PLUCKED STRINGS - A REVIEW

Neville H. Fletcher
Department of Physics
University of New England
Armidale N.S.W. 2351 Australia

Introduction

The behaviour of plucked strings is, generally speaking, of comparatively little interest to the players of bowed-string instruments, though it is, of course, of direct concern to the makers and players of guitars and harpsichords. Despite the truth of this statement, it is worthwhile for all of us to have a good understanding of the physics of plucked strings for several reasons - the phenomena involved are all relevant to the more complex case of bowed strings, the physical conditions are more easily specified and studied, and the mathematical analysis is nearly linear and therefore comparatively simple to carry out.

It is not the purpose of this paper to set out a complete analysis of the plucked string, for that is not yet available, but rather to discuss some of the more important points which are of use in other contexts. Most of the analysis is available in various places in the literature, to which references are given, but I have not tried to be complete about this - my survey is meant to be informative rather than scholarly.

Simple Strings

Let us begin our discussion with the simplest possible case - a 'perfect' string - and then add in features until we approach the behaviour of a real string on a musical instrument. In this way we will appreciate the relative importance and effect of each physical addition. I will necessarily use a little mathematics to make the discussion more precise but this need not be followed in detail to understand the conclusions.

Suppose we have a string of length L made from wire of radius r and density ρ and fixed with tension T between rigid supports. If we take the x-axis along the string and concern ourselves with transverse vibrations in the y and z directions, then the equation governing these[1] is

$$\pi r^2 \rho \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2} \tag{1}$$

with a similar equation for the z displacement. It actually makes sense to use the elastic stress S in the string rather than the tension T, since this is more closely related to the physical state of the string material, and

$$S = T/\pi r^2 \tag{2}$$

This stress can be related to the natural length L_o and stretched length L of the string by

$$S = Y(L - L_o)/L_o \tag{3}$$

where Y is the Young's modulus of the string material.

The condition that the string supports must be rigid means that $y = 0$ and $z = 0$ at $x = 0$ and L and, with these boundary conditions, the possible modes of vibration of the string (its 'normal modes') can be found. These have frequencies ν_n which are integer multiples of the fundamental frequency ν_1

$$\nu_n = n\nu_1 = \frac{n}{2L}\sqrt{\frac{S}{\rho}} \tag{4}$$

and the general form

$$y_n = a_n \sin(n\pi x/L)\cos(2\pi\nu_n t + \phi_n) \tag{5}$$

It will later be convenient to introduce exponential notation to write this as

$$y_n = a_n \sin(n\pi x/L)\exp(-2\pi i\nu_n t + i\phi_n) \tag{6}$$

and consider only the real part.

Because all these vibrations can coexist on the string without interference, a general vibration of the string has the form

$$y = \sum_n y_n \tag{7}$$

where the amplitudes a_n and phases ϕ_n of each string mode are determined by the way in which the string is set into vibration.

In our case, suppose that the string is plucked at a distance ℓ from one end and released at time $t = 0$. Then all the phases ϕ_n are zero and a Fourier analysis shows that the amplitudes a_n are given by

$$a_n = \frac{2a}{(n\pi)^2} \frac{L^2}{\ell(L-\ell)} \sin\left(\frac{n\pi\ell}{L}\right) \tag{8}$$

where a is the initial pluck deflection.

It is useful to note several things about this result. The first is the occurrence of n^2 in the denominator, so that the amplitudes of the upper modes are small compared with the fundamental. The second is the form of the sine term which shows that if $\ell/L = 1/N$, where N is an integer, then a_N, a_{2N}, a_{3N},... are all zero.

In a later section we shall see that the string does not radiate much sound itself, but that the radiation is accomplished by transmission through the end supports to the soundboard. It is therefore useful to know the form of the deflecting force on these supports. In the y direction it is simply

$$F = T\, dy/dx \tag{9}$$

and, since y is given in components by (7), (6) and (8), we can write the nth component of F as

$$F_n = \frac{2aT}{n\pi} \frac{L}{\ell(L-\ell)} \sin\left(\frac{n\pi\ell}{L}\right) \tag{10}$$

Again if $\ell/L = 1/N$ the corresponding harmonics are missing, and the height of the intervening maxima, where the sine term is near unity, decrease as $1/n$. Well below the first missing harmonic, however, the sine can be expanded to nearly cancel some of the other factors and this gives

$$F_n \approx 2aT/L, \qquad n \ll N \tag{11}$$

This behaviour is illustrated in fig. 1. The sideways force on the bridge actually has the asymmetric square-wave form shown (very different from the triangular-wave deflecting force produced by a bowed string), the ratio of the durations of the positive and negative segments being as $\ell : (L-\ell)$. The spectrum of the deflecting force shows the characteristics that we have discussed above.

The plucking situation we have assumed above is one in which the plectrum is ideally sharp and hard. This is approximately true of a quill plectrum in a harpsichord and of a plastic or ivory plectrum for an instrument like the guitar. Often, however, a string may be plucked by a leather plectrum or simply with the fingers and this modifies the plucking condition considerably.

The essential difference in the case of a soft broad plectrum is that the initial shape of the string, before it is released, is not two straight segments meeting at a sharp angle, but rather two straight segments joined by a smooth curve. If the width of the soft plectrum is δ, then modes with wavelengths

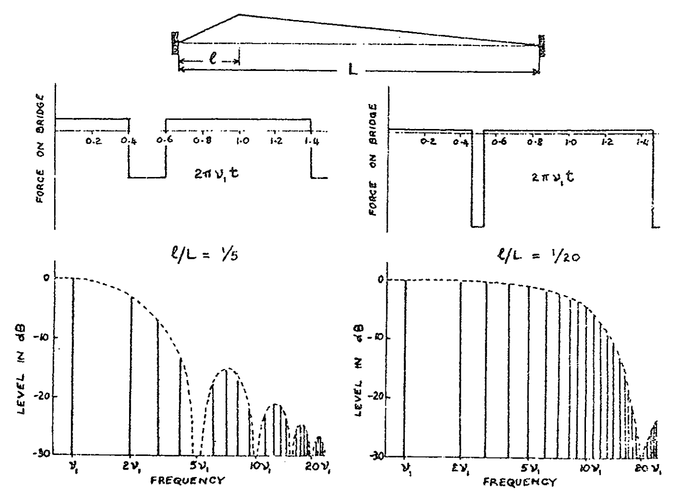

Fig. 1 *The form of the transverse force on the bridge and the frequency spectrum of this force for the case of a string plucked at (a) one-fifth and (b) one-twentieth of its length from one end.*

shorter than about 2δ are very little excited. This is equivalent to a high-frequency cut-off in the string spectrum at a mode number $n \simeq L/\delta$. In typical cases of finger-plucked instruments this might limit the spectrum to $n < 50$ which will greatly modify the sound compared with that from a sharp plectrum.

String Stiffness

Real strings, particularly rather thick strings made of metal, have stiffness in addition to their tension. It is fairly easy to include this in our original equation (1) which then has the more complicated form[2]

$$\pi r^2 \rho \frac{\partial^2 y}{\partial t^2} = \pi r^2 Y \frac{(L-L_o)}{L_o} \frac{\partial^2 y}{\partial x^2} - \tfrac{1}{2}\pi r^4 Y \frac{\partial^4 y}{\partial x^4} \qquad (12)$$

where we have used (2) and (3) to re-express the tension in terms of the string extension and the elastic modulus Y. The last term is the restoring force due to stiffness.

There does not seem to be any way to solve this equation exactly, but it is quite easy to find an approximate solution if the string is not too stiff[2]. The mode frequencies are

$$\nu_n \simeq n\nu_1^0[1 + \beta + \beta^2 + \frac{n^2\pi^2}{8}\beta^2] \qquad (13)$$

where ν_1^0 is the fundamental frequency (4) of the same string without stiffness, and

$$\beta = \frac{r^2}{L^2} \cdot \frac{L_o}{(L-L_o)} \qquad (14)$$

The first terms β and β^2 in (13) simply increase the fundamental frequency, and with it the frequencies of all the overtones, because of the added stiffness. The final term in (13) depends on n^2 and so increases the frequencies of higher overtones much more than lower overtones. The string is thus inharmonic, since ν_n is no longer an integer multiple of ν, and this gives the sound a slightly bell-like character.

This effect is well known in piano strings and causes the octaves to be stretched slightly more than their normal 2 : 1 ratio to bring the intervals to a minimum-beat condition of in-tuneness[3].

From (14) it is clear that, if we want to make the overtones as nearly harmonic as possible, which is generally desirable if we are to make an instrument to play in consort with sustained-tone instruments whose overtones are generally in exact harmonic relation, then we should make r/L as small as possible (i.e. use very thin strings) and make $(L-L_o)/L_o$ as large as possible (i.e. use a string material which will stretch a lot without breaking - this is generally the same as possessing a low Young's modulus).

These considerations would lead us to select fine-gauge strings made from gut or nylon for all our plucked-string instruments but, as we see presently, other criteria may lead us to modify this choice in many cases.

Non-linearity

All our treatment so far has been linear, in that the string displacement y in equations like (12) occur only to the first power. It is this linearity which allows us to treat all the normal modes y_n as independent, and to add them together as in (7) to obtain a general solution.

In reality there is a complication, for the string length L in (12) is greater than the distance between the supports because the vibrations themselves stretch the string[4]. In fact we should replace L by

$$L \rightarrow L[1 + \frac{1}{8}\sum_n a_n^2 + \frac{1}{8}\sum_n a_n^2 \cos(4\pi\nu_n t + 2\phi_n)] \qquad (15)$$

so that not only is L increased, which raises the frequencies of all the modes, but new driving terms are produced at twice the mode frequencies[5].

These effects couple all the modes together at sum and difference frequencies and, if the modes are not exactly harmonic, beat-like effects can be produced. The interactions also tend to generate components to fill in the spectral gaps predicted by the plucking-point relation (8).

Again the effect is reduced for given amplitudes a_n if $(L - L_o)/L$ is large suggesting the use of a string material with low Young's modulus.

Bridge and Soundboard

In our treatment so far we have assumed the end supports of the string to be rigid. Vibrational energy is therefore confined to the string and the solutions (5) of their more complex forms show no decay in time. We have not yet considered energy radiation from the string, apart from stating that it is an inefficient process, nor have we considered other loss mechanisms. As the next step in our development we recognise that the end supports are not rigid[6] and one at least generally consists of a light bridge attached to a soundboard, the prime purpose of which is to take vibrational energy from the string and impart it to the soundboard, from which it can be more efficiently radiated.

If we define the mechanical impedance of a support to be the ratio between the force applied to it and the velocity with which it moves, then we will always be dealing with supports with high impedance. The impedance is generally a complex quantity, in that the support does not move exactly in phase with the force, and its real and imaginary parts have different significance.

If the support has an impedance whose imaginary part is mass-like, so that the velocity lags behind the applied force, then its effect is to raise slightly the frequencies of the normal modes. On the other hand, if the imaginary part is spring-like, the normal mode frequencies are lowered. For strings and bridges of normal construction the shifts will only be small, but they will generally be slightly different for vibrations in the y and z directions, leading to further complications of the type discussed in the previous section. The impedance of the support will generally change with frequency so that the shifts for different modes may also be different.

We will be concerned, however, primarily with the real part of the impedance, or rather with the real part of its reciprocal, the admittance. This we call the conductance and denote by G, a quantity which is always positive, representing energy transfer from the string through the bridge to the soundboard. Contributions to G come partly from unavoidable losses in the bridge and soundboard material, similar to those which we discuss presently for string material, and partly from losses by sound radiation. This latter useful component is maximized, relative to the non-useful losses, by careful selection of soundboard material and by making the bridge and soundboard structure as light as possible. On the other hand the soundboard must be mechanically strong and must have its resonances so distributed that G is fairly smooth (and perhaps nearly constant) over the range of frequencies generated by the strings. We shall not go into soundboard design here but simply assume G to have some constant small value independent of frequency. The bridge design will generally be such that G is a maximum for either the y or z polarization of string vibrations, depending on the normal playing method for the instrument, and we shall consider only this mode, denoting it by y.

In real instruments strings are often terminated at one end in a rigid bridge, the nut, and the impedance of this is effectively infinite ($G = 0$). The same approximation is also appropriate for an instrument like the guitar in which the string is stopped against a rigid metal fret. In the case of a string stopped by the finger, however, as in a violin playing pizzicato, the conductance of the soft finger tip material is large and losses here may dominate over the smaller losses through the bridge. In what follows we deal predominantly with the first case for which $G = 0$ at the inactive support.

For a given string mode n, the force component F_n is given by (10). The velocity component imparted to the bridge and thence to the soundboard is then

$$v_n = \alpha G F_n \tag{16}$$

where α is a constant allowing for any impedance change due to lever-like action of the bridge, as in the violin.

We are here basically concerned with string material, so let us suppose the length L of the string and its fundamental frequency ν_1 to be determined by instrument design. The plucking ratio ℓ/L may also be taken to have been determined by design or by the tone quality desired. Using (4) to eliminate T we find velocity amplitudes which vary as

$$v = \text{const} \times G\, a\, r^2 \rho\, \nu_1 \tag{17}$$

where r and ρ refer to string radius and density and a to pluck displacement.

Because radiated intensity varies as v^2, we see from (17) that, if a is fixed at some constant value determined by mechanical clearances or by the musically acceptable limit to the 'twang' caused by increased string tension, the intensity of the radiated sound is increased by using thick strings of dense material. If, on the other hand, it is the total plucking force which is fixed by other considerations, then $ar^2\rho$ is fixed so no freedom is available. Within reasonable limits the former situation applies to many instruments.

In a plucked-string instrument, of course, all the energy is supplied to the string by the initial plucking action and thereafter the vibrational energy decays. If the pluck displacement is a, then the initial energy is

$$E_0 = a^2 T[L/2\ell(L-\ell)]$$

or

$$E_0 \propto r^2 \rho a^2 \tag{18}$$

if L, ℓ and ν_1 are fixed. Now the rate of energy loss to the bridge is

$$-dE/dt = v^2/G \propto G a^2 r^4 \rho^2 \nu_1^2$$

or

$$dE/dt \propto -G r^2 \rho \nu_1^2 E \tag{19}$$

so that the energy, and hence the radiated intensity, falls as $\exp(-t/\tau_1)$ with a characteristic decay time

$$\tau_1 = \text{const}/G r^2 \rho \nu_1^2 \tag{20}$$

where the constant depends on string length and on frequency.

Generally speaking, long decay times are desirable, which at this stage argues for thin strings of light material. However, as we shall see later, energy loss to the soundboard is not the major damping mechanism in most practical cases, so we need not pursue this conclusion.

Air Damping

As mentioned before, a vibrating string is not a good sound radiator. The reason for this is that the string is a dipole source, producing a compression in front and a rarefaction behind, as it moves, and its radius is very small compared with the sound wavelength involved so that these effectively cancel each other. This does not mean, however, that the string has little interaction with the air. Indeed viscous flow of air around the moving string is often the major cause of damping of its vibrations.

The rather difficult problem of viscous drag on a vibrating string was solved long ago by Stokes[7] who showed that the force on the wire has two components. One is an additional mass-like load which is of little importance except for a small lowering of the mode frequencies; the second produces a simple exponential damping.

The analysis is complicated and the results depend both on wire radius and on frequency. Combining the damping terms with an expression for initial energy however we are led to the decay time τ_2 for this mechanism as[8,9]

$$\tau_2 \propto \rho r^2 \,, \quad \nu \ll r^{-2} \tag{21}$$

$$\tau_2 \propto \rho r \nu^{-1/2} \,, \quad \nu \gg r^{-2} \tag{22}$$

for r in millimetres and ν in Hertz. In this case clearly a long decay time requires a thick wire of dense material.

Internal Damping

Our string material has so far been characterized by its radius, its density and its Young's modulus, but more can be said than this. All real materials show an elastic behaviour in which, when a stress is applied, an instantaneous strain occurs and then, over some characteristic time τ, the strain increases slightly. This second elongation may be moderately large or extremely small and the time τ may be anything from less than a millisecond to many seconds. In visco-elastic materials the second elongation increases slowly but without limit.

Such behaviour can be represented, when we use the exponential form of equation (6), by making the Young's modulus for the material complex

$$Y = Y_1 + iY_2 \tag{23}$$

According to a relaxation formula due to Debye, Y_2 has a peak at the relaxation frequency $\omega = 1/\tau$. Equation (23) can however be used in the more general case where many relaxation times contribute, both Y_1 and Y_2 varying with frequency. This behaviour is simple to understand, Y_1 being contributed by normal elastic bond distortions and Y_2 by relaxation processes like dislocation motion or the movement of kinks in polymer chains. Typically Y_2/Y_1 may be less than 10^{-4} in hard crystals, rather larger in metals, and perhaps as large as 10^{-1} in some polymer materials, though in such cases it may also depend on temperature. One elastic constant is really inadequate to describe even isotropic materials but we shall neglect this added complication here.

If the complex form (23) is substituted into (12), neglecting any other losses, we find that the major contribution comes from the second term. We can in fact solve this equation formally to obtain a set of complex mode frequencies ω_n, the imaginary parts of which represent damping of the vibrations. In this way we easily find the decay time for this internal damping

$$\tau_3 = \frac{1}{\pi\nu}\,\frac{Y_1}{Y_2} \tag{24}$$

Clearly internal damping is a material property independent of string radius, length or tension. It is generally negligible for solid metal strings but may become the prime damping mechanism for gut or nylon strings or, even more particularly, for strings of nylon overspun with metal. The decay time due to this mechanism is clearly shortest at high frequencies if, as is often the case, Y_1 is nearly independent of frequency.

Decay Time

When, as will generally be the case in practice, all these energy-loss mechanisms occur simultaneously, the resultant decay time τ is given by

$$\tau^{-1} = \tau_1^{-1} + \tau_2^{-1} + \tau_3^{-1} \tag{25}$$

This relation is indicated schematically in fig. 2 on the assumption that G and Y_2 are independent of frequency. The curves show the behaviour of the various τ_i as functions of frequency on the assumption that we are dealing with a single string whose pitch is raised by reducing its length. Also indicated are the directions in which the various curves would move in response to increases in various string parameters. The curve for the resultant decay time τ is a smoothed lower envelope to the individual decay-time curves τ_i.

Overspun Strings

Compound strings consisting of a metal or nylon core overspun with heavy wire or metal tape are common in the lower range of many instruments. The functions of this construction are well-known - the overspinning adds extra mass to the string without appreciably increasing its stiffness, thus keeping its length, tension and inharmonicity within reasonable bounds.

In terms of our discussion, the overspinning can be treated simply as an addition to the mass and hence to the effective density of the core material (unless, as in the lower strings of a piano, a large increase in diameter is involved). From a practical point of view it should be noted that overspun strings in poor condition may have relatively large internal damping (effective Y_2) while from the point of view of theory certain other minor modifications are also necessary.

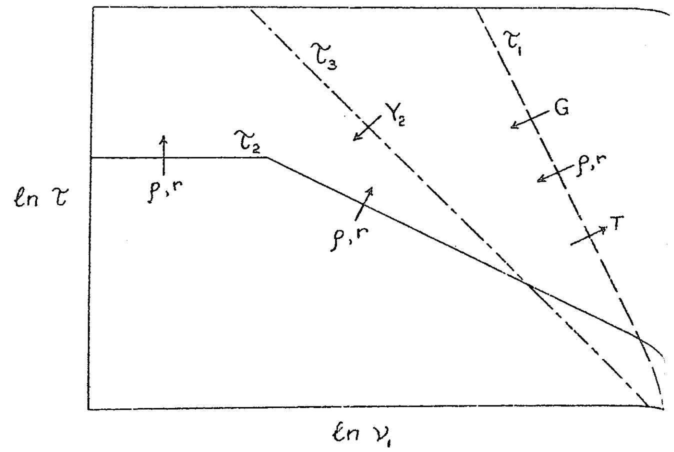

Fig. 2 *Schematic behaviour of the decay times τ_i caused by various mechanisms, as functions of the fundamental frequency ν_1 of the string, which is assumed to be varied by changing only the string length. τ_1 is determined by loss through the bridge to the soundboard, τ_2 by air damping and τ_3 by internal damping. Arrows indicate the directions in which the curves would be shifted by increase in string radius r, density ρ, tension T and imaginary part of its Young's modulus Y_2 and by the mechanical conductivity G of the bridges.*

Conclusions

Our analysis shows the various factors that can influence the sound of a plucked string, as far as string itself is concerned. Designers of practical musical instruments make use of this freedom to achieve both their basic design and a high degree of finesse in its realization. An example of the application of this procedure to the harpsichord has been given elsewhere[9].

To summarize our discussion we begin with the basic spectrum of a string plucked with a sharp hard plectrum at a pre-determined point, a fraction $1/\beta$ along its length. Spectral components are of nearly constant amplitude out to the harmonic of number $\beta/$ after which the energy falls at an overall rate of 6dB/octave with, superposed on this, a succession of minima or zeros at harmonics numbered $n\beta$ $(n = 1, 2, \ldots$

Use of a soft plectrum of width δ on a string of length L introduces a rather sharp cut-off for harmonics with numbers above L/δ, with consequent decrease in the brilliance of the tone.

Decay time affects the subjective loudness of the sound and the relative decay time of the different overtones affects also the brilliance and general tone quality. For thin metal strings the decay time is determined mostly by air viscosity and this gives a decay time for upper partials which varies as $\nu^{-\frac{1}{2}}$.

For instruments with gut or nylon strings, internal damping in the string material becomes dominant for most overtones and the decay time for upper partials varies as ν^{-1}. Such strings therefore have a much less brilliant sound than do metal strings.

If a finger tip is used to stop the string, if a felt pad is used against it, or if the bridge is so light and the string so heavy that end losses predominate, then the decay time for upper partials varies as ν^{-2} giving a very pronounced decrease in brilliance.

Rules such as these, or more quantitative results derived from the equations we have developed, can serve as useful guides in the design or adjustment of musical instruments, but it must always be the educated ear that acts as the final arbiter of what is appropriate to a given musical need.

Acknowledgements

This paper is an extended version of a talk given at the Catgut Acoustical Society Technical Conference in Montclair, N.J., in April 1975. Those who heard the original will recognise that I have profited greatly from discussion with other members of the Society during the weekend. In particular I would like to thank Jont Allen for several most useful comments.

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REFERENCES

1. See, for example, P.M. Morse 'Vibration and Sound' (New York: McGraw-Hill, 1948) 71-147
2. P.M. Morse, ref. 1, 166-170
3. O.H. Schuck and R.W. Young 'Observations of the vibrations of piano strings' J. Acoust. Soc. Amer. 15 1-11 (1943)
4. A detailed discussion in terms of coupling between transverse and longitudinal string waves (a more rigorous procedure) is given by P.M. Morse and U. Ingard 'Theoretical Acoustics' (New York: McGraw-Hill, 1968) 856-863
5. E. Lieber 'Moderne Theorien über die Physik der schwingenden Saite und ihre Bedeutung für die musikalische Akustik' Acustica 33 324-335 (1975)
6. P.M. Morse, ref. 1, 133-147
7. G.G. Stokes 'On the effect of internal friction of fluids on the motion of pendulums' Trans. Cambridge Phil. Soc. 9, 8 (1851) Reprinted in 'Mathematical and Physical Papers of Sir George Stokes' Vol.3 (Cambridge: Cambridge Univ. Press, 1922) 1-140
8. N.H. Fletcher 'Scaling rules for organ flue pipe ranks and for the string choirs of harpsichords' J. Acoust. Soc. Amer. 59, S50 (1976) [Abstract only]
9. N.H. Fletcher 'Analysis of the design and performance of harpsichords' Acustica (to be published)

MUSICAL OVERTONES AND THE ORIGINS OF VIBRATION THEORY

Sigalia Dostrovsky, Department of Physics, Barnard College, New York, N.Y.

Higher vibrational modes posed a perplexing problem in the seventeenth century. Although they could be heard, contemporary theory was unable to predict them; in fact, it even seemed to deny their existence. Nevertheless, higher modes could be studied because they were a musical phenomenon. In contrast with the situation today, when the physics of music is a part of the general subject of acoustics and vibration theory, most seventeenth century studies of vibration were studies of musical sound. Natural philosophers, many of whom had musical experience, listened to pitch, loudness, and timbre to do experiments on sound; they paid attention to the properties of musical instruments to learn about vibration; and they used the traditional Pythagorean ratios to obtain relative frequencies. I have recently written about the ways in which music led to the origins of vibration theory ("Early Vibration Theory: Physics and Music in the Seventeenth Century," Archive for History of Exact Sciences 14 (1975), 169-218). In this essay I shall summarize some of the main ideas associated with the discovery of higher modes.

Almost all seventeenth century discoveries in the physics of sound and vibration resulted from the realization that the sensation of pitch is appropriately quantified by vibrational frequency (that pitch corresponds to frequency). At least since the time of ancient Greece musical intervals had been represented by ratios obtained from relative lengths of similar strings, at the same tension, sounding these intervals. These length ratios formed the basis for the arithmetical music theories of antiquity and the middle ages. Towards the end of the Renaissance, when arithmetical dogmatism in music was being criticized, the ratios seemed arbitrary: why, for example, length rather than tension or thickness? To demonstrate this problem, around 1590, Vincenzo Galilei (the father of Galileo) did possibly the first experiments in acoustics. Soon afterwards, Mersenne and others understood that the traditional ratios are naturally determined by the vibrational frequency. Galileo expressed the idea clearly in the Two New Sciences (1638)

> "The length of strings is not the direct and immediate reason behind the forms (ratios) of musical intervals, nor is their tension, nor their thickness, but rather, the ratios of the numbers of vibrations and impacts of air waves that go to strike our eardrum...."

It was the identification of pitch with frequency that introduced the paradox of the overtones, because it implied that one object was vibrating simultaneously with a number of frequencies. Mersenne, who had chosen music as his specialty in the new science, discussed the problem in the Harmonie Universelle (1636), his encyclopaedic work on music and musical instruments. In his discussion of stringed instruments, Mersenne remarked that he had no trouble hearing at least four overtones (whose pitches he identified as the octave, the twelfth, the double octave, and the seventeenth). But he found it difficult to account for them:

> "(Since the string) produces five or six tones..., it seems that it is entirely necessary that it beat the air five, four, three, and two times at the same time, which is impossible to imagine, unless one says that half the string beats it twice while the whole string beats it once and that, in the same time, third, quarter, and fifth parts beat it three, four, and five times, a situation that is against experience...."

Higher modes were puzzling to Mersenne even when they were produced separately, as in the trumpet marine. This popular stringed instrument used nothing but overtones (before they became a familiar part of violin technique). It was played by bowing the string while touching it at one of the points that later would be identified as nodal points. Mersenne identified the locations of these points but he was surprised that the string produced an ugly tone when it was touched at other places, since a viol string sounded good no matter where it was stopped. Although Mersenne himself never understood higher modes, he appreciated the importance