Most owners who inherit prop must work too. These benefit from more income than workers starting with nothing, and can therefore afford to survive and reproduce on lower pay. Consequently they will be low bidders when jobs are scarce and the alternative is no pay at all. At an extreme, they will be the only bidders able to survive on pay below the maint threshold. Prop inheritance for some therefore tends to lower pay for all. Among inheritors, moreover, those able to survive and reproduce on least pay are those willing to invade principal by depleting the inheritance over their lifetimes. Then the dynamic of pay equilibrium should tend to deplete prop inheritance as it arises.

Pay equilibrium analysis also suggests constraints on the cost-effectiveness of social overhead capital and social transfer payments. Social overhead capital, in forms such as streets and public buildings and bridges and harbors and parks, is a social form of inheritance. Like the individual form, it lowers the threshold costs for maint and reproduction and consequently the lowest bids for pay. Social transfer payments move prop to the neediest, unlike social overhead capital which spreads it among all, or prop inheritance which aims it at the chosen beneficiaries of donors. But whoever is more benefited becomes a lower bidder for pay, since he now needs less for maint and reproduction. In either case, as with individual prop inheritance, equilibrium pay tends to fall as social overhead capital and social transfer payments rise.

This line of thought suggests that most workers, regardless how laws are written, will reach maturity with little property. Other arguments point toward the same conclusion. The fact of generational survival indicates that adults invest in the young. In order to invest cost-effectively, they should seek to maximize the ratio of self to prop of donees at maturity. The reason is that a fool and his money are soon parted. Those with highest skills will gain in prop, and conversely. Thus the donor who best converts his means of giving into self of donees at maturity will most enrich the next generation. But investment in self requires selectiveness. Although any amount of prop may be given to any donee, until market triage determines the amount he is capable of holding, individual talent determines the potential limit of self. Optimum education and guidance cost so much per pupil, and no more, before the next increment is better spent on someone else. It follows that a large fortune, in order to be invested efficiently in the next generation, must be invested in the self of many. If biology drives creatures to invest nepotistically, as seems likely, then the wealthiest donors must find what investment efficiencies they can within the circle of kin.

Both theory and evidence, then, suggest that most owners reach maturity with little prop and inherit little at any age. But evidence suggests that they acquire prop, and then divest it, over adulthood. This acquired amount may be called 'interim prop', producing 'interim profit', over the adult span. The defining feature of interim profit, under the assumption of pay equilibrium, is that it affords no waste and no inheritance. The point here is that the pay from which it is drawn is enough for equilibrium between the work force and the job market and no more. Under market efficiency, then, we would expect most waste to be drawn from 'inherited prop' rather than from interim prop.

Some owners acquire or inherit enough for independence, and need not compete for pay. These are the few that enable social transfer payments and waste. Most owners, however, are subject to the constraints of pay equilibrium. One such constraint, we saw, is the tendency to own negligible prop at maturity, and again at the end of life. This can be notated

$$K(M) \approx K(\lambda) \approx 0, \quad \text{implying} \quad J(M) \approx H(M) \quad \text{and} \quad J(\lambda) \approx H(\lambda) \approx 0, \tag{61}$$

where $M$ is age of maturity, meaning independence from parental gift, and $\lambda$ again is age at death.

The main Ricardian idea is that pure pay equilibrates to the level just sufficient for maint and generational replacement to meet market demand. Since pure pay is maint plus sold work, that would mean that all sold work (discretionary pay) is invested directly or indirectly in the young. Although some may be invested in interim prop, and some possibly in adult insumption, all investment including earnings must be yielded back in parental gift by the end. Thus interim prop is tantamount to a trust fund for the education and nurture of the young, growing through retained earnings until divested and converted into self of donees. Likewise eventual gains in sold work due to earlier adult insumption must be invested in the trust fund or in the young directly. Meanwhile any part of discretionary pay invested immediately in the young also produces a return in the sense that the young supplement it with their own insumed work, as crops from seed and fertilizer, so that growth of the young is an exponential function of parental gift received. Both the trust fund and the young themselves, that is, grow exponentially from the shares of discretionary pay allotted to them. The rate of growth is the productive rate of capital, meaning the rate of return. In the case of the young, the equation is

$$H(x) = \int_{x_0}^{x} \gamma_{-}(z)\, \mathrm{e}^{\bar{\gamma}_z{}^{(x-t)}}\, \mathrm{d}z, \quad x \leq M, \tag{62}$$

where $H(x)$ is self of a donee at age $x$, $x_0$ is the age of first parental gift (say at ovulation), $\gamma_{-}(x)$ is total parental gift received by the donee from all donors, and $z$ is a dummy variable of age. $\overline{r_z}$ in this case is the future average value of $r$ between ages $z$ and $x$. The meaning is that the young augment their endowment with insumed work equal to their entire thrift work $H(x)\, r_\theta$, and also grow at the free growth rate, so that overall growth is $\gamma_- + H(x)r$. (62) assumes no losses in self due to maint, and is therefore valid only to the age where pay begins. This illustrates what is meant by the statement that parental gift, like investment in interim prop, can be treated in the equilibrium pay model as direct or indirect current investment in the next generation compounding at rate $r$ over the average period of investment.

For easiest math, consider all cohorts simultaneously under the growth simplifications, so that thrift growth alone explains differences between cohorts. Also suppose that all investments in prop, in self of adults, and in self of the young thrift-grow at the same rate $r_\theta$. The pay equilibrium idea is that all such investment funnels through to insumption by the young at the end. That is,

$$G \int_M^\lambda w_\sigma(x)\,\mathrm{d}x = \int_{x_0}^M \imath(x)\,\mathrm{d}x, \tag{63}$$

where 'growth factor' $G$ is the effect of compound growth at rate $r_\theta$ between the moment of sold work invested and the moment of maturity of the young at age $M$.

$G$, in turn, can be clarified if we know the maximum period of compounding. This period begins when the parental generation reaches age $M$, even if their young will not be born for years to come. It cannot begin earlier, since that generation has previously invested only in itself through insumed work, and cannot begin later under the assumption of pay equilibrium. It continues until the next generation reaches independence. Thus the maximum period of investment is the period between maturities of successive generations. This equals the 'generation length' or 'mean age of reproduction' $T$, the average period between births of successive generations, much as the repeated ceiling-to-ceiling distance in a skyscraper will equal the floor-to-floor distance. $T$ is average age of both parents over all births, including later births as well as first ones.

We will see that the adult span $\lambda - M$ in America today is about twice the generation length $T$, and that the span from ovulation to maturity $M - x_0$ is about three fourths of $T$. Thus the period of investment in the young means different things from different perspectives. At the household scale, for example, the adult span is folded over to show roughly two adult generations, typically parents and grandparents, investing ultimately or directly in a single generation of young over period $T$. This overlapping of donor generations makes it defensible to suppose for simplicity that aggregate sold work of the household is age-independent, even though we will see that sold work varies strongly over the adult span. Under that assumption of constancy,

$$G = \frac{\sum w_\sigma \int_0^T \mathrm{e}^{r_\theta (T-x)}\,\mathrm{d}x}{T \sum w_\sigma} = \frac{\int_0^T \mathrm{e}^{r_\theta x}\,\mathrm{d}x}{T} = \frac{\mathrm{e}^{r_\theta T} - 1}{r_\theta T}, \tag{64}$$

where $\Sigma w_\sigma$ is aggregate sold work of the household including grandparents.

By (32), also, the right-hand side of (63) equals $H(M)$, under the growth simplifications, if we suppose that sold work, and hence maint, are absent before age $M$. This allows combination of (63) and (64) in

$$\frac{\lambda - M}{T}\left(\frac{\mathrm{e}^{r_\theta T} - 1}{r_\theta}\right)\overline{w}_\sigma = H(M), \quad \text{where } \overline{w_\sigma} = \frac{1}{\lambda - M}\int_M^\lambda w_\sigma(x)\,\mathrm{d}x. \tag{65}$$

The second equation clarifies $\overline{w}_\sigma$ as average sold work over the adult span. The meaning of (65) is that the aggregate sold work of an adult over that span, compounding at rate $r_\theta$ over a period of $T/2$ on average, is just enough to bring one successor to maturity. This does not rule out population growth, since all cohorts might grow at a common rate as in Fisher's (1930) model.

Further define

$$\imath_{\text{young}} = \int_{x_0^M} \imath(x) = \frac{H(M)}{\mathrm{d}x}, \text{ and note } \quad w_\sigma = \int_M^\lambda w_\sigma(x) \tag{66}$$

at closure, if we continue to assume no maint or sold work before age $M$. These definitions allow restatement of (63) and (65) as the 'pay equilibrium rule'

$$Gw_\sigma\,\mathrm{d}x = \imath_{\text{young}}\,\mathrm{d}x = H(M). \tag{67}$$

The meaning is that the aggregate sold work of all adult cohorts, multiplied by the growth factor, equals aggregate insumption by the young, and equivalently produces self equal to $H(M)$ over each differential instant $dx$. (Since the number of cohorts is $\lambda/\mathrm{d}x$, each cohort value including $H(M)$ is a differential.)

Meanwhile these reflections on Ricardo's (1817) idea may inform estate tax policy. Wise governments enact taxes which are perceived as fair, if only to lessen the cost of enforcement, and which can be collected efficiently. Large estates enable scale economies in collection, and wealthy testators may accede to a policy which preferentially steers their wealth toward the many, and particularly toward the self of many young, rather than toward the prop holdings of a chosen few who presumably have already earned the amounts to which their talents and stations in life befit them.

## 11. A generational model of return

Overlapping generations theory has accustomed economists to the idea that the rate of return can be modelled from vital statistics. TRE follows this precedent, although with radically different premises and conclusions. Getty (2001) offers several such models, each deriving thrift return of average-risk assets at several percent per year. Let us try a new one.

Models need simplifying assumptions, preferably as few and mild as possible. We will again assume the growth simplifications, under which all age growth is thrift growth and all time growth is free growth, so that age-correlated differences between cohorts at a single instant are explained by thrift growth alone. We will also assume pay equilibrium as described in Section 10, and other simplifications as we go along.

Economic life begins with the parental investments of ovulation and gestation, at one year before birth after rounding. The young then grow by parental gift and insumed work, as described in (62), through the period of dependence on parents. We will assume for simplicity that parental gift ends when pay begins. Judging from data, this shift centers at about age 20. Then the period of dependence lasts 21 years including the year before birth. We will suppose that life continues another 58 years, ending at age 78, and that workers are productive to the end. Retirement relocates sold work from the shop to the home, where retirees earn their keep in literal or imputed pay by caring for themselves and one another.

Age-earnings profiles show that pay rises into the early fifties before declining (e.g. Table 2). This is proof of adult insumption to that age at a minimum, since it reflects rising skills, and since skill development is insumption. It may continue longer, since erosion of old skills overall does not preclude simultaneous acquisition of new ones. But it seems realistic to say that once schooling ends and full-time work begins, nearly all skill development is practical experience rather than book learning. And practical experience is insumed work. Thus we will assume that $\tau_{H-}$ is small for adults, implying $\eta_H \approx \sigma_H$ by (60). We will also neglect non-maint expense, since it seems awkward to gauge and intuitively minor. By this simplification and the last, adult work yield is equal to pay. Since in fact there is some neg work yield and non-maint expense, work yield of adults is somewhat overstated on these grounds alone. Pay, on the other hand, is understated in the market data from which we will model because it excludes most imputed pay. In any case, it turns out that our main findings will depend on the shape of the age/earnings profile, rather than on absolute dollar amounts.

It is likely that entry-level pay equals maint only, since the new worker is largely an apprentice working to learn. His starting pay may be even less than his needs if he is supported by parents or others as a continuing cost of education. By the sold work rule, then, all his work is initially insumed work. The ratio of sold work gradually

*Table 2.* Pay/age date for full-time workers in 1999

| Age | Males ($) | Females ($) | Average[a] ($) |
|---|---|---|---|
| 18–24 years | 13 215 | 10 590 | 11 902 |
| 25–34 years | 35 282 | 23 872 | 29 577 |
| 35–44 years | 46 000 | 26 632 | 36 316 |
| 45–54 years | 53 411 | 28 683 | 41 047 |
| 55–64 years | 49 646 | 25 876 | 37 761 |
| 65 years and older | 31 910 | 13 739 | 22 825 |
| All ages | 40 257 | 23 551 | 31 904 |

[a] Unweighted.

rises thereafter. Since the young worker learns easily, but has little experience in prop management, he invests preferentially in self. This preference reverses as his prop skills rise and his remaining lifespan for payout from further learning shrinks.

Now let us look at a recent age-earnings profile. Data in Table 2 are reproduced or derived from Table 677 of the *Statistical Abstract of the United States: 2001* (SAUS 01).

I have assumed equal numbers of male and female workers, since those not working in the office are assumed to be working as productively at home. Pure pay in the TRE sense is surely higher, since figures in Table 2 include nothing for imputed pay. However, let us take them at face value. Since the midpoint of the youngest bracket is 21 years, say that the pay of 20-year-olds averages \$11 000. And since the midpoints of the next four brackets are 30, 40, 50 and 60 years, suppose that those cohorts receive the average pay for those brackets. Suppose next that 70-year-olds earn \$23 000, rounded upwards from the average of \$22 825 for all workers over 65. Since expected age of death increases with age, from 78 at age 20 to 80 at age 50, and so forth, let us finally provide for an 80-year-old cohort earning \$22 000. This provision will enable valuation of middle-aged cohorts without much affecting valuation of younger ones.

These data and estimates, with the simplifying assumption $\eta_H \approx \sigma_H$, allow valuation of self as a function of adult age by (5). The equation in this case becomes

$$H(x) \approx \int_x^\lambda \sigma_H(x)\, \mathrm{e}^{-\gamma_\theta^{(z-M)}}\, \mathrm{d}z, \quad x \geq M, \tag{68}$$

where $z$ again is a dummy variable of age. Note that we discount by $r_\theta$, not by $r$, since we are comparing pay of different cohorts at a single instant in time under the growth simplifications. Table 3 shows results under a range of assumptions for $r_\theta$.

Values past age 60 are omitted because of uncertainty of pay/age data after age 65. Note that values are higher at age 30 than at 20 for all values of $r_\theta > 0.02$, showing that adult insumption exceeds maint through that age.

$H$ (20) can also be derived from (65) if the generation length $T$ is known. Table 70 in SAUS 01, with other sources, shows data from which $T$ can be gauged at 28 years in the United States today (see Getty 2001). Then we can set $\lambda = 78$, $M = 20$ and

*Table 3*. Self/age profile for adults (in thousands)

| $r_\theta$ | 0.02 (\$) | 0.025 (\$) | 0.03 (\$) | 0.035 (\$) | 0.04 (\$) |
|---|---|---|---|---|---|
| $H(20)^a$ | 1070 | 947 | 840 | 750 | 672 |
| $H(30)^b$ | 1045 | 976 | 890 | 816 | 754 |
| $H(40)^a$ | 905 | 871 | 812 | 756 | 708 |
| $H(50)^b$ | 729 | 691 | 656 | 624 | 594 |
| $H(60)^a$ | 426 | 409 | 393 | 378 | 364 |

[a] By Simpson's rule.

[b] By the trapezoidal rule.

*Table 4.* $H(20)$ derived from equation (65) (in thousands)

| | | | $H(20)$: $r_\theta$ ($) | | | | |
|---|---|---|---|---|---|---|---|
| $\bar{\mu}$ ($) | $\bar{w}_\sigma$[a] ($) | $H(20)_{\max}$[b] ($) | 0.02 | 0.025 | 0.03 | 0.035 | 0.04 |
| 16 | 16 | 928 | 1243 | 1334 | 1454 | 1576 | 1711 |
| 17 | 15 | 986 | 1165 | 1260 | 1363 | 1478 | 1604 |
| 18 | 14 | 1044 | 1087 | 1176 | 1272 | 1379 | 1497 |
| 19 | 13 | 1102 | 1010 | 1092 | 1182 | 1281 | 1390 |
| 20 | 12 | 1160 | 932 | 1008 | 1091 | 1182 | 1283 |
| 21 | 11 | 1218 | 845 | 924[c] | 1000 | 1084 | 1176 |
| 22 | 10 | 1276 | 777 | 840 | 909 | 985 | 1069 |
| 23 | 9 | 1334 | 699 | 756 | 818[c] | 887 | 962 |
| 24 | 8 | 1392 | 622 | 672 | 727 | 788[c] | 855 |
| 25 | 7 | 1450 | 544 | 588 | 636 | 690 | 748 |
| 26 | 6 | 1508 | 466 | 504 | 545 | 591 | 641[c] |

[a] 32 minus $\bar{\mu}$.

[b] 58 times $\bar{\mu}$.

[c] Best fit with Table 3.

$T = 28$ in (65). Meanwhile average sold work $\overline{w}_\sigma$, by the sold work rule, is average pure pay less average maint. Average pure pay for all ages, if pay equals pure pay on average, is shown in Table 2 at \$31 904. Let us round this to \$32 000. By the maint preponderance rule, maint cannot be less than half of that. Table 4 therefore evaluates $H$ (20) from equation (65) under assumptions of $\overline{w}_\sigma$ not greater than \$116 000. For comparison, Table 4 also shows the maximum possible value of $H$ (20) as 58 times average maint $\bar{\mu}$. The rationale here is the third duplication rule (32), which shows that self at any age, in the absence of free growth, equals aggregate expected maint less insumption over the remaining lifespan. It therefore cannot exceed expected aggregate maint over that span.

Best fits with calculations of $H(20)$ shown in Table 3 are marked, and show that higher values of $r_\theta$ fit when maint is also higher. Note that all shown values of $\bar{\mu}$ less than \$119 000 are ruled out in Table 4, since all corresponding values of $H(20)$ exceed their maxima. Since $H(20)$ varies inversely as $r_\theta$ in Table 3, although directly in Table 4, and since values shown for $H(20)$ exceed their maxima in both tables when $r_\theta = 0.02$, $\bar{\mu}$ of \$18 000 or less is ruled out at all values of $r_\theta$. For the same reason, values of $r_\theta$ of 0.02 or less are ruled out at all values of $\bar{\mu}$.

We could say more if we knew more about the curve of maint over age. How much do we know? Earlier we defined an individual's 'vital consumption' loosely as whatever he needs to survive and work. It is typically a part of his insumption when he is young, and typically equals his maint after maturity. But maint cannot exceed pay, as we saw, and is therefore less than vital consumption when pay is less. That is possible if some adult cohorts are subsidized by other ones, or by their own prop yield.

Youngest adult cohorts may benefit from continuing parental gift, for example, and oldest ones from social security. Thus it is possible that some youngest and oldest adults are selling work services whose market value would not support them independently.

Milton Friedman (1957) has proposed that consumption is roughly constant over the adult span, so that individuals save until retirement and dissave after. Meanwhile the last day rules (54) show that pure pay converges to maint at the end of life. If Friedman (1957) is right, then, and if maint approximates vital consumption on average over all cohorts, after netting out subsidies, average maint should not be much less than pure pay of oldest cohorts, and might be somewhat higher. Pure pay of the 80-year-old cohort, and equivalently its maint, was modelled at \$22 000. Then if average maint for all adult cohorts is estimated at something between \$21 000 and \$24 000, from Friedman’s (1957) idea, best fits occur when $r_\theta$ is something between 0.025 and 0.035.

The third duplication rule allows a check. We reasoned that adult insumption ought to decline with age. Suppose that there is a cutoff age, so that all thrift work is sold work thereafter. Then pure pay less maint would give the whole of thrift work, rather than the sold part only, and cumulative future maint would give self by the third duplication rule. If these algorithms were applied before cutoff, however, they would understate thrift output and overstate self, and thus doubly understate $r_\theta$. We therefore write

$$r_{\theta,\min} = \frac{w_\theta(x)_{\min}}{H(x)_{\max}} \approx \frac{\sigma_H(x)}{\bar{\mu}(\lambda(x) - x)}, \tag{69}$$

where $\lambda(x) - x$ is expected remaining lifespan at age $x$, and where $\bar{\mu}$ in this case is average expected maint over that period only.

The oldest cohort for which we have reasonably clear pay data is that of 60-year-olds (Table 2), for whom we have interpolated pay and pure pay at \$37 800 after rounding. Table 5 calculates $r_{\theta,\min}$ under a range of assumptions for average expected maint $\bar{\mu}$ over the years left. Expected remaining life at age 60 for Americans of both

*Table 5.* $r_{\theta,\min}$ for 60-year-olds by equation (43)

| $\bar{\mu}$ (\$) | $w_\theta$ (60)[a] (\$) | $H(60)$[b] (\$) | $r_{\theta,\min}$[c] |
|---|---|---|---|
| 20 000 | 17 800 | 430 000 | 0.041 |
| 21 000 | 16 800 | 452 000 | 0.037 |
| 22 000 | 15 800 | 473 000 | 0.033 |
| 23 000 | 14 800 | 495 000 | 0.030 |
| 24 000 | 13 800 | 516 000 | 0.027 |

[a] \$37 800$-\bar{\mu}$.

[b] $21.5\bar{\mu}$.

[c] $w_\theta(60)/H(60)$.

sexes and all races is 21.5 years. (All life expectancy statistics in this study are derived from SAUS 01, Table No. 98.)

The likeliest range of $\bar{\mu}$ for cohorts aged 60 or more, consistent with data and assumptions and Friedman's (1957) idea, is \$21 000–\$23 000. Corresponding values for $r_{\theta,\min}$ range from 0.037 to 0.03. The fact that all shown values for $H(60)$ exceed the corresponding ones in Table 3 can be explained as noise. Note for example that remaining life at age 60 is 20 years in Table 3, but 21.5 years in Table 5, and that values for $H(60)$ would be closer if these periods agreed. But it is also possible all that unsold work continues past age 60. $r_\theta$, in that case, would exceed $r_{\theta,\min}$. All tables and equations might fit if $r_\theta$ were 0.035, for example, where maint averages \$24 000 over all adult ages, but a little under \$22 000 after age 60, or if $r_\theta$ were 0.03 and maint averages \$23 000 over each period.

Then best fits across the three tables point to a range of 0.03 to 0.035 for $r_\theta$, and a range of \$21 000–\$23 000 for average maint. The obvious cautions should be noted. All stocks and flows are probably too low, since pure pay in the TRE sense includes much unpaid work not reflected in Table 2. But rates, such as rate of thrift return, need not be distorted if the shape of the pay/age curve is right. If all data in that table are wrong in like proportion, $r_\theta$ will no more be affected than if values were given in dinars rather than dollars.

There is plenty of room for further refinement. I did not discount future expected yield (pure pay) by the probability of reaching successive ages of such yield in Table 3, since pay/age data did not seem complete enough to warrant so much fuss. Values would have been somewhat less for younger adult cohorts, and more for older ones, if I had. And I did not provide for the likelihood that cohorts may differ in risk, and hence vary in $r_\theta$ as a function of age. The object is a first approximation only.

## 12. Modelling the factor index

Tradition models the ratio of self (human capital) to total capital, which TRE calls the 'factor index,' as the ratio of pay to income, and finds two thirds to three fourths as a typical value. TRE sadly invalidates that simple algorithm, since pay includes maint as well as sold work, but excludes unsold work including free growth of self. Thus tradition is right only if these errors offset.

The equilibrium pay model allows an independent approach. Table 3 models $H(x)$ as a function of adult age. Then it should be possible to find average $H(x)$ over all ages, under simplifying assumptions, and compare with data for average prop ownership. Since values in Table 3 are capitalized from pay/age data in Table 2, which miss unpaid work at home and are consequently too low, projections for the factor index by this method can probably be taken as minima.

Let us again prefer crude and straightforward methods. The equilibrium pay model suggests that average-risk $r_\theta$ lies in the range (0.03, 0.035). Assume the values for $H(x)$ shown in those columns in Table 3, and also set $H(0) = 0$ and $H(80) = 0$. This gives curves for $H(x)$ over the entire lifespan under the assumptions $r_\theta = 0.03$

and $r_\theta = 0.035$. Solution for average $H(x)$ for all years by Simpson's rule gives $\overline{H(x)} = \$626\,000$ if $r_\theta = 0.03$, and $\overline{H(x)} = \$579\,000$ if $r_\theta = 0.035$. As before, I have neglected mortality between ages 0 and 80.

Meanwhile SAUS 01, Table 689, shows aggregate net worth of the household and non-profit sectors at about \$42 trillion in 1999. Round this up to \$50 trillion to allow for social overhead capital, and divide by the 1999 population number of 280 million to get average prop per man, woman and child at about \$179 000. That would bring average total capital per individual (self plus prop) to \$805 000 and \$758 000, respectively, under the assumptions $r_\theta = 0.03$ and $r_\theta = 0.035$, so that the factor index would come to 78% and 76%, respectively. To repeat, these estimates are probably low unless I have undervalued prop more than self. By first approximation, anyhow, the factor index seems to fall near or above the top of the range expected by tradition.

## 13. A path not followed

TRE is a hard slog, both for its math-density and for its high ratio of inference to evidence. In some ways, its novelties of approach may make it tougher reading for economists than for laymen uninvested in tradition. Part of my duty is to help critics pick holes in it. The centerpiece of TRE, and the natural focus of criticism, is the duplication rules. Section 5 began with a review of the argument for them. It seems to me that the conclusion cannot be escaped if we accept that maint is final yield as well as expense. But logic does not force us to do so. We could claim self-consistently that the taste satisfactions from maint are free goods concomitant with the expense, just as some taste satisfactions are free goods concomitant with insumption. Since free goods are excluded from final yield, that would mean that maint has expense value alone, and no final yield value.

Such an interpretation would take us from the frying pan to the fire. If maint is not final yield, it becomes impractical to recognize self as a form of capital. The problem is the exhaust principle, and the fact that self stands at the end of the value-added chain. These combine to show that the liquidation of self in senescence cannot be transferred to a later link, as wheat to flour to bread to self, and must be exhausted in equal final yield alone. Without maint, there is no source for this final yield. Senescence satisfies no tastes *per se*, except to ascetics yearning for the next world, and therefore cannot find any compensating final yield because of its position at the end of the chain. Maint, if recognized as final yield, evades this dilemma because it is not a product of the chain. It is an expense used up internally in providing the true product flows of insumption and waste.

If I had excluded self from capital, and maint from final yield, TRE might scarcely have been more shocking than it is. But it would have been much less informative. I chose as I did because the aim of economics is to predict behavior. Owners behave as if maint were an economic end in itself, and provide insumption for the young and themselves as if investing in something of productive value. Thus I derived the duplication rules, and with them the growth principles and simplifications, and the generational models of thrift return and of the factor index. Along the way we found the maint preponderance rule, and the pay equilibrium model developed from insights of Ricardo (1817).

Many of these rules and models are testable, at least potentially. The growth principles are tested from data shown in Table 1, where the prod index for large-cap equities is calculated at about 0.95. Also the thrift return model can be tested against data for return to the business sector, cap-weighting debt and equity claims on that sector, or to the housing sector if apartment rentals are chosen as the guide.

The path avoided would have led to none of these inferences. It would have led to paradox. No self would imply no output by self, meaning work, and thus no sold work or insumed work. But evidence that pay exceeds work expense (maint plus non-yield expense) implies sold work, under market efficiency, and age-earnings profiles imply adult insumption whose main apparent source is insumed work. Self is real, and maint is its true liquidation in final yield as well as its expense recovered in product prices.

## 14. Conclusions

TRE is a structure of tautologies which aims to wring the most inference from the least assumption and measurement. Its chief surprise is the duplication rules, loosely equating maint with senescence. Its practical contributions include the growth principles, which support the teaching of Robert Solow (1969) that growth at the scale of closure is wholly productivity growth, and the pay equilibrium model predicting the social rate of thrift return from vital statistics and the shape of pay-earnings profiles. The former warns policy makers that inducements to save are unlikely to help growth, and the second can guide investment decisions. 3% or 3.5% per year, plus the expected free growth rate, looks to be the appropriate discount rate for capitalizing average-risk disinflated future yields.

Since this rate is inferred from no data other than vital statistics and the shape of the age-earnings profiles, as just noted, it is inferred for all times and places where such data are the same. Insofar as that shape is governed by biology, then, thrift return is a function of biology alone. Note that it is not a function of the factor index, unless that index affects the shape of the pay-earnings profile. This rooting in biology could explain why interest rates are recorded in the range of several percent per year, rather than per second or per millennium, in most observations over history.

## Appendix. Algorithms for disinflation and interpolation

Inflation is deterioration in the value of the nominal dollar, and therefore creates a kind of spurious growth. The defining equation is $g_{\text{nom}} = g + i$, where $g_{\text{nom}}$ is nominal growth rate and i is inflation rate. We must first distinguish current dollar inflation from constant dollar inflation. Even in dollars of the current instant, the flow of nominal growth includes this spurious growth. Capital and yield, however, need no disinflation if they are expressed in current dollars. If we express values in terms of a past constant dollar, however, then we must further disinflate all stocks and flows, but not rates. Therefore $r = g + \varepsilon = g_{\mathrm{nom}} - i + \varepsilon = r_{\mathrm{nom}} - i$, whether expressed in current or constant dollars. Table A-20 in SBBI 2001 finds real $r$ for large-caps by this method, allowing for differences in accounting. Meanwhile Tables A-3 and A-15 show $g_{\mathrm{nom}}$ and $i$ for large caps, so that we can find real g by subtracting values in A-15 from values in A-3. That is the method used in my own Table 1.

Table 1 also finds the stasis return rate $r_0$ by linear interpolation. Linear interpolation means

$$f(x) = f(x_a) + \frac{x - x_a}{x_b - x_a}[f(x_b) - f(x_a)], \quad \text{where } x_a \leq x \leq x_b. \tag{70}$$

Therefore

$$f(x) = 0 \quad \text{when } f(x_a) = -\frac{x - x_a}{x_b - x_a}[f(x_b) - f(x_a)]. \tag{71}$$

Let $x$ mean time, and substitute growth rate $g$ for $f(x)$. Then $g = 0$ when

$$g_a = \frac{t - t_a}{t_b - t_a}(g_b - g_a). \tag{72}$$

Now suppose that rate of return $r$ also varies linearly over time in the interval $(t_a, t_b)$. That would mean

$$\frac{t - t_a}{t_b - t_a} = \frac{r - r_a}{r_b - r_a}, \tag{73}$$

implying

$$g_a = -\frac{r - r_a}{r_b - r_a}(g_b - g_a) \tag{74}$$

at the moment when $g$ is zero. Let $r_0$ be the value of $r$ at that moment. If we further assume that $g_a$ is always negative, we can express the last result as

$$\mathrm{abs}(g_a) = \frac{r_0 - r_a}{r_b - r_a}[g_b + \mathrm{abs}(g_a)]. \tag{75}$$

This can be rearranged to give (38).

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