Question 30 part C.

An electron microscope has a magnetic lens for focusing a wide beam of electrons to a point. Theoretically, a beam of electrons can be focused to a point by a magnetic field region with the shape shown in Diagram 1, on condition that all the electrons are moving at the same speed.

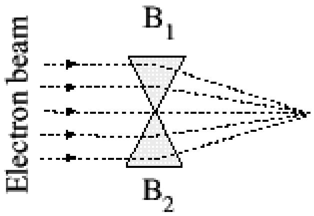

Diagram 1

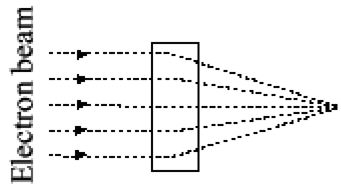

Diagram 2

(i) If the electrons have a velocity of 1.6×10^6 m s^{-1}, and the magnetic field strength of the magnetic lens is 0.12 T, what deflecting force acts on the electrons?

(ii) Explain how the triangular shape of the magnetic field region in Diagram 1 focuses the electrons.

(iii) If the magnetic field region were not triangular, but had the shape shown in Diagram 2, how could you focus the electron beam?

Simple version

i) The magnitude of the magnetic force due to a field **B** acting on a charge q travelling at velocity **v** is qvB sin ϕ where ϕ is the angle between **v** and **B**. The question does not inform us the direction of B, but, unless the field lines formed circles about the axis of symmetry, the focussing would not work. So let's assume that B_1 is into the page and B_2 is out of the page. In this case ϕ is (initially) 90° so the magnitude of the force is

$$|qvB| = 31 \text{ fN}.$$

ii) Let us assume that the fields are uniform, and that the lens has a long focal length, so that vertical velocity components are much smaller than the horizontal velocity $v_x \cong v$.

The vertical acceleration a_y is approximately constant while the electrons are in the field, and is directed towards the axis of symmetry. The vertical velocity component v_y is $a_y\Delta t$, where Δt is the time spent in the field. Because Δt is proportional to the longitudinal length L of the field region, the electrons further from the axis will acquire a greater vertical velocity, so $|v_y| \propto \Delta t \propto L \propto |y|$. So the vertical velocity is proportional to the displacement from the axis, and directed towards it. Therefore all of the electrons reach the axis at the same *time*. Provided that their horizontal velocity components a the same, then they all intersect the axis at the same *place*, called the focus.

iii) Instead of using $L \propto |y|$, we could make $|\mathbf{B}| \propto |y|$. Again we would achieve $|v_y| \propto |y|$.

Note that in both cases the thin lens approximations have been used. In practice, these approximations do not hold, but focussing is achieved using a combination of fields that are inhomogeneous and whose confinement shape is not triangular.

More serious answer

i) The magnetic force due to a field **B** acting on a charge q travelling at velocity **v** is $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$, so the magnitude is

$$F = qvB \sin \phi \quad \text{where } \phi \text{ is the angle between } \mathbf{v} \text{ and } \mathbf{B}.$$

The question does not inform us the direction of B, but, unless the field lines formed circles about the axis of symmetry, the focussing would not work. So let's assume that B_1 is into the page and B_2 is out of the page. In this case ϕ is (initially) 90° so the magnitude of the force is

$$|qvB| = 31 \text{ fN}.$$

ii) Let us assume that the fields are uniform, and make the thin lens approximations, ie that the lens thickness and diameter are much smaller than the focal length. This means that the total angle of deflection θ of the beam is <<1 radian. Let x and y be horizontal and vertical coordinates, and place the origin at the centre of the lens.

Because $\theta << 1$, the magnetic force is always approximately perpendicular to the initial direction of **v**, so the x component v_x is unchanged, and $v \cong v_x$. The magnetic force is therefore always approximately vertical, so $a_y = qvB/m$ and is approximately constant while the electron is in the field. Because of the triangular shape of the field regions ($\Delta x = k|y|$) where k is a constant of the triangle (twice the tan of its half angle), the electrons are within the field for a time

$$\Delta t = \Delta x/v_x = k|y|/v_x.$$

So the vertical velocity component when the electron leaves the field is

$$v_y = a_y\Delta t = qvBk|y|/mv_x \cong qBk|y|/m = Ky,$$

where $K = q|B|k/m$ is a negative constant. In other words, the vertical velocity component when the electrons leave the field is proportional to their distance from the axis, and towards the axis. Therefore they all reach the axis at the same time and, because v_x is uniform, they reach they all cross the axis at the same place.

The focal length is $y/\theta = yv_x/v_y = v/K$

iii) In this case, we could make the field proportional to y, i.e. $B = B_o y$. If the thickness of the lens is L, an analysis similar to that given above leads in this case to

$$v_y = a_y\Delta t = qvB_o yL/mv_x = K'y,$$

where $K' = qB_oL/m$ is a negative constant, and so the same argument about focussing follows.

Note that in both cases the thin lens approximations have been used. In practice, these approximations do not hold, but focussing is achieved using a combination of fields that are inhomogeneous and whose confinement shape is not triangular.