The Determination of Shear Modulus of Tonewood

Shear moduli are normally difficult for the amateur to measure. They are not of primary importance to the instrument maker but may become so in the future. Shear deformation is important in some vibration modes and increasingly at high frequencies in other modes. They are required for Finite Element calculations. However, there is a simple method of measurement that can be used if high precision is not required. It uses a torsion pendulum made of parts relatively easily obtained. The determination of shear modulus depends on a knowledge of the oscillating inertia which is made to be much larger than that of the specimen so that oscillating frequency is about one per second. The specimen must be accurately made and since it forms the slender suspension, the lateral dimension appears in the 4th power thus magnifying errors. The frequency of oscillation is low enough for the swings to be counted and timed with a stop clock.

Pendulum Setup

A stout rod fixed vertically to a firm base forms the framework for the pendulum. Laboratory clamps enable the parts to be assembled. A small tap wrench forms the top support for the specimen; if a cylindrical rod, directly; if a wire, via a pin vice; if rectangular in cross section a small aluminium adapter piece is used so that the axis of the specimen coincides with the axis of the apparatus. Another tap wrench is clamped to the lower end of the specimen, having determined its centre of balance and using spacers for adjustment. The lower edges of the sample are bevelled to fit the tap wrench jaws. The ends of this tap wrench are machined to take the inertia weights which are an equal distance from the specimen axis. In this study the specimens are wood samples so below the torsion bar another clamp holds a cork through which passes a needle positioned on the pendulum axis. This axis is determined with a plumbob, the base having been levelled. A machinists scriber is used to indent the specimen end to coincide with the axis and the needle is used to align the pendulum on the instrument axis and is then partially withdrawn during operation. The alignment is necessary and allows the specimen top clamping to be properly adjusted. All this setting up is done before the inertia weights are added. During operation the needle at the bottom provides lateral stability without any significant damping. A card on which is marked a scale, is supported on the handle attached to the needle in this pendulum. A vertical line on the inertia weight passes over the scale which is marked with a zero line and marks representing a suitable swing amplitude and 0.368 of this amplitude. The time taken for the number of complete oscillations between these amplitude limits (or any other limits) enables the frequency to be found, but, in this case, as a bonus, the Q value which is πN; N being the number of swings between these two amplitude limits. A photograph, figure 1, shows the set up of the pendulum.

Pendulum Parameters

The inertia weights are mild steel discs 81 mm dia., 12.7 mm thick weighing 0.5 Kg each. They slide onto the 6.35 mm dia.

reduced ends of the torsion bar being located against shoulders that separate their centres by 178.5 mm in this case. The inertia of each weight was found by calculation using the equation;

$$I_M = M(h^2/12 + r^2/4 + R^2)$$

where M is the mass of each weight, h the length of the cylindrical weight, and r its radius. R is the distance of the centre of the weight to the pendulum rotation axis. The total inertia is twice this value plus the inertia of the weight support bar i.e. the tap wrench; the inertia of the specimen was ignored without serious error.

The inertia of the support bar was found by difference since it was of an awkward shape for calculation. The equation;

$$G_{eff} = 8\pi \; I \; l_w f^2/r_w^4$$

where G_{eff} is the shear modulus, l_w and r_w are the length and radius of the wire specimen used, f the oscillation frequency and I the total inertia acting. By determining the frequency with and without the inertia weights we can use the identity;

$$G_{eff} = 8\pi \; I_1 \; l_w f_1 2/r_w^4 = 8\pi \; I_2 \; l_w f_2^2/r_w^4$$

which reduces to;

$$I_1/I_2 = f_2^2/f_1^2$$

where $I_1 = I_{weights} + I_{torsion\ bar}$
$I_2 = I_{torsion\ bar}$
f_1 = frequency with weights added
f_2 = frequency with torsion bar only.

For the 0.5 Kg weights used we have the following results;

Total inertia weight	I_M	f_1	f_2	I_2
1 Kg	0.0084424	0.230	1.47	0.0002119

The total inertia I_1 is then 0.008654 Kg m^2

Orthotropic Materials

The determination of shear moduli for orthotropic materials i.e. wood, the subject of this study, makes use of a procedure outlined by J.Bodig and B.A.Jayne (1). The elastic properties of anisotropic materials are dealt with in detail by R.F.S.Hearmon (2).

The principal directions in wood where the properties have limiting values are L, along the grain parallel to the axis of the tree, R, radial and normal to the tree axis, and T, transverse i.e. normal to the axis and the radial direction.

Elastic moduli, E_L, E_R, and E_T measured in these directions are characteristic of the material. They are related through Poisson's ratio.

Shear moduli occur on the three principal planes and these concern us in this paper. G_{LR} and G_{LT} can be determined on samples cut from violin tonewood using the method of Bodig and Jayne. Figure 2 shows the principal directions and planes.

Specimens were cut in the longitudinal direction with rectangular cross section, one with the long side of the rectangle in the R direction and the second with the long side in the T direction. The frequency of oscillation of the two samples was determined and substituted in turn in the equation;

$$G_{ij} = 4\pi^2 Ilf^2/k_1ab^3$$

where I is the imposed inertia, l is the sample length, f is the frequency of oscillation, a the long side and b the short side of the cross section and which should have a ratio not less than 4:1 and k_1 is a constant obtained from a table, originally due to St.Venant, of k_1 versus $a/b(G_{LT}/G_{LR})^{1/2}$. For accurate results the values of a and b should be constant over the length of the sampe and known precisely. The value of k_1 for the two samples should be similar in value. A graph of k_1 is shown in figure 3. G_{LR} and G_{LT} are obtained by substituting k_1 back into the equation.

The remaining shear modulus G_{RT} requires two specimens cut with the length in the radial direction and the long side of the cross section, of one specimen, in the transverse direction so that G_{RT} would be the dominant modulus. The other sample would have the long side of the cross section in the L direction. The G_{LR} modulus determined here should agree with the previous determination. Two lengths could be joined by glueing to make a specimen long enough.

<u>Example of Calculation</u>

The following is the tabulation of the results of the calculation of G_{LR} and G_{LT} for spruce using the experimental determination of frequency for the two samples required. Figure 4 shows the necessary geometry.

Sample	$f(s^{-1})$	l(m)	a(m)	b(m)	a/b	ab^3
G_{LR}	0.348	0.40	0.0083	0.0021	3.952	0.769×10^{-10}
G_{LT}	0.500	0.40	0.0084	0.0030	2.80	2.268×10^{-10}

$$G_{LR} = (4\pi^2 Ilf^2)/(k_1a_Rb_T^3) = 0.2178\times10^9/k_1 \qquad G_{LT} = (4\pi^2 Ilf^2)/(k_1a_Tb_R^3) = 0.1507\times10^9/k_1$$

$$G_{LT} = (0.1507/0.2178)G_{LR} \qquad (G_{LT}/G_{LR})^{1/2} = 0.8318$$

Sample	$a/b(G_{LT}/G_{LR})^{1/2}$	k_1(from graph)
G_{LR}	3.287	0.268
G_{LT}	2.329	0.245
		Av. 0.256

$$G_{LR} = 0.2178X10^{9}/0.256 = 0.85X10^{9}\ Pa$$
$$G_{LT} = 0.1507x10^{9}/0.256 = 0.59x10^{9}\ Pa$$

Discussion

It must be remembered that this method measures a dynamic shear modulus which is that operating during vibration. G_{LR} would dominate in quarter cut plates vibrating in mode 1. Tests carried out on 10 samples of European spruce showed no significant correlation with density for either shear modulus. The results fell in the range 0.5 to 1.0x10^{9} Pa. For 11 samples of European maple a correlation of both shear moduli with density was found. G_{LR} was generally 50% higher than G_{LT} and both had a similar trend line to E_R, with change in density. G_{LR} for the maple was about twice that for the spruce. More samples need to be measured for a difinitive result. As for the Q values, the tentative results are: for spruce the values ranged from 45 to 60 (Av. 51.7) for Q_{LR}, and ~~50~~ 44 to ~~90~~ 53 (Av. 47.0) for Q_{LT}; for maple the values ranged from 45 to 72 (Av. 56.8) for Q_{LR}, and 47 to 53 (Av. 50.3) for Q_{LT}.

References

(1) J.Bodig and B.A.Jayne: Mechanics of Wood and Wood Composites, Van Nostrand Reinhold, 1982.

(2) R.F.S.Hearmon: Introduction to Applied Anisotropic Elasticity, O.U.P. London, 1961.

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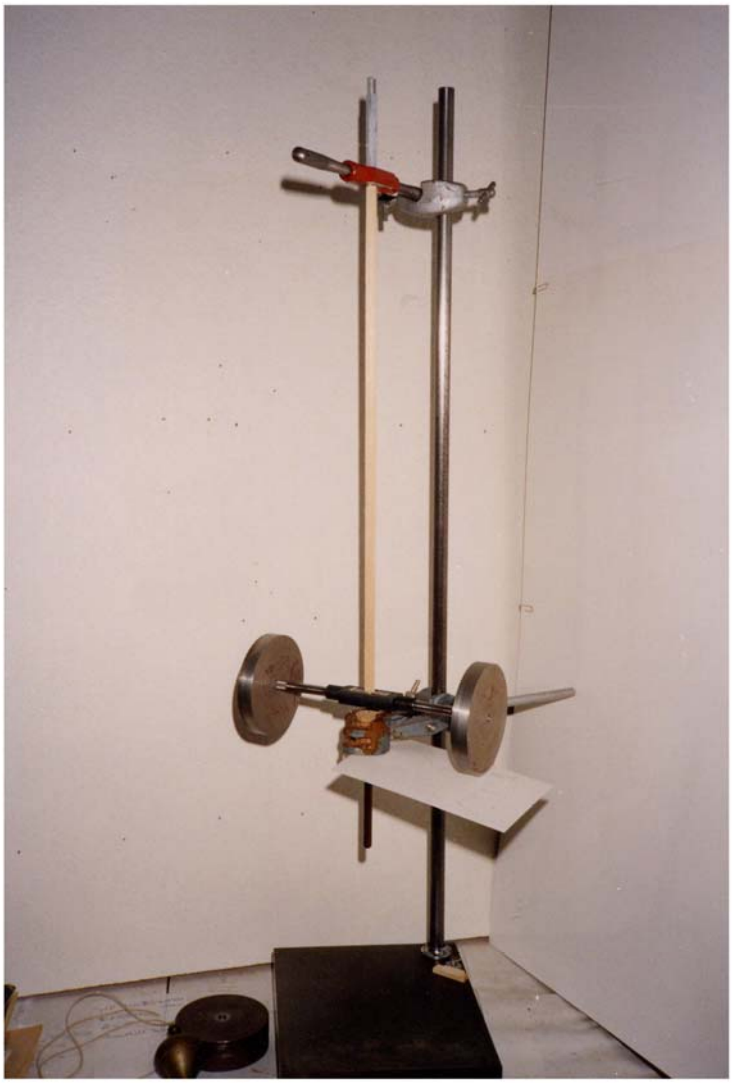

Figure 1. Torsion Pendulum setup.

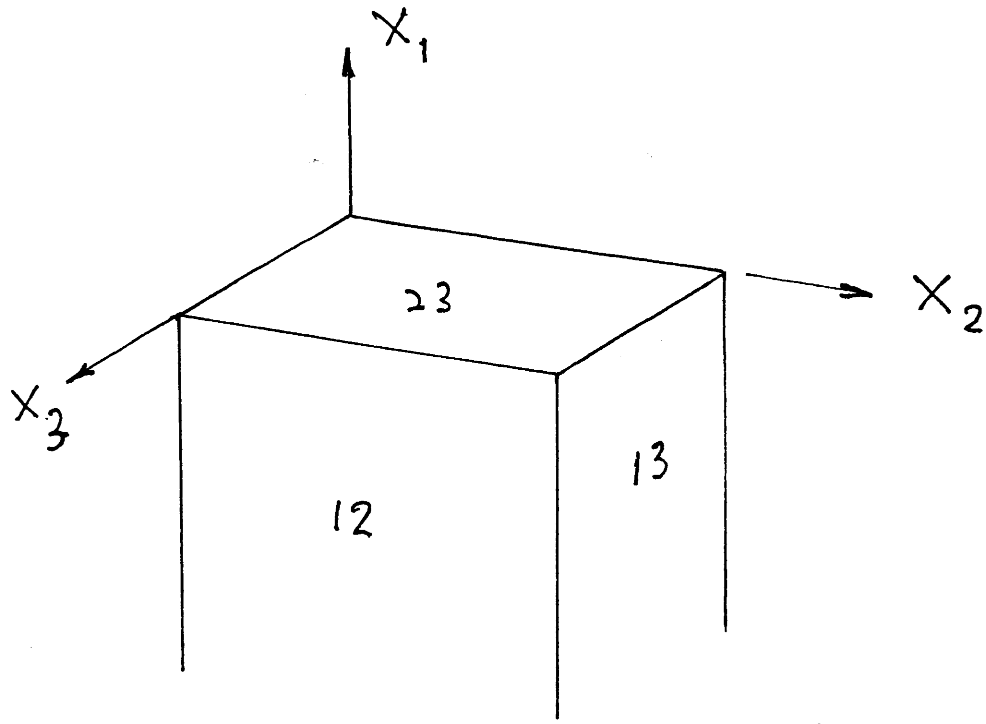

Figure 2. Principal Directions and Planes in an orthotropic material.

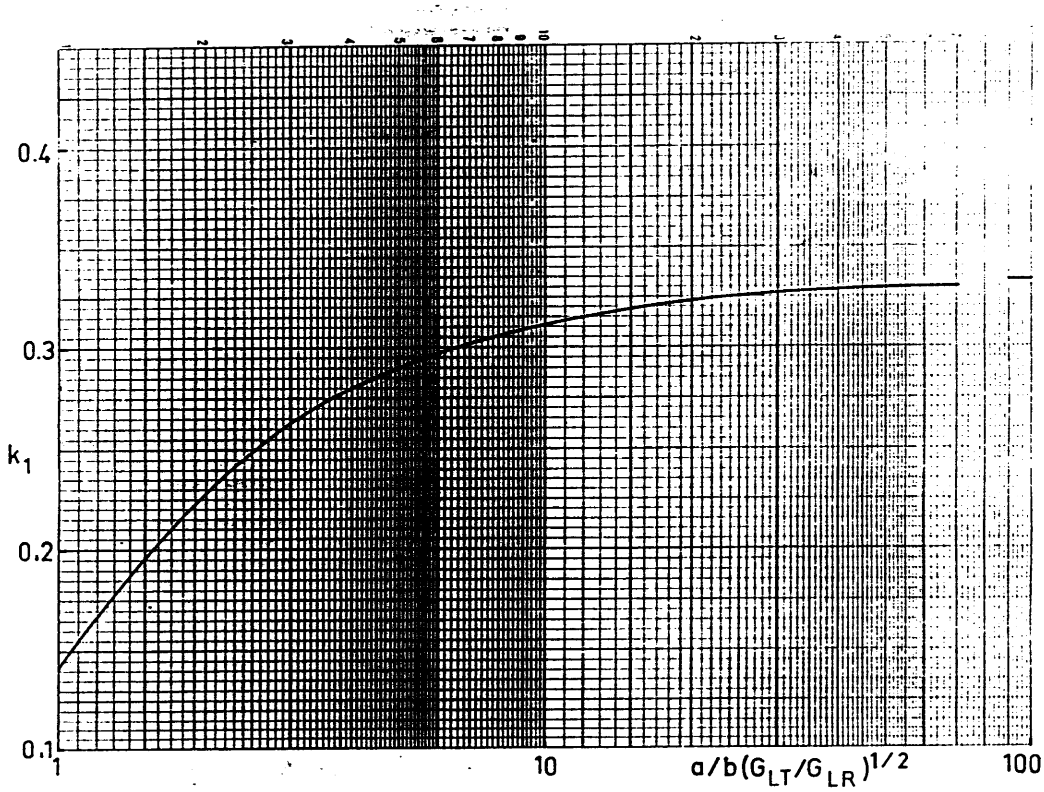

Figure 3. Graph of k_1 versus $a/b(G_{LT}/G_{LR})^{1/2}$ from Bodig and Jayne.

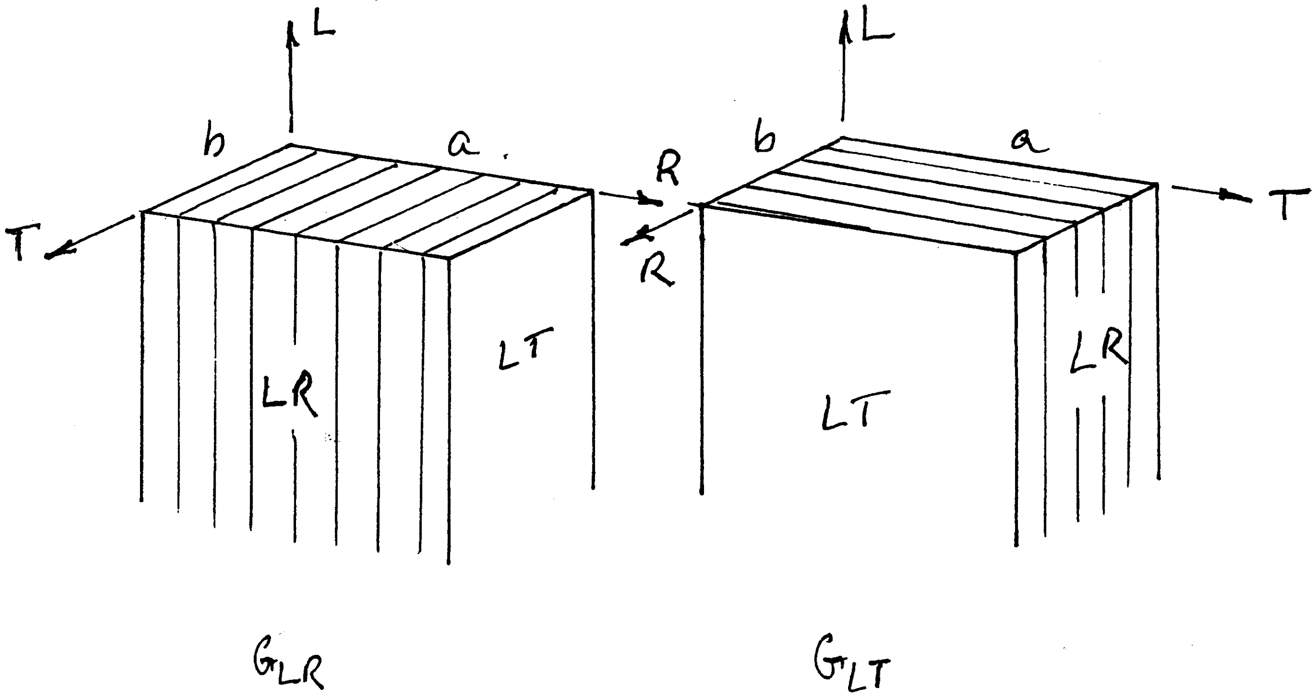

Figure 4. Principal Directions and Shear planes in Spruce.