THE VIRTUAL BOEHM FLUTE — A WEB SERVICE THAT PREDICTS MULTIPHONICS, MICROTONES AND ALTERNATIVE FINGERINGS

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ABSTRACT. We report a web service for flute players, 'The Virtual Boehm Flute', that provides alternative fingerings which may be easier to play, less awkward to finger and/or more in tune for different circumstances. It also provides possible fingerings for multiphonics (chords). It uses an expert system that predicts the playability of notes from features in the input impedance spectra, based on the playability of 957 impedance minima as determined by an expert flutist. Used in conjunction with a theoretical model, developed from detailed experimental measurements, it can predict the acoustic impedance spectrum for 39,744 different acoustic configurations of the flute. The resulting database provides, via a musician-friendly interface, the predicted possible notes and multiphonics for any selected fingering, and all the possible fingerings predicted to play a desired note or multiphonic. The service is at http://www.phys.unsw.edu.au/music/flute

1. INTRODUCTION

A particular combination of keys pressed on a woodwind instrument is called a fingering and corresponds to an acoustic configuration with specific tone holes closed or open. One might expect that a flute with 17 tone holes would have 2^{17} possible configurations, but the number is smaller because of linkages and clutches. In this paper we describe a database and web service that allow flute players to search all 39,744 acoustic configurations of the modern flute, both C and B foot. We begin by explaining why it is interesting to look at so many.

A few dozen fingerings are 'standard': beginners on the instrument learn one or perhaps two 'standard' fingerings for each of the few dozen notes in the normal playing range. More advanced players learn dozens of alternative fingerings that have different properties of pitch, stability and timbre at different playing loudness, or that may be used to facilitate awkward, fast passages and trills (rapid alternations between notes). Players of contemporary flute music are required to use many more fingerings. Some of these produce multiphonics, or chords, in which two or more notes are sounded simultaneously. Others produce microtones: notes with pitch intermediate between those of the equal tempered scale. Yet others are used to produce notes with unusual or contrasting timbres. The composer Berio [1] was one of the early users of these techniques.

Of the 39,744 possible fingerings, only a fraction are given in advanced texts for flutists or for composers writing for the instrument [2,3], so presumably many playable chords and other possibilities remain unknown. Further, searches cannot be conducted easily. A composer wishing to use multiphonics or interesting effects of contrasting timbres (or a player required to play them) has hitherto had no easy way of finding out which chords are possible and how they may be played. The Virtual Boehm Flute aims to overcome these problems.

2. FLUTE ACOUSTICS

Much information about the acoustical properties of the flute for a given fingering may be determined from the spectrum of the acoustical impedance $Z(f)$, the ratio of acoustic pressure to volume flow of air, measured at the embouchure hole (the 'input') of the flute. For any fingering, the flute plays notes whose frequencies are close to those of the resonances or standing waves in the tube of the instrument for that fingering. The flute is played with the embouchure hole open to the atmosphere, and so its resonances correspond closely to the minima of the acoustic impedance at the embouchure. The acoustical principles of the flute are reviewed by Fletcher and Rossing [4].

Standard fingerings

In many standard fingerings, all the holes are closed down to a certain point, and (nearly) all open beyond that. In a crude approximation, the flute with such a fingering acts like a tube, open at both ends, whose length L is approximately that between the embouchure hole and the first open tone hole. The minima in $Z(f)$ correspond to standing waves with wavelengths of $2L/n$, where n is an integer. These resonances give rise to a harmonic series. Thus the flute can operate using one of these resonances as the fundamental, and producing harmonics that are supported by the higher resonances. Vibrations with frequencies in harmonic ratios together produce a periodic wave and are usually recognised as a single note. In practice, the standing waves propagate a little past the first open hole, and the geometry near the embouchure is complicated, so accurate calculations of each resonance frequency are rather more involved.

Cross fingerings and multiphonics

Cross fingerings are fingerings in which one or more tone holes are closed downstream from the first open hole. An open hole acts like a low impedance shunt (actually an inertance, the acoustic analogue of an inductance). Some of the travelling wave is reflected at the first open hole, and some is transmitted, only to be reflected at the next open hole or series of open holes. These two reflections can give rise to two different standing waves, which the player may be able to excite

simultaneously in superposition. As the lengths involved are not in general simple harmonic ratios, these two different resonances sound together as a chord or multiphonic (see Fig 1).

There are several constraints on producing them. First, the jet becomes increasingly non-linear as one blows harder, so mode locking occurs [5]. Consequently, multiphonics can usually be produced only at relatively low dynamic levels. Further, the impedance of a single, large, open hole is so low at low frequencies that little power in the wave is transmitted beyond it. Consequently, there are few multiphonics in the low range of the instrument, and those that occur at the lowest frequencies are usually those that use the smallest holes in the instrument as the first reflection. Multiphonics have been studied acoustically by several authors who have examined the relationship between the spectra of the notes produced and the input impedance spectrum of the instrument [6,7], the relationship among the fundamental frequencies of the notes produced [8], and the behaviour of the sound spectra in phase space [9,10]. Backus [7] studied multiphonics by relating the sound spectrum to the instrument's impedance spectrum. He reported heterodyne components from the interaction, indicating a non-linear superposition of the two notes.
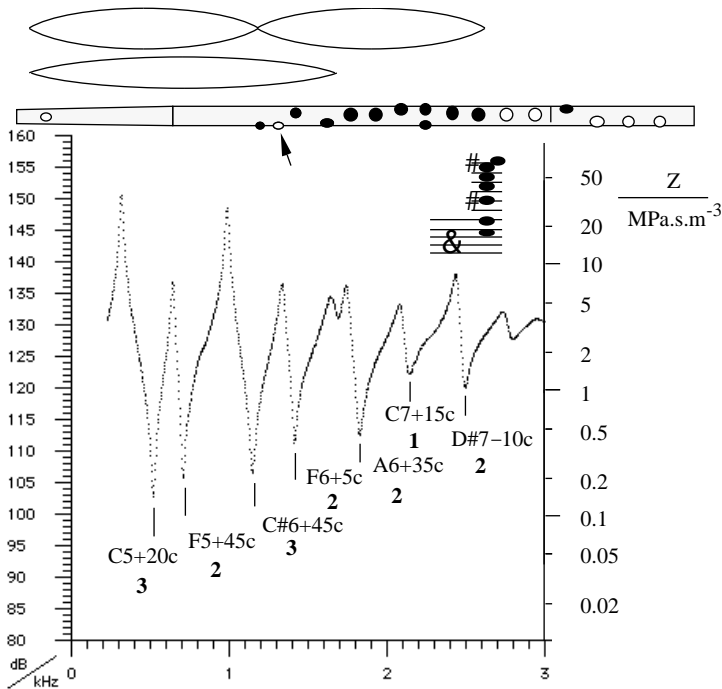

Figure 1. A sketch of the configuration of a flute that will play a multiphonic with notes close to C5 and F5, among others. On the flute schematic, black and white indicate closed and open holes respectively. When the small hole arrowed is closed, this fingering plays F5, whose standing wave is sketched at the top of the figure. When open, this hole produces a reflection whose standing wave is approximately that of C5 (the second standing wave sketched). The wave has a substantial end effect: the inertance of the small hole behaves like an extra length of bore, as indicated. The graph is the measured impedance spectrum for this fingering in M , or MPa.s.m^{-3}. (The dB scale is 20 $\log_{10}(Z/M$).) The playabilities predicted by the expert system are shown for each of the identified minima (bold numbers: 3 is easiest, 0 is impossible).

3. A THEORETICAL MODEL FOR *Z(f)*

Waveguide models have been used to calculate *Z(f)* for a range of orchestral wind instruments: e.g. [11-15]. These models take advantage of the fact that the wavelengths of the sounds of interest are rather longer than the diameter of the instrument. Consequently, the important waves in the bore are predominantly planar.

To perform such a calculation, one starts from the downstream end of the flute and works back towards the embouchure. The acoustic impedance spectrum at the end of a pipe is that of the radiation field, which is known [16]. This is then used as the load impedance for a section of waveguide leading to the first tone hole. The input impedance of the waveguide is calculated using a transfer matrix. The tone hole is also a (very short) section of waveguide whose load is either another radiation load (if open) or open circuit (if closed). These two waveguides in parallel form the load for the next segment of the bore. The process continues to the embouchure.

This approximation has limitations, of course, because the instrument is clearly not one dimensional, particularly at the junctions between the bore and a tone hole or the embouchure. However, the effects of these complications can be included by adding extra elements, such as an end correction to account for the junction between pipes of different cross sectional areas. We have measured the parameters describing these effects independently, using progressively more complex geometrical systems (a single cylinder, open or closed, simple branched tubes of varying lengths in which the single side branch had the same diameter, a cylindrical flute head, a cylindrical head with a small number of holes, a real head joint, a real flute). The components were then combined into a complete model for the whole flute [17,18] and tested against the fingerings in our database of experimental measurements [19,20]. The average rms difference between log10 of the calculated *Z(f)* and log10 of the measured *Z(f)*, averaged over 40 standard fingerings each covering the frequency range from approx. 200 Hz to 4 kHz in 1402 steps, was ±0.073. The model could thus predict *Z(f)* for any fingering with sufficient accuracy for our purposes.

4. FROM *Z(f)* TO PLAYING FREQUENCIES

There are several reasons why playing frequencies in a flute do not coincide with those of the measured minima in *Z(f)*. Although these differences are 'only' a few percent or less, this means that they may be a substantial fraction of a semitone. Flutists raise the temperature and humidity of the air in the instruments, and thus raise the pitch overall. They can also vary the pitch by varying the extent to which the lower lip covers the embouchure hole. They also vary the speed of the jet.

We elected to include all these factors in a single, empirical function. Two flutists were asked to play the flute used in the experimental study. They were asked to use their normal embouchure and to avoid correcting the pitch when and if the instrument was out of tune. They played each note over the range from B3 to E7 using standard fingerings and maintaining the note for several seconds while a pitch measurement

was made using a commercial tuning meter. The use of only the impedance minimum of the fundamental to estimate playing frequency is a crude approximation for notes at the bottom of the range, for which several harmonic minima may all contribute to the playing régime [5, 20]. However, this approximation should be valid for quietly played notes, where the jet behaviour is least non-linear. Flutists are used to correcting for the variation of pitch with loudness, so this approximation should not greatly reduce the utility of the model. The difference between the measured frequency f of the note played, and the frequency fm of the minimum that corresponded to the fundamental of the note played, was calculated using the following relationship, which corresponds to three straight line segments in a plot of pitch correction vs pitch.

$$f = \beta f_m^{\alpha}$$

where
$$\alpha = 1.0293, \beta = 0.8588 \text{ if } f_m < 350 \text{ Hz};$$
$$\alpha = 0.9769, \beta = 1.1674 \text{ if } 350 \leq f_m < 1700 \text{ Hz}; \quad (1)$$
$$\alpha = 1.026, \quad \beta = 0.8074 \text{ if } f_m \leq 1700 \text{ Hz}.$$

The correction never exceeded 35 cents.

We neglect variation among flutes: the measurements were made on a standard, production model, prepared in a standard way [20], so this is the flute being modelled. Different flutes and different players will give different results, but in this context it is worth noting that players vary frequency by more than 10 cents (a tenth of a semitone) in different circumstances.

5. PREDICTING PLAYABILITY FROM $Z(f)$.

Quantifying extrema in $Z(f)$.

The frequency range studied was 0.2 to 4.0kHz. This covers the range of all playable notes on the instrument and furthermore, $Z(f)$ has very little structure above about 3 kHz. A set of parameters (Z_m, f_m, f_m) were calculated for each extremum (maximum or minimum) and stored. f_m denotes the frequency corresponding to that extremum, Z_m denotes the magnitude of $Z(f)$ at frequency fm and Dfm denotes the bandwidth. Q f_m/ f_m was also evaluated as a variable that might influence playability.

Measured 'playability' of notes

The presence of a minimum in $Z(f)$ does not necessarily mean that a note can be played at that pitch. The 'playability' of minima was measured as follows. An experienced flutist ranked the notes corresponding to each of the 957 minima present in measured $Z(f)$ data for 76 selected fingerings on one flute into four levels of playability, from 3 (most readily playable) to 0 (impossible). Some playabilities are indicated in Fig. 1.

Predicting playability from parameters of $Z(f)$.

What features in $Z(f)$ are related to playability? As well as the sets (Z_m, f_m, f_m) corresponding to each minimum, the influence of other parameters was also examined, particularly the presence of higher minima that are harmonics of the minimum studied, the impedance at these minima, and the proximity and magnitude of nearby minima and maxima.

Three methods were tried to relate playability to these parameters. Linear regression yielded little insight because many of the parameters are strongly correlated. The neural net method was unacceptably slow for these data, even when only subsets of the parameters were used.

The successful method used decision trees, developed using the C5.0 algorithm suite. Decision trees are an artificial intelligence technique described by Quinlan [21,22] The set of expert decisions was used to train a two-tiered system; the first decision tree predicts whether a given impedance minimum is playable or unplayable based on its physical parameters (using C5.0), and the second decision tree ranks playable minima on a continuous scale of 0 to 3 via a conditional set of linear equations (using Cubist, the continuous form of C5.0).

When presented with the discrete expert data (i.e. whether each of the 957 impedance minima are simply playable or unplayable), C5.0 evaluates a decision tree relating physical minima parameters to a decision of playability. Cross-validation was used to test the performance of the decision tree with unseen data. In this process the expert data are randomly divided into ten subsets, and in each iteration a single subset is withheld from the C5.0 algorithm and used as a test set. Using this technique, a decision tree may be pruned to remove spurious dependence on any of the minima parameters that do not improve the error rate of the tree. The decision tree which demonstrated the least error rate (5.2% during cross-validation) is shown in Fig. 2.
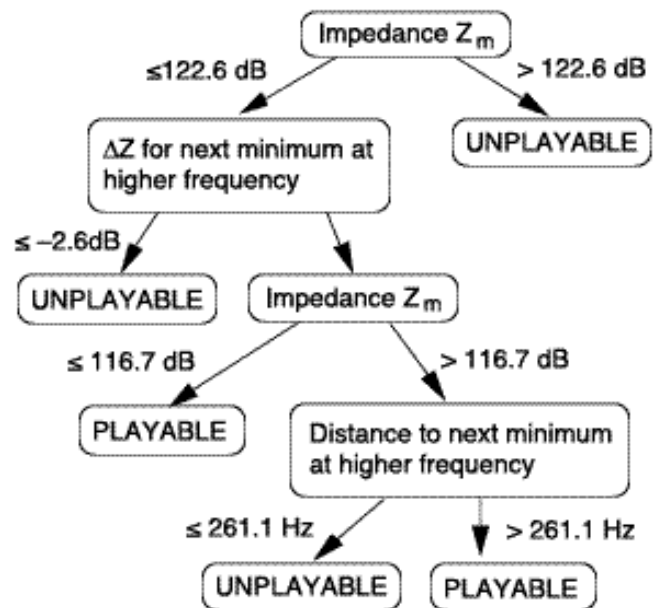

Figure 2. A schematic diagram of the C5.0 decision tree.

Similarly, playable expert data were presented to Cubist to evaluate a set of linear equations relating minima parameters to a degree of playability P ($0 \leq P \leq 3$). Whereas the parameter dependence and error rate of the discrete decision tree are well-behaved, the composition of Cubist's output is somewhat fuzzy. This is for two reasons: (i) the physical minima parameters have a wide range of correlations with the expert flutist's discrete scale of playability ($P = 1, 2$ or 3), and (ii) the minima parameters themselves are strongly correlated, and can therefore appear interchangeably in relationships. Nevertheless a rough estimate of playability is useful to a

musician, and we found it possible to rank the playability of a minimum with frequency f_m on this continuum scale using only the following rule.

$$P = 3.0 \quad \text{if } Z_m < 103.2 \text{ dB} \quad \text{else}$$

$$P = 0.6 + 0.77 \log \frac{Z(f_+^-)}{Z_m} - 0.57 \log \frac{Z(f_-^-)}{Z_m} + 1.6 \times 10^{-3} (f_+^+ - f_m) \quad (2)$$

$$- 2 \times 10^{-4} (f_m - f_-^+) + 8 \times 10^{-5} (f_m - f_-^-) + 0.005 N + 0.018 H$$

where f_+^+ and f_+^- denote the frequencies of the nearest maxima above and below f_m respectively. f_-^- denotes the frequency of the closest minimum below f_m.

The harmonicity of higher minima was included in the study because of their possible involvement in 'mode locking' of the non-linear oscillation régime of the jet [5]. Minima at frequencies above f_m were deemed to be harmonic if their frequency was in the range $n(1\pm0.05)f_m$, where n is a positive integer. The harmonic number N was the total of such harmonic mimima. The harmonicity function H was the average of $\log(Z)/n$ for the harmonic minima. The small coefficients of N and H in equation (2) seem at first glance to suggest that harmonicity was not very important in determining playability. However because of the various correlations among the input variables, including those in equation (2), one should be cautious in regarding any of these coefficients as simple weighting factors.

6. THE VIRTUAL BOEHM FLUTE

The following process is performed to predict the playable notes of any fingering: (i) calculate an impedance spectrum for a given fingering using the developed physical model, (ii) extract the physical parameters of each minimum from the spectrum, (iii) use the developed expert system to determine which minima are playable and their degree of playability, and (iv) correct the pitch of playable minima for playing conditions. For any fingering, pairs and triplets of playable notes that are not harmonically related are predicted as possible multiphonics. These steps are repeated for each of the 39,744 B foot and C foot fingerings, the entire process requiring approximately 12 hours to compute on an Intel Pentium III PC. The resulting data are stored in a substantially sized relational database (there are in the order of 150,000 possible notes). To access these data in a manner which is useful and intuitive for a musician, a web interface was developed following the principles of Greenspun [23] and Nielsen [24]. This web service, titled 'The Virtual Boehm Flute', provides three tools for flutists and composers. These are shown, as they appear on the screen, in Fig. 3.

The first tool allows the user to input a fingering, using a graphical interface that represents the keys on a flute in a way that is obvious to flutists. The Virtual Boehm Flute returns a prediction of all possible notes, with predicted pitches and playabilities, and a list of multiphonic possibilities.

The second tool is used for alternative fingerings and microtones. The user enters the desired note, and some details about the flute s/he is using. The database is then searched for all possible fingerings that predict notes within half a semitone

Figure 3. The entry page for The Virtual Boehm Flute, showing the three available tools.

of the note sought. These may be ranked by playability or pitch. Ranking them by pitch allows the user to seek microtone fingerings for a desired pitch.

Alternative fingerings are very useful to musicians: players often practise a single phrase many times because of the awkwardness or poor intonation of the standard fingerings for a particular series of notes. Combinations of fingerings are often particularly awkward in the higher registers. The alternative fingering tool allows players to include certain keys (that might be already used in the preceding or succeeding note) or to exclude keys, so that all fingers move in the same direction. For example, the rapid alternation (trill) between the notes F6 and A6 is awkward using standard fingerings. A search for an alternative fingering for F6 in which the standard keys closed for A6 were included yields the fingering (known in text to a flute player as (Th 1 2 3 | 1 – tr2 D#) which yields a comfortable, easy trill.

This tool can also find fingerings that are easier to play, or have better intonation than those given as standard, particularly in the fourth octave. One of the authors (a player of reed instruments who rarely plays flute) is unable to play F7 with the standard fingering (– 2 – | – – 3 tr2 D# C#). The Virtual Boehm Flute suggests a fingering (1 2 – | – 2 tr2) with which he can play it either softly or loudly.

The third tool searches the database of multiphonic combinations that are input by the user. It may be used, for example, by a composer who wishes to include a chord for the flute, but who needs to know if the chord is possible. Traditionally, composers are expected to supply the fingering when multiphonics or other peculiar effects are required.

All tools allow the user the possibility of running the theoretical model for the selected fingering to produce $Z(f)$, whose minima may be identified with notes using the mouse.

The Virtual Boehm Flute is widely used by flutists around the world, whose comments have been highly favourable. It is located on our music acoustics site at

<http://www.phys.unsw.edu.au/music/flute/virtual>.

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