

A-level Physics Tutor Guides

A-level Physics  
COURSE NOTES

THERMAL PHYSICS

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# Thermometry

## What is temperature?

**Temperature** is a **measure of the average kinetic energy** of the particles making up the substance being examined.

Temperature must not be confused with heat.

**Heat** is the amount of **vibrational energy contained in a particular mass**.

## Fixed Points

For any temperature scale, it is necessary to have two fixed points. These are temperatures at which particular physical properties manifest themselves eg melting, boiling.

Common fixed points:

**ice point** - temperature when pure water ice is in equilibrium with liquid water at standard atmospheric pressure (76mm Hg).

**steam point** - temperature when pure liquid water exists in equilibrium with water vapour at standard atmospheric pressure.

**triple point\*** - temperature when pure water ice, pure liquid water and pure water vapour exist in equilibrium.

\* there are triple points for many other substances besides water

Temperature scales are divided into a specific number of degrees between the two fixed points.

The gap between the upper and lower fixed points is called the **fundamental interval**.



As a matter of historical interest, the **Fahrenheit** scale was founded on the following fixed points:

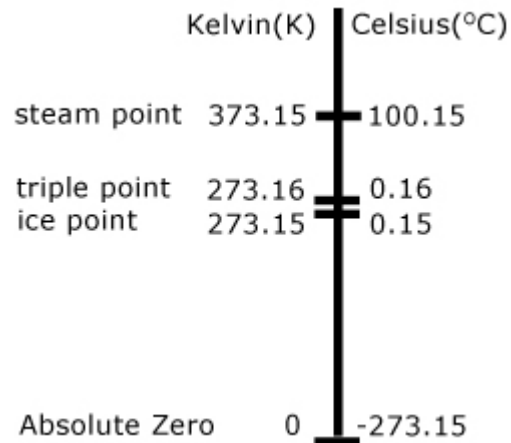
lower fixed point 0 deg.(0°F) - the coldest salt mixture temperature attained

higher fixed point 100 deg.(100°F) - horse blood temperature

### The Kelvin (Absolute) temperature scale

This scale uses the the triple point of water as the upper fixed point and absolute zero (zero molecular motion) as the lower fixed point.

The triple point of water is assigned the temperature 273.16K while the the ice point is 273.15K and the steam point 373.15K.



By definition one degree **Kelvin** is:  $1 / 273.16$  (approximately 0.00366) of the difference between absolute zero and the triple point of water.

### The Celsius temperature scale

The Celsius scale was introduced to replace the **Centigrade** scale. The Centigrade scale had the melting point of ice as its lower fixed point and the boiling point of water as the upper fixed point, both measured at standard atmospheric pressure.

However, unlike the triple point of water, the melting point of ice cannot be measured with enough precision to make it an absolute marker.

The Celsius scale is closely related to the Kelvin scale.

By definition one degree **Celsius** is:  $1 / 273.16$  (approximately 0.00366) of the difference between absolute zero and the triple point of water.

This is the same definition as for a degree Kelvin. So a temperature change of 1K equates to a change of 1°C.

The two scales are also defined by the equation:

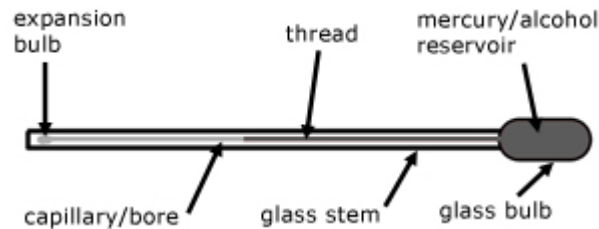
$$\theta = T - 273.15$$

$\theta$  temperature in °C

$T$  temperature in K

## Thermometers

### Liquid in glass



The thermometer works by an expanding liquid in a vacuum, moving against a scale.

There are a number of disadvantages to this instrument:

- 1.) The glass itself expands and contracts and leading to under and over reading of temperatures.
- 2.) Parallax errors mean readings are only  $0.1^{\circ}\text{C}$  accurate.
- 3.) The diameter of the bore is not consistent.
- 4.) Their large thermal capacity means that they do not react quickly and they may affect the temperature they are trying to measure.

This is how the two liquids used in thermometers, alcohol and mercury compare:

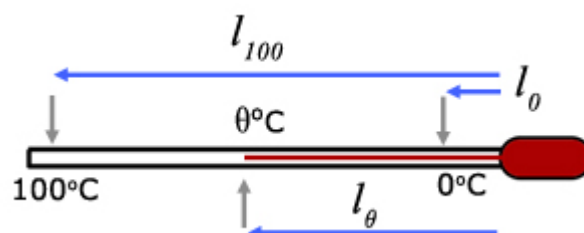
#### **alcohol**

- transparent, must be used with a dye
- heat conduction poor
- sticks to glass - concave meniscus
- temperature range  $\sim 150^{\circ}\text{C} \dots -114.9^{\circ}\text{C}$

#### **mercury**

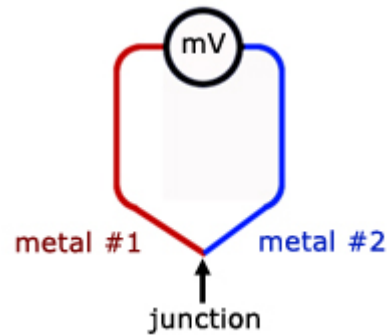
- opaque
- is a metal and therefore a good heat conductor
- does not stick to glass - convex meniscus
- temperature range  $\sim 356^{\circ}\text{C} \dots -39^{\circ}\text{C}$

The temperature on a liquid in glass thermometers can be calculated by making certain measurements.



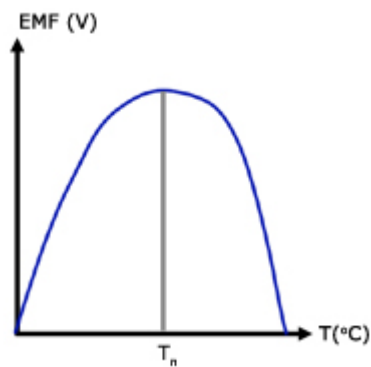
$$\theta = \frac{I_{\theta} - I_0}{I_{100} - I_0} \times 100$$

### Thermocouple



Thermocouples work on a principle called the **thermoelectric** or **Seebeck Effect**.

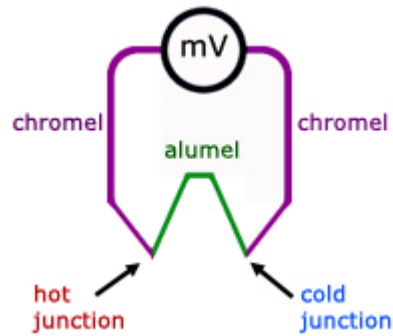
When two different metal wires are twisted together at a junction, an EMF (electromotive force) is generated across the loose ends. The magnitude of this EMF relates to the temperature at the junction.



On the graph, the value of T where the EMF is maximum is called the '**neutral temperature**'. The gradient (d(EMF)/dT) anywhere on the curve is called the **thermoelectric power**.

Measured with a high resistance millivoltmeter, EMF values are in the range 1mV - 4mV/100°C.

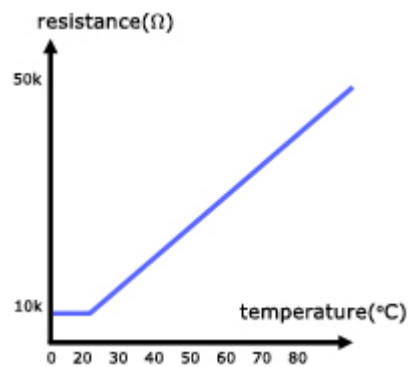
A more convenient and efficient setup is to have two junctions instead of one, but still have just two metals. The reference cold temperature is usually melting ice.



Typical pairs of metals and temperature ranges:

metals	temperature range <sup>°C</sup>
chromel/alumel	~1100 max.
Pt/Pt-Rh	1100 - 1700
Fe/Constantan	95 - 760
Cu/Constantan	200 - 350

### Resistance thermometer

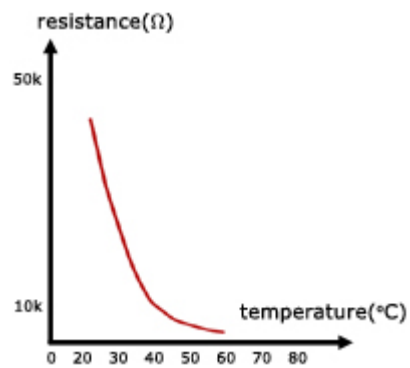


The property of metals that their resistance is temperature-dependent makes them ideal as thermometers. The metal of choice is platinum as a result of its high melting point(1773<sup>°C</sup>) and large resistance temperature coefficient\*.

\* $\alpha$ (alpha) a big increase in resistance for a small rise in temp.

In practice resistance thermometers are either thin films of platinum on a substrate or platinum wire wound around a former.

## Thermistor



Semiconductors like metals have resistance that is temperature-dependent. So they too make ideal thermometers.

The difference is, as temperature rises, the resistance of metals increases, but the resistance of semiconductors decreases.

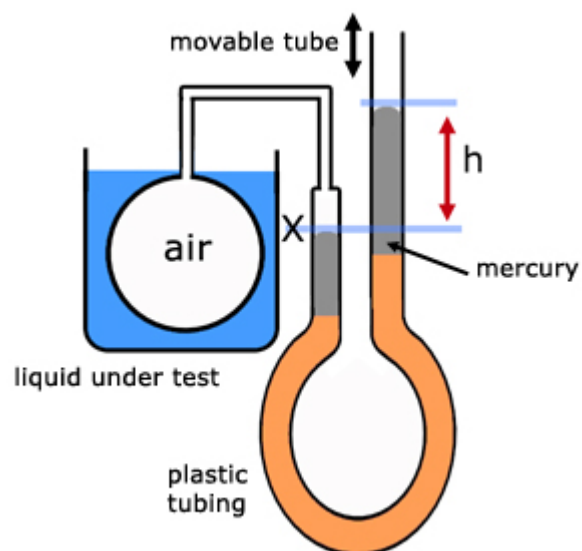
Semiconductors have large resistance temperature coefficients, but they are negative. This means that there is a big decrease in resistance for a small rise in temperature.

Typical temperature range of thermistors is  $-70^{\circ}\text{C} \dots 300^{\circ}\text{C}$  .

The thermal capacities of thermistors are small. So they absorb little heat energy and do not appreciable affect the temperature they are measuring.

Thermistor resistance is  $\sim 1\text{k}\Omega$  .

## Constant-volume gas thermometer





In its original state the glass bulb is full of air and the mercury levels are the same. A mark(X) is made against the glass to record this.

When the bulb is placed in a hot liquid for a temperature reading, the air in the bulb expands, pushing the mercury down on the left and up on the right.

To get the air in the flask back to its original volume, the movable tube is lowered until the mercury is at the level previously marked.

There is now a level difference(head)  $h$  between the two tubes. This is a measure of the pressure of the gas without taking account of atmospheric pressure  $p_A$ . So accounting for atmospheric pressure, the pressure  $p_\theta$  of the gas at temperature  $\theta$  is:

$$p_\theta = p_A + h$$

note, all pressures expressed in mm of mercury

It follows that the temperature of the gas,  $\theta$  is given by:

$$\theta = \frac{p_\theta - p_0}{p_{100} - p_0}$$

where  $p_0$  and  $p_{100}$  are pressures at  $0^\circ\text{C}$  &  $100^\circ\text{C}$  respectively.

Temperatures using the constant-volume gas thermometer can be measured to two decimal places. However there are several sources of error that prevent further accuracy:

1. the capillary tube air is not heated
2. the volume of the bulb increases with temp.
3. air is not an 'ideal gas'

## Calorimetry

### Heat (thermal) Capacity C

By definition,

**heat capacity (C) is the heat energy required to raise the temperature of a body by one degree** ( $^{\circ}\text{C}$  or K).

$$\Delta Q = C\Delta\theta$$

where,

- $\Delta Q$  is the heat energy added to the body
- $\Delta\theta$  is the temperature rise of the body
- $C$  is the heat capacity of the body

The units of heat capacity are Joules per degree. Since Kelvin and Celsius degrees are equivalent the units are:  $\text{JK}^{-1}$  or  $\text{JC}^{-1}$

### Specific Heat Capacity c

By definition,

**specific heat capacity (c) is the heat energy required to raise the temperature of unit mass by one degree** ( $^{\circ}\text{C}$  or K).

$$\Delta Q = mc\Delta\theta$$

where,

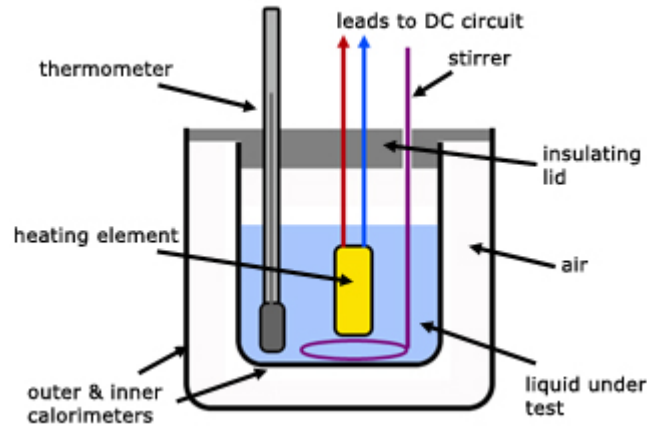
- $\Delta Q$  is the amount heat energy concerned
- $m$  is the mass of the body
- $\Delta\theta$  is the temperature rise of the body
- $c$  is the specific heat capacity of the body

The units of specific heat capacity are  $\text{Jkg}^{-1}\text{K}^{-1}$  or  $\text{Jkg}^{-1}\text{C}^{-1}$

### Determination of Specific Heat Capacity by experiment

These two methods concern the heating up a known mass and measuring the temperature rise for a known amount of electrical energy used.

Specific Heat Capacity of a **liquid** by an electrical method



The heat energy supplied by the electrical element is given to the liquid and its container, producing a temperature rise  $\Delta\theta$ .

The heater current ( $I$ ) and voltage ( $V$ ) are monitored for a time ( $t$ ).

energy supplied by heater =  $VIt$

energy absorbed by liquid and container =  $m_L c_L \Delta\theta + m_C c_C \Delta\theta$

where,

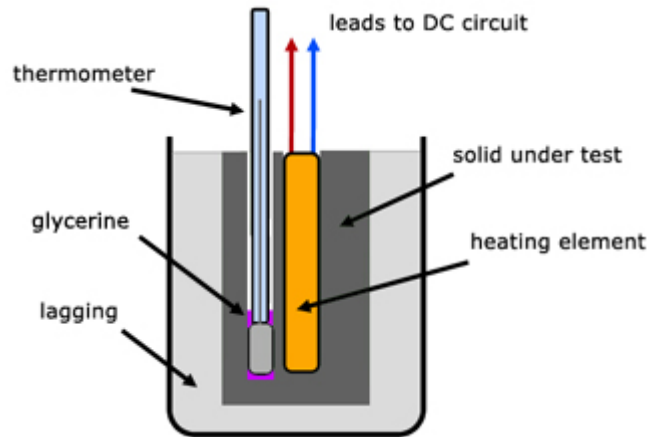
- $m_L$  mass of liquid
- $m_C$  mass of container
- $c_L$  specific heat capacity of liquid
- $c_C$  specific heat capacity of container

Equating the two quantities,

$$VIt = m_L c_L \Delta\theta + m_C c_C \Delta\theta$$

$m_L$ ,  $m_C$ ,  $c_C$  are known and  $V$ ,  $I$ ,  $t$ ,  $\Delta\theta$  are all measured. So the specific heat capacity of the liquid ( $c_L$ ) can be calculated.

Specific Heat Capacity of a **solid** by an electrical method



The method is very similar to that for a liquid except that there is no container. The solid under test is a lagged cylinder with holes drilled for the thermometer and the heater element. A little **glycerine** is added to the thermometer hole to improve thermal contact.

Heat energy supplied by the electrical element is given directly to the solid, producing a temperature rise  $\Delta\theta$ .

$$VIt = m_s c_s \Delta\theta$$

where,

$m_s$  - mass of solid

$c_s$  - specific heat capacity of solid

$m_s$  is known and  $V$ ,  $I$ ,  $t$ ,  $\Delta\theta$  are measured. So the specific heat capacity of the solid ( $c_s$ ) can be calculated.

note: more accurate results can be obtained by applying a '**cooling correction**'.

This is based on **Newton's Law of Cooling**, which states :

**The rate of cooling is proportional to the excess temperature over the environment.**

Unfortunately space does not allow a more in-depth treatment of this issue.

## Latent Heat

Latent heat is the energy involved when a substance changes state.

Latent heat energy ( $L$ ) has two components:

$\Delta U$  - the increase/decrease in internal PE

$\Delta W$  - the external work involved in expansion(+ $\Delta W$ ) and contraction(- $\Delta W$ )

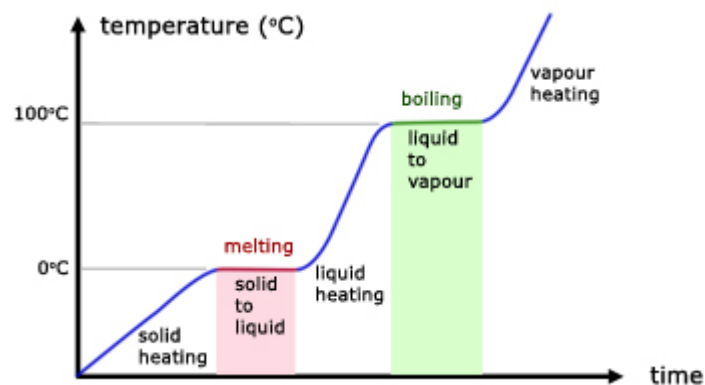
This can be summarized as:

$$L = \Delta U + \Delta W$$

The phase changes involving latent heat energy are:

phase change	action	symbol
solid to liquid	melting	$L_F$
liquid to solid	fusion	$L_F$
liquid to vapour	vaporization	$L_V$
vapour to liquid	condensation	$L_V$
solid to vapour	sublimation	$L_S$
vapour to solid	sublimation	$L_S$

The graph illustrates the temperature changes when a solid(eg ice) is heated from below its melting point, to above boiling.



Note that the changes of state occur in the flat areas. There is no temperature rise here and hence no increase in KE.

latent heat must be **absorbed** from the surroundings (and given to the substance) for the substance to **melt** or **boil**.

Latent heat is **given out** to the surroundings (from the substance) when the substance **condenses or freezes**.

### Specific Latent Heat Capacity $l$

By definition,

the **latent heat of fusion** of a substance is the energy involved in changing the state of unit mass of the substance at the melting/freezing point.

the latent **heat of vaporization** of a substance is the energy involved in changing the state of unit mass of the substance at the boiling point.

This may be summarized by the equation:

$$\Delta Q = ml$$

where,

- $\Delta Q$  is the amount heat energy concerned
- $m$  is the mass of substance
- $l$  specific latent heat of fusion/vaporization

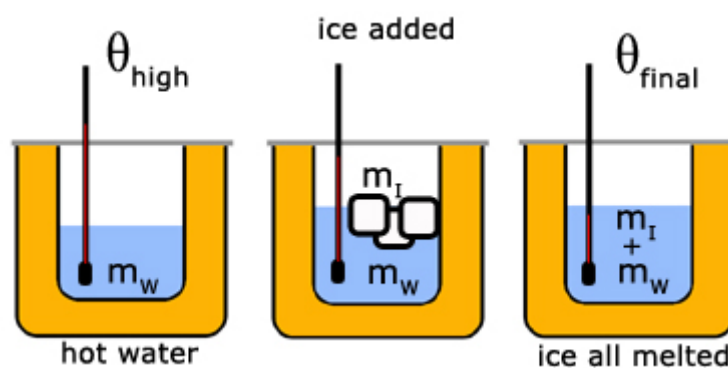
The units of specific heat capacity are  $\text{Jkg}^{-1}$ .

### Determination of Specific Latent heat Capacity by experiment

There are a number of different methods for finding  $l$  for different substances and different phase changes.

Here we will briefly look at two methods concerning fusion and vaporization.

#### The specific latent heat of ice by the 'method of mixtures'



Ice cubes are added to hot water of known temperature in a copper calorimeter. The mixture is stirred until all the ice has melted and a final reading of temperature taken.

$$\begin{aligned}
 & \text{heat energy lost from hot water in cooling from } \theta_{\text{high}} \text{ to } \theta_{\text{final}} + \text{heat energy lost from calorimeter in cooling from } \theta_{\text{high}} \text{ to } \theta_{\text{final}} \\
 = & \text{heat energy used to melt ice at } 0^{\circ}\text{C} + \text{heat energy used to increase temperature of melted ice from } 0^{\circ}\text{C to } \theta_{\text{final}}
 \end{aligned}$$

$$\begin{aligned}
 & m_{\text{w}}c_{\text{w}}(\theta_{\text{high}} - \theta_{\text{final}}) + m_{\text{C}}c_{\text{C}}(\theta_{\text{high}} - \theta_{\text{final}}) \\
 = & m_{\text{I}}l + m_{\text{I}}c_{\text{w}}(\theta_{\text{final}} - 0)
 \end{aligned}$$

where,

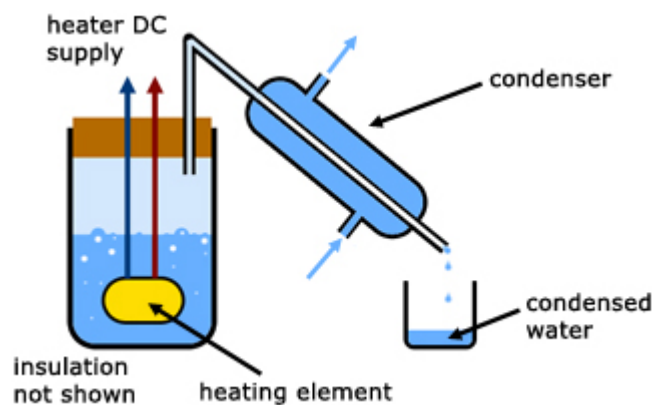
$m_{\text{L}}$  mass of water  
 $m_{\text{I}}$  mass of ice  
 $m_{\text{C}}$  mass of calorimeter

$c_{\text{L}}$  specific heat capacity of liquid water  
 $c_{\text{C}}$  specific heat capacity of calorimeter

$\theta_{\text{high}}$  temperature of the hot water  
 $\theta_{\text{final}}$  temperature of mixture

$l$  specific latent heat of fusion of ice

Hence  $l$  can be calculated from the knowns and measured values.

The specific latent heat of vaporization of a liquid

Water is heated electrically until it boils. The condensed water ( $m$ ) is collected over time ( $t$ ). Heating element readings of voltage ( $V$ ) and current ( $I$ ) are recorded.

In the steady state,

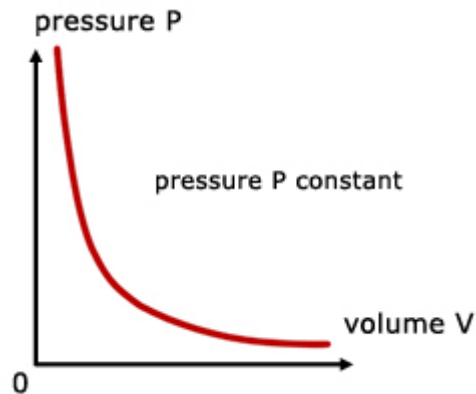
electrical energy supplied = heat energy to produce steam

$$VIt = ml$$



## The Gas Laws

### Boyle's Law



**For a fixed mass of gas at constant temperature and pressure, the pressure is inversely proportional to the volume.**

$$p \propto \frac{1}{V}$$

making the proportionality into an equality,

$$pV = k$$

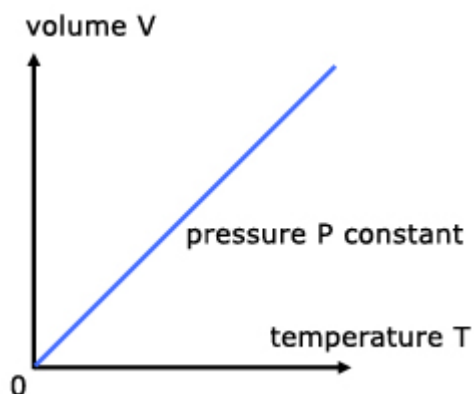
where ***k*** is a constant

Now, consider a fixed mass of gas at one temperature at different pressures and volumes,

$$p_1V_1 = k \quad p_2V_2 = k$$

eliminating the constant ***k***

$$p_1V_1 = p_2V_2$$

Charles' Law

**For a fixed mass of gas at constant temperature and pressure, the volume is directly proportional to the temperature(K).**

$$V \propto T$$

making the proportionality into an equality,

$$V = mT$$

where ***m*** is a constant

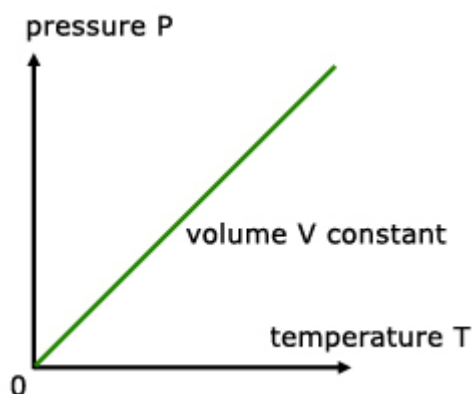
$$\frac{V}{T} = m$$

Now, consider a fixed mass of gas at one pressure at two different temperatures and volumes,

$$\frac{V_1}{T_1} = m \quad \frac{V_2}{T_2} = m$$

eliminating the constant ***m***,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Pressure Law

**For a fixed mass of gas at constant volume and pressure, the pressure is directly proportional to the temperature(K).**

$$p \propto T$$

making the proportionality into an equality,

$$p = nT$$

where  $n$  is a constant

$$\frac{p}{T} = n$$

Now, consider a fixed mass of gas at one volume at two different temperatures and pressures,

$$\frac{p_1}{T_1} = n \quad \frac{p_2}{T_2} = n$$

eliminating the constant  $n$ ,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Combined gas equation

The three gas law equations, with constants  $k, m, n$  are :

$$pV = k \quad \frac{V}{T} = m \quad \frac{p}{T} = n$$

These can be combined into one equation:

$$\frac{pV}{T} = K$$

where  $K$  is a new constant

Now, consider a fixed mass of gas at two different temperatures, volumes and pressures,

$$\frac{p_1 V_1}{T_1} = K \quad \frac{p_2 V_2}{T_2} = K$$

eliminating the constant  $K$ ,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

The Mole(mol)

**A Mole is the amount of substance that has the same no. of particles as there are atoms in 12g of carbon 12.**

or

**A Mole of anything contains the Avagadro number of particles.**

$$\text{Avagadro's Number (N}_A\text{)} = 6.022 \times 10^{23} \text{ mol}^{-1}$$

A Mole is a very large number.

Consider the Earth's population at present  $6 \times 10^9$ . A Mole of people would be equivalent to  $10^{14}$  Earths !

The **mass** of one mole of a substance is the relative molecular mass(RMM) of a substance expressed in grams.

eg 1 mole of molecular hydrogen (H<sub>2</sub>) has a mass of 2g

The Ideal Gas Equation

The combined gas equation,

$$\frac{pV}{T} = K$$

can be modified to take account of the amount of gas in units of moles. This is done by making the constant **K** a function of the number of moles *n* of gas present .

$$K = nR$$

**R** is the **Universal Gas Constant** (= 8.31 JK<sup>-1</sup>mol<sup>-1</sup> )

The 'equation of state' for an ideal gas is then given by:

$$pV = nRT$$

An 'ideal gas' is not a perfect model, but it is a good approximation.

The concept is based on the assumption that gas internal energy is only kinetic in nature. The equation is accurate for real gases at low pressures and at temperatures well above liquefaction.

Units

**V** - volume **cubic metres**  $\text{m}^3$

**p** - pressure **Pascals** Pa (1 Pa = 1 Newton per square metre)

**T** - temperature **Kelvin** K

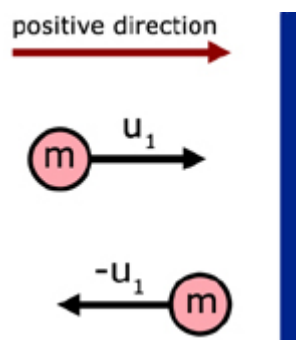
## The Kinetic Theory of Gases

### Main assumptions of the Kinetic Theory

1. all the molecules of a particular gas are equal
2. collisions between molecules and their container are completely elastic
3. collisions between molecules themselves are completely elastic
4. the size of actual molecules is negligible compared to molecular separation
5. the laws of Newtonian mechanics apply
6. extremely large numbers of molecules mean that statistical methods can be applied
7. between collisions molecules move in straight lines at constant speed
8. the motion of molecules is random
9. gravitational effects are negligible
10. the time for each collision is negligible

### Derivation of equations

The first step in understanding this theory is to appreciate the momentum change when a particle rebounds from a collision with a wall.

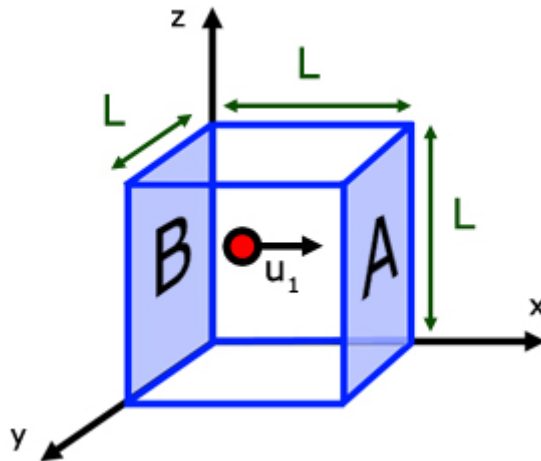


initial mtm. in the x-direction      =  $mu_1$

final mtm. in the x-dir.              =  $m(-u_1) = -mu_1$

momentum change in the x-dir.      =  $mu_1 - (-mu_1) = \mathbf{2mu_1}$

Consider a volume of gas in a cuboidal shape of side  $L$ .



We have seen how the change in momentum of a molecule of gas when it rebounds from one face, is  $2mu_1$ .

The distance the molecules travels between collisions is  $2L$ .

It collides with face A. Moves a distance  $L$  to collide with opposite face B, before returning to face A.

Since speed = distance/time,      time = distance /speed

Therefore the time ( $t$ ) it takes for the molecule to traverse this distance  $2L$  is given by:

$$t = \frac{2L}{u_1}$$

The rate of momentum change ( $dp/dt$ ) in this time interval is given by:

$$\text{rate of momentum change} = \frac{\text{momentum change}}{\text{time for change to take place}}$$

$$\begin{aligned} \frac{dp}{dt} &= \frac{2mu_1}{t} \\ &= \frac{2mu_1}{\left(\frac{2L}{u_1}\right)} \\ &= 2mu_1 \left(\frac{u_1}{2L}\right) \end{aligned}$$

$$\underline{\underline{\frac{dp}{dt} = \frac{mu_1^2}{L}}}$$



From Newton's 2nd Law, applied force is equal to the rate of change of momentum.

The molecule therefore exerts a force  $F$  on the wall, given by:

$$F = \frac{mu_1^2}{L}$$

Since pressure ( $p$ ) = force/area , the pressure on wall A produced by a single molecule is,

$$\begin{aligned} p &= \frac{F}{L^2} \\ &= \frac{\left(\frac{mu_1^2}{L}\right)}{L^2} \\ &= \left(\frac{mu_1^2}{L}\right) \frac{1}{L^2} \\ &= \frac{mu_1^2}{L^3} \\ \underline{p} &= \underline{\frac{mu_1^2}{L^3}} \end{aligned}$$

Now, consider all the molecules ( $N$ ) in the cube and each of their x-component velocities ( $u_1 u_2 u_3 \dots u_N$ ).

The pressure on wall A becomes:

$$p = \frac{m}{L^3} N \overline{u^2}$$

where,

$\overline{u^2}$  is the mean square velocity of molecules in the x-direction

The density ( $\rho$  rho) of the gas is given by:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

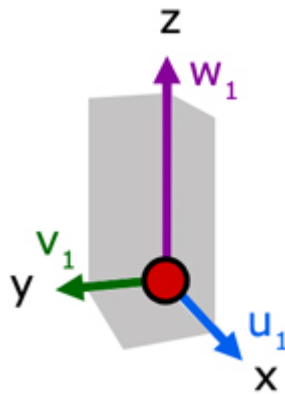
$$\rho = \frac{Nm}{L^3}$$

substituting for  $\frac{Nm}{L^3}$  into the equation for pressure  $p$

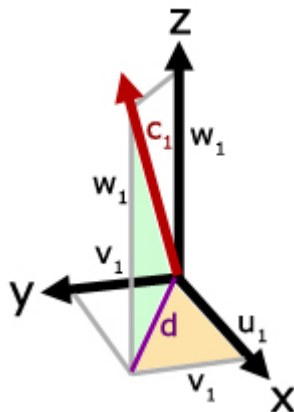
$$p = \frac{m}{L^3} N \overline{u^2}$$

$$p = \rho \overline{u^2}$$

Up to now we have considered the velocity of a molecule in one direction. Now, let us consider the resultant velocity of a molecule in three dimensions.



In the diagram, molecule velocity ( $c_1$ ) is resolved into  $x$ ,  $y$  and  $z$  directions. The value of each component is respectively,  $u_1$ ,  $v_1$  and  $w_1$ . Side  $d$  is the projection of  $c_1$  on the  $x$ - $y$  plane.



Using Pythagoras' Theorem,

$$d^2 = u_1^2 + v_1^2$$

$$c_1^2 = d^2 + w_1^2$$

substituting for  $d^2$  in the second equation,

$$c_1^2 = u_1^2 + v_1^2 + w_1^2$$

The equation represents component velocities for one molecule.

Replacing these velocities with mean square velocities, the equation now applies for all molecules.

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

We make the assumptions that there are very large numbers of molecules and their motion is random. So we can say that mean square velocity components are equal to one another.

$$\overline{u^2} = \overline{v^2} = \overline{w^2}$$

So molecular velocity  $c$  in terms of components becomes,

$$\overline{c^2} = 3\overline{u^2}$$

$$\overline{u^2} = \frac{1}{3}\overline{c^2}$$

Substituting for  $\overline{u^2}$  into the equation for pressure previously obtained,

$$p = \rho\overline{u^2}$$

$$p = \frac{1}{3}\rho\overline{c^2}$$

Molecular KE and temperature

$$p = \frac{1}{3} \rho \overline{c^2}$$

Multiplying the Kinetic Theory equation for pressure by  $V$ , (the volume of the gas) we obtain:

$$pV = \frac{1}{3} \rho V \overline{c^2} \quad (i)$$

but density  $\rho$  (rho) is given by:

$$\rho = \frac{M}{V}$$

making the mass  $M$  the subject, (where  $M$  is the mass of gas)

$$M = \rho V$$

substituting for  $\rho V$  into the Kinetic Theory equation (i),

$$pV = \frac{1}{3} M \overline{c^2} \quad (ii)$$

With some simple arithmetic and a more detailed description of  $M$ , this equation can be amended into a more useful form:

$$\frac{1}{3} = \frac{2}{3} \times \frac{1}{2}$$

If  $N$  is the total number of molecules and  $m$  is the mass of one molecule:

$$M = Nm$$

Now, substituting for  $1/3$  and  $M$  into equation (ii),

$$pV = \frac{2}{3} N \left( \frac{1}{2} m \overline{c^2} \right)$$

The ideal gas equation is,

$$pV = nRT$$

where,

$n$  is number moles of gas  
 $R$  is the Universal Gas Constant  
 $T$  is the temperature in kelvin

Eliminating  $pV$  between the last two equations,

$$\frac{2}{3} N \left( \frac{1}{2} \overline{mc^2} \right) = nRT$$

Making  $\left( \frac{1}{2} \overline{mc^2} \right)$  the subject of the equation,

$$\frac{1}{2} \overline{mc^2} = \frac{3}{2} \frac{nR}{N} T \quad (\text{iii})$$

The Avagadro Number  $N_A$  is by definition the number of molecules per mole. It is obtained by dividing the total number of molecules by the number of moles of matter:

$$N_A = \frac{N}{n}$$

We can now modify equation (iii) to include  $N_A$  by substituting for  $n/N$ ,

$$\frac{1}{2} \overline{mc^2} = \frac{3}{2} \frac{R}{N_A} T$$

By definition the Boltzmann's constant  $k$  is given by:

$$k = \frac{R}{N_A}$$

So the final form of the equation is:

$$\underline{\underline{\frac{1}{2} \overline{mc^2} = \frac{3}{2} kT}}$$

$\frac{1}{2} \overline{mc^2}$  is called the **average translational KE of a molecule**

So the average kinetic energy of gas molecules is proportional to the temperature.

This can also be said in the converse: temperature is a measure of the average kinetic energy of gas molecules.

## Thermodynamics

### The Zeroth Law of Thermodynamics

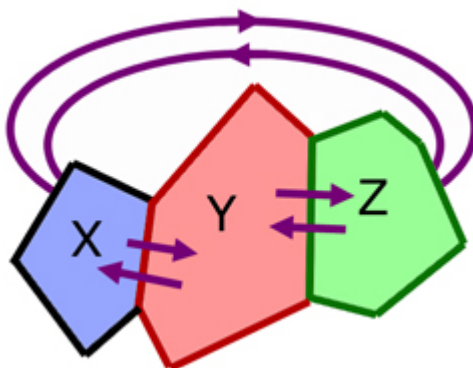
This states:

**If two systems are in thermal equilibrium with a third system, they are also in thermal equilibrium with each other.**

To understand this concept we must first appreciate what **thermal equilibrium** is.

Consider a body at a high temperature in contact with a body at a low temperature. Heat is transferred from the high temperature body to the lower temperature body until the temperatures are equalized.

When this equal, constant temperature is reached and maintained, the two bodies are said to be in thermal equilibrium.



Consider three bodies, **X**, **Y** and **Z**.

**Z** is in thermal equilibrium with **Y**.

**X** is in thermal equilibrium with **Y**.

Then **Z** is in thermal equilibrium with **X**.

To try to visualize this further, consider a hot cup of tea.

After about twelve hours, the saucer, the cup and the tea will all be at the same temperature.

The **saucer** is in equilibrium with the **cup**.

The **cup** is in equilibrium with the **tea**.

Therefore the **tea** is in equilibrium with the **saucer**.

They are each in thermal equilibrium with each other.

### The First Law of Thermodynamics

This states:

**The change in the internal energy\* (  $\Delta U$  ) of a system is equal to the amount of heat supplied (  $\Delta Q$  ) to the system, minus the amount of work (  $\Delta W$  ) performed by the system on its surroundings.**

\*Only internal energy changes can be measured. Absolute values of internal energy are not defined.

This can be stated as the equation:

$$\Delta U = \Delta Q - \Delta W$$

note:

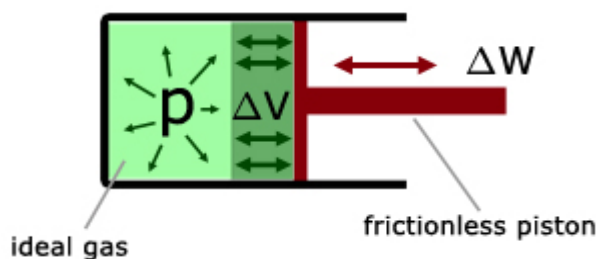
$\Delta Q$  is positive when heat is absorbed by the system

$\Delta Q$  is negative when heat flows away from the system

$\Delta W$  is negative when work is taken out of the system

$\Delta W$  is positive when work is put into the system

This can be better understood if we consider a mass of gas in a piston arrangement (frictionless piston, an ideal gas in the cylinder).



Work is done when the volume of the gas changes.  
By considering dimensions it can be shown that :

$$\text{work (} \mathbf{W} \text{)} = \text{constant pressure (} \mathbf{p} \text{)} \times \text{change in volume (} \mathbf{\Delta V} \text{)}$$

$$[\mathbf{J}] = \left[ \frac{\mathbf{N}}{\mathbf{m}^2} \right] \times [\mathbf{m}^3] = [\mathbf{Nm}] = [\mathbf{J}]$$

$$W = p\Delta V$$

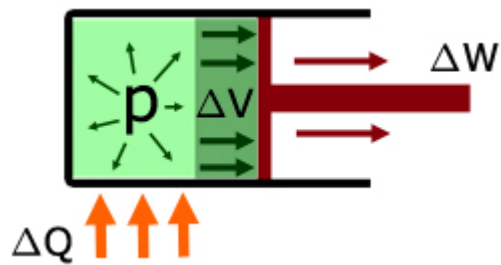
When heat energy  $\Delta Q$  is supplied to the gas :

the temperature of the gas increases

work  $\Delta W$  is done by the gas expanding to move the piston

the internal energy of the gas increases  $\Delta U$

the volume of the gas increases  $\Delta V$



$$\Delta U = \Delta Q - \Delta W$$

note  $\Delta W$  is negative - work is removed from the system

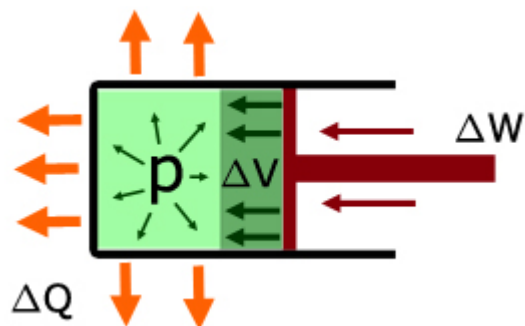
When no heat is applied and work is done externally by pushing the piston inwards to compress the gas:

the temperature of the gas increases

work  $\Delta W$  is done compressing the gas

the internal energy of the gas increases  $\Delta U$

the volume of the gas decreases  $\Delta V$





$$\Delta U = -\Delta Q + \Delta W$$

note:

$\Delta W$  is positive - work is done on the system

$\Delta Q$  is negative - heat is removed from the system

### The Second Law of Thermodynamics

There are a number of statements outlining this law.

#### The Clausius Statement

**No process is possible whose sole result is the transfer of heat from a body of lower temperature to a body of higher temperature.**

#### The Kelvin Statement

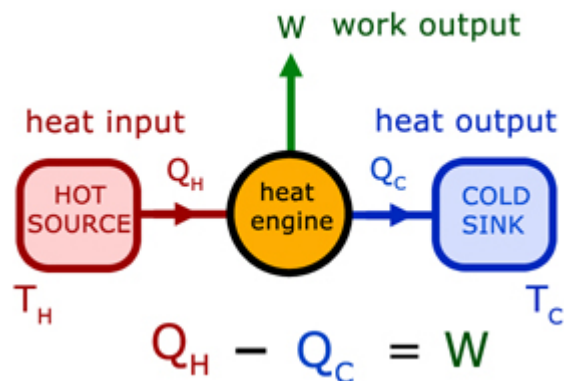
**No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work.**

#### A general statement

**It is not possible to convert heat continuously into work without at the same time transferring some heat from a warmer body to a colder one.**

### Heat engines & heat pumps

**Heat engines** are devices that use the flow of energy from a hot source to a cold sink to produce work.



The **Thermal Efficiency** ( $\eta$  - eta) is given by:

$$\eta = \frac{T_H - T_C}{T_H} \times 100$$

This also applies to heat pumps.

In reality the efficiency of an actual engine (eg a petrol engine) is much less than theory predicts. There are two main reasons:

- 1) frictional effects
- 2) heat is absorbed and emitted over a range of temperatures

**Heat pumps** are devices that use work to alter the energy flow from cold to hot (eg a refrigerator).

