Excitation Mechanisms in Woodwind and Brass Instruments

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Summary

A general analysis is given of the behaviour of musical instruments driven by a reed mechanism. Attention is concentrated on the reed mechanism itself and a clear distinction is drawn between the behaviour of reeds striking inwards (as in the clarinet, oboe and organ pipe) and reeds striking outwards (as in the trumpet and other brass instruments). Impedance curves are calculated for reed generators of each type and it is shown that sounding of the instrument requires that the acoustic admittance of the reed, as seen from inside the mouthpiece, should have a negative real part of larger magnitude than the real part of the pipe admittance. This implies that there is a minimum permissible blowing pressure for each reed configuration. A reed striking inwards must operate at a frequency below the reed resonance and below but close to the frequency of an impedance maximum for the pipe. A reed striking outwards must operate at a frequency above and close to the reed resonance and above and close to an impedance maximum for the pipe. Brief consideration is given to other matters, including non-linearities and their role in limiting oscillation amplitude and in generating harmonics.

Anregungsmechanismen in Holzblas- und Blechblasinstrumenten

Zusammenfassung

Das Verhalten von Musikinstrumenten, die durch einen Zungenmechanismus angeregt werden, wird in allgemeiner Form analysiert. Dabei wird dem Zungenmechanismus selbst besondere Aufmerksamkeit gewidmet. Es wird streng zwischen dem Verhalten nach innen arbeitender Zungen (wie bei der Klarinette, Oboe und Orgelpfeife) und dem nach außen arbeitender (wie bei der Trompete und anderen Blechblasinstrumenten) unterschieden. Für Zungengeneratoren beider Typen werden Impedanzkurven berechnet und es wird gezeigt, daß zur Schallabstrahlung des Instruments die vom Innern des Mundstücks aus gesehene Zungenadmittanz einen negativen Realteil haben muß, der betragsmäßig größer als der Realteil der Rohrimpedanz sein sollte. Dies impliziert einen minimalen zulässigen Blasdruck für jede Zungenkonfiguration. Eine nach innen arbeitende Zunge muß bei Frequenzen unterhalb der Zungenresonanz und kurz unterhalb einer Frequenz, für die die Rohrimpedanz ein Maximum aufweist, betrieben werden, eine nach außen arbeitende Zunge bei Frequenzen kurz oberhalb der Zungenresonanz und kurz oberhalb eines Impedanzmaximums des Rohrs. In kurzer Form werden auch andere Probleme, z.B. Nichtlinearitäten und ihre Rolle bei der Begrenzung der Schwingungsamplitude und bei der Erzeugung von Harmonischen betrachtet.

Les mécanismes d'excitation du son dans les instruments à vent (bois et cuivres)

Sommaire

On a procédé à une analyse générale du fonctionnement des instruments de musique à anche, en centrant l'attention sur le mécanisme de l'anche elle-même. Une claire distinction est établie entre le comportement des anches travaillant vers l'intérieur, comme dans la clarinette, le hautbois et le tuyau d'orgue, et celui des anches travaillant vers l'extérieur, comme dans la trompette et les autres cuivres. On a calculé les courbes d'impédance pour des générateurs à anches de chaque type et montré que l'émission sonore ne peut avoir lieu que si l'admittance acoustique de l'anche, vue de l'intérieur de l'embouchure, possède une partie réelle négative d'amplitude absolue supérieure à la partie réelle de l'admittance du tuyau. Cette condition implique qu'il existe une valeur minimale admissible pour la pression du souffle sur chaque configuration d'anche. Une anche travaillant vers l'intérieur doit fonctionner à une fréquence inférieure à celle de sa résonance et inférieure également, mais de peu, à la fréquence du maximum d'impédance du tuyau. Une anche travaillant vers l'extérieur doit opérer à une fréquence un peu supérieure à celle de sa résonance et un peu supérieure également à celle du maximum d'impédance du tuyau. D'autres questions sont examinées plus brièvement, comme celle des non-linéarités et de leur rôle dans la limitation des oscillations et dans la génération des harmoniques.

1. Introduction

The behaviour of musical wind instruments in which the sound is excited either by an air-driven reed or by an air-driven vibration of the player's lips has been the subject of study for more than a century, and very considerable advances in understanding have been made. A recent informal book by Benade [1], a more formal treatment by Nederveen [2] and a collection of reprints edited by Kent [3] serve to summarize the present situation.

Somewhat surprisingly, though the theory of wave propagation and resonance effects in the bores and finger holes of such instruments has been examined in detail, comparatively little attention has been given to the actual mechanism by which the sound is produced and coupled to the tube of the instrument. There are, in fact, several notable papers on reed mechanisms [4], [5], [6], [7] but the whole picture is not entirely clear, particularly in relation to the lip-driven brass instruments.

It is the purpose of the present paper to attempt to remedy this situation to some extent, not by treating a particular example in great detail but rather by developing a more general theory and investigating its implications for particular classes of instruments. Many of the results are not original, but they do not appear to have been presented in this form before.

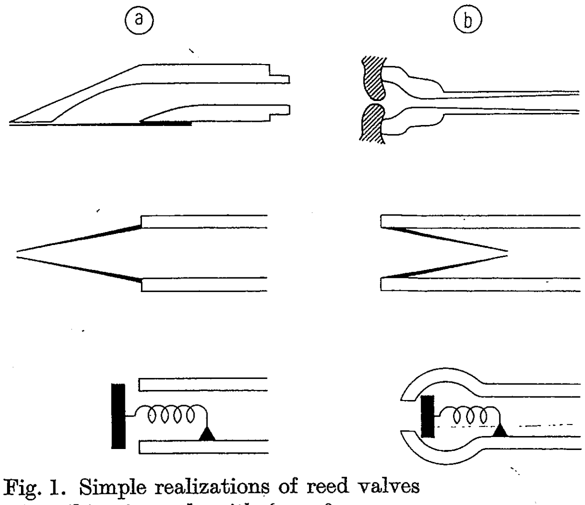

Fig. 1. Simple realizations of reed valves
(a) striking inwards with $\xi_0 > 0$,
(b) striking outwards with $\xi_0 < 0$.
All are assumed to be blown from the left and to communicate with a musical horn on the right.

2. Phase relations

The first explicit discussion of reed-driven wind instruments seems to be that of Helmholtz [4], who distinguished two cases as follows.

"The action of reeds differs essentially according as the passage which they close is opened when the reed moves against the wind towards the windchest, or moves with the wind towards the pipe. I shall say that the first strike inwards, and the second strike outwards. The reeds of the clarinet, oboe, bassoon, and organ all strike inwards. The human lips in brass instruments, on the other hand, are reeds striking outwards".

Helmholtz then analyzes the two cases in some detail. Because his results are a useful introduction we repeat them here but employ a rather different notation and method of development. The analysis is correct, as far as it goes, but omits a number of important features of the real problem.

Suppose the pressure in the mouthpiece of the instrument is $p \exp(\mathrm{j}\,\omega t)$ and that the coordinate describing the reed opening towards the air supply is ξ, so that the size of the opening varies like

$$\xi_0 + \xi \exp \mathrm{j}\,(\omega t + \varphi) \qquad (1)$$

where ξ_0 must be taken as positive for a reed striking inwards and negative for a reed striking outwards as shown in Fig. 1. Because the reed is a damped mechanical resonator driven in the ξ direction by the mouthpiece pressure, we can write

$$m_\mathrm{r}(-\omega^2 + \mathrm{j}\varkappa_\mathrm{r}\,\omega_\mathrm{r}\,\omega + \omega_\mathrm{r}^2)\,\xi e^{\mathrm{j}\varphi} = s_\mathrm{r}\,p \qquad (2)$$

where m_r and s_r are respectively the effective moving mass and the area of the reed, ω_r is its resonance frequency and $\varkappa_\mathrm{r}$ its damping coefficient. From this equation we immediately find for the phase angle φ,

$$\tan\varphi = \varkappa_\mathrm{r}\,\omega\,\omega_\mathrm{r}/(\omega^2 - \omega_\mathrm{r}^2) \qquad (3)$$

and φ varies from 0 for $\omega \ll \omega_\mathrm{r}$ to π for $\omega \gg \omega_\mathrm{r}$.

If we suppose the magnitude of the blowing pressure to be much greater than p, then the phase of the air flow into the instrument is φ for a reed striking inwards and $\varphi - \pi$ for a reed striking outwards.

Now the input impedance of a cylindrical pipe of length l and cross-section S which is damped predominantly by wall effects has, as is shown in the Appendix, the form

$$Z_\mathrm{p} = \left(\frac{\varrho c}{S}\right)\left(\frac{1 + \mathrm{j}H\tan kl}{H + \mathrm{j}\tan kl}\right) \qquad (4)$$

over that part of the frequency range where the height of the impedance maxima is H relative to the reference level $\varrho c/S$. ϱ is the density of air, c the speed of sound in air and $k = \omega/c$. This form of the result is useful since, by making both l and H to be slowly-varying functions of ω, it adequately represents the input impedance of a real musical instrument. If we write Z as $|Z| \exp(\mathrm{j}\,\theta)$ then

$$\tan\theta = [(H^2 - 1)/2H]\sin 2kl \qquad (5)$$

and the phase of the acoustic flow relative to the pressure is $-\theta$.

Now for the instrument to function in a steady manner, the phase of the acoustic flow must be constant throughout and therefore $\tan\theta = -\tan\theta$, which relation accommodates the requirements for both reed geometries. Using eqs. (3) and (5) this implies

$$\sin 2kl = 2\varkappa_r [H/(H^2-1)]\,\omega_r\,\omega/(\omega_r^2-\omega^2) \quad (6)$$

which is essentially the condition derived by Helmholtz. In order that the reed be a generator rather than a load, it is necessary that both sides of eq. (6) be positive for reeds striking inwards and negative for reeds striking outwards. In the first case, therefore, which applies to woodwind instruments, we must have $\omega < \omega_r$, while for brass instruments with lip drive $\omega > \omega_r$. The necessary requirement that the left side of eq. (4) be no greater than unity implies only that the fractional shift from reed resonance, $(\omega_r - \omega)/\omega_r$, be greater than about $\varkappa_r/H$ which is typically of order 10^{-2}.

Because eq. (6) contains none of the dynamics of the problem, it is necessary to appeal to other arguments to proceed further. If the reed is not very heavy, so that it is influenced by the pipe, and ω is not nearly equal to ω_r, then eq. (6) implies that $\sin 2kl = \pm\varepsilon$ where ε is a small positive quantity and the plus sign applies to woodwind and the minus to brass instruments. The solutions to this are $\omega \approx n\pi c/2l$ or $l \approx n\lambda/4$, $n = 1, 2, 3, \ldots$, but those with even values of n must be ruled out because they correspond to situations with a pressure node at the reed so that no drive can take place. When the non-zero value of ε is included we find

$$\omega = (2n+1)\,\pi c/2l \mp \delta = \omega_p \mp \delta \quad (7)$$

where δ is a small quantity determined by ε and now the minus sign applies to woodwind and the positive sign to brass instruments. The ω_p are resonance frequencies for a pipe stopped at the end where the reed is fitted.

3. Flow dynamics

The next extension of the theory is due to Backus [5] who included a treatment of air flow dynamics in a clarinet-like system but simplified matters, consistently with the conclusions drawn from eq. (6), by taking $\omega_r = \infty$, which is equivalent to neglecting the mass of the reed. This is largely justified for the actual case of a clarinet but restricts the generality of the results. This particular assumption was relaxed in later treatments, also for a clarinet-like system, by Worman [6] and by Wilson and Beavers [7]. All these workers, except Worman, were concerned with small-amplitude oscillations just above the self-excitation threshold so that the theories are essentially linear. They do however, give some information about excitation of the various pipe modes and, as for the predictions of Helmholtz's theory, agree with experiment when checked against appropriately circumscribed cases.

The primary innovation of these treatments relates to the dynamics of flow past the reed and in particular to inclusion of the effect of mouthpiece pressure on this flow. We shall develop this rather more generally so that it applies to reeds striking in either direction.

Suppose that the blowing pressure applied to the reed is p_0 and the pressure in the mouthpiece is p. Then in a mechanically static situation the volume flow U through the reed will be given by the Bernoulli result

$$U = |\xi|\,b\,[2(p_0-p)/\varrho]^{1/2} \quad (8)$$

where $|\xi|$ and b define the dimensions of the reed opening and ϱ is the air density. While this result is adequate for simple reed geometry, many practical situations are more complex, so we write

$$U = D|\xi|^\alpha (p_0-p)^\beta \quad (9)$$

where $\alpha \approx 1$ and $\beta \approx \frac{1}{2}$. In fact Bacus [5] has shown empirically that for a typical clarinet reed and mouthpiece $\alpha \approx 4/3$, $\beta \approx 2/3$.

The total equation of flow must take into account the fact that ξ depends on p and also that there is a mass-like load

$$M(\xi) = \varrho a/|\xi|\,b \quad (10)$$

associated with the air in the reed tip channel (or lip aperture) whose length we take as a. We can therefore write, using eq. (9),

$$p_0 - p = D'|\xi|^{-\alpha/\beta}\,U^{1/\beta} + M(\xi)\,(\partial U/\partial t) \quad (11)$$

where we have written D' for $D^{-1/\beta}$. We might also add a small resistive term $R(\xi)\,U$ to account for viscous losses in the reed tip channel but, since the effect of such a term is obvious, we omit it in the interests of simplicity. Because eq. (11) is clearly non-linear we can no longer simply add complex exponentials as in eq. (1) but must take

$$\xi \to \xi_0 + \sum_n \xi_n \cos(n\omega t + \varphi_n) \quad (12)$$

where the sums here and subsequently are over $n = 1, 2, 3, \ldots$ Again $\xi_0 > 0$ for an inward striking and $\xi_0 < 0$ for an outward striking reed. Similarly we write

$$U \to U_0 + \sum_n U_n \cos(n\omega t + \psi_n) \quad (13)$$

and, taking atmospheric pressure as reference,

$$p \to \sum_n p_n \cos(n\omega t + \zeta_n)\,. \quad (14)$$

The equation obeyed by ξ is, by analogy with eq. (2),

$$s_r^{-1} \sum_n m_r \left[\frac{d^2}{dt^2} + \varkappa_r \omega_r \frac{d}{dt} + \omega_r^2\right] \xi_n \times \times \cos(n\omega t + \varphi_n) = \sum_n p_n \cos(n\omega t + \zeta_n) \quad (15)$$

and the mean reed opening ξ_0 is given in terms of the "unblown" opening ξ_0' by

$$\xi_0 = \xi_0' - (s_r/m_r \omega_r^2) p_0 . \quad (16)$$

We have neglected non-linear terms in eq. (15), which is a reasonable approximation unless the reed opening actually closes. We then immediately have the results

$$\tan(\varphi_n - \zeta_n) = \varkappa_r \omega_r n \omega/(n^2\omega^2 - \omega_r^2) \quad (17)$$

where $0 \leqq \varphi_n - \zeta_n \leqq \pi$, and

$$p_n = s_r^{-1} m_r \xi_n [(\omega_r^2 - n^2\omega^2)^2 + + (\varkappa_r \omega_r n \omega)^2]^{1/2} \equiv K_n \xi_n . \quad (18)$$

Clearly the reed deflection is essentially $p_n s_r/m_r \omega_r^2$ for $n\omega \ll \omega_r$, peaks to $1/\varkappa_r$ times this value at the resonance frequency ω_r, and then decreases as $p_n s_r/m_r n^2 \omega^2$ for higher frequencies. $m_r \omega_r^2/s_r$ is just the static stiffness of the reed.

Eqs. (14) to (16) are in fact not complete and it is worthwhile pointing out what has been omitted in addition to the resistive term referred to in eq. (11). In eq. (14) we have assumed that the static component of the mouthpiece pressure p is simply atmospheric pressure whereas we should really make allowance for the finite resistance offered by the mouthpiece and instrument bore to steady flow. The only effect is a small reduction in the effective blowing pressure p_0 which could easily be included but which we neglect for simplicity. The second simplification is that we have omitted from the right hand side of eq. (15) a small term of the form $-\varepsilon(U/\xi)^2$ which describes the action of the flow in producing a Bernoulli pressure in the smallest constriction of the reed opening. The significance of this term depends on the reed geometry but is usually very small compared with the direct effect of mouthpiece pressure. In the interests of simplicity and to avoid introduction of the extra parameter ε we omit this term from the subsequent discussion. Thirdly we have assumed the impedance of the wind channel leading to the reed to be zero, so that the blowing pressure is always p_0. Neglect of these effects means that the reed will not oscillate in the absence of a resonator, which is not strictly true of real reeds.

Substituting eqs. (12), (13) and (14) in eq. (11), and assuming $\sum \xi_n < |\xi_0|$ and $\sum U_n < U_0$, we can collect up terms for various Fourier components. The zero-frequency equation differs only in second order from eq. (9) with $p = 0$. The general equation for the nth harmonic can be expressed as

$$\begin{aligned} p_n \cos\zeta_n = {} & p_0 \left[\frac{\alpha \xi_n}{\beta \xi_0} \cos\varphi_n - \frac{1}{\beta}\frac{U_n}{U_0}\cos\psi_n\right] + \\ & + \frac{\varrho a}{b|\xi_0|} n\omega U_n \sin\psi_n - \frac{\alpha}{4\beta}\left(\frac{\alpha}{\beta}+1\right) \times \\ & \times \left[\sum_m \frac{\xi_m \xi_{n-m}}{\xi_0^2} \cos(\varphi_m + \varphi_{n-m}) + \right. \\ & \left. + \sum_m \frac{\xi_m \xi_{m\pm n}}{\xi_0^2} \cos(\varphi_m - \varphi_{m\pm n})\right] - \\ & - \frac{1}{4\beta}\left(\frac{1}{\beta} - 1\right)\left[\sum_m \frac{U_m\ U_{n-m}}{U_0^2} \cos(...) + ...\right] \\ & - \frac{\alpha}{2\beta^2}\left[\sum_m \frac{\xi_m U_{n-m}}{\xi_0 U_0} \cos(\ldots) + \ldots\right] + \\ & + \frac{\varrho a \omega}{2b|\xi_0|}\left[\frac{m U_m \xi_{n-m}}{\xi_0} \sin(\ldots) + \ldots\right] + \\ & + \text{higher terms} \end{aligned} \quad (19)$$

together with a similar expression with cos → sin and sin → − cos throughout. These expressions are written out only to second order and only the first of the second order terms is given explicitly, the others following a similar pattern. Clearly the non-linearity of the flow couples together all the partials of the oscillation, and it is this coupling which justifies our original assumption that these partials are all harmonically related [8].

First of all let us consider just the linear terms, which will be the most important under conditions of small excitation. These give, using eq. (18),

$$\begin{aligned} p_n[(p_0\alpha/\beta\xi_0 K_n)\cos\varphi_n - \cos\zeta_n] = {} & \\ = U_n[(p_0/\beta U_0)\cos\psi_n - & \\ - (\varrho a/b|\xi_0|) n\omega \sin\psi_n] & \end{aligned} \quad (20)$$

together with a similar equation with cos → sin and sin → − cos. These are generalizations of the corresponding equations of Backus [5], with allowance made for the finite resonance frequency of the reed, as was done by Worman [6] and by Wilson and Beavers [7], and with the sign of ξ_0 taken explicitly into account.

Since the modes are uncoupled in the linear approximation, we can take the phase $\zeta_n = 0$. If necessary this phase can be restored later. We also drop the multiplier n and write simply ω for the frequency.

From eq. (20) we then find

$$\tan\psi = \frac{(p_0/\beta U_0)\, A + \omega(\varrho a/b|\xi_0|)(B - 1)}{(p_0/\beta U_0)(1 - B) - \omega(\varrho a/b|\xi_0|)\, A} \quad (21)$$

where

$$A = m_r (p_0 \alpha/\beta \xi_0)\, \omega\, \omega_r \varkappa_r/(K^2 s_r) \quad (22)$$

$$B = m_r (p_0 \alpha/\beta \xi_0)(\omega_r^2 - \omega^2) K^2 s_r \quad (23)$$

and we take ψ to lie between $-\pi/2$ and $\pi/2$. We can also define the reed generator admittance Y_r, as measured from inside the mouthpiece, as

$$Y_r = -U/p. \tag{24}$$

We shall later find this to be a more useful quantity than the generator impedance $Z_r = Y^{-1}$. At frequency ω this admittance has phase angle ψ and magnitude

$$[Y_r] = \frac{(1-B)}{(p_0/\beta U_0)\cos\psi - \omega(\varrho a/b\,|\xi_0|)\sin\psi} \tag{25}$$

where, because of the restriction placed on ψ, $[Y_r]$ is allowed to be negative. Note that Y_r does not become zero when $B = 1$ because of a balancing zero in the denominator. Note also that eqs. (21) and (25) cease to be valid in the limit $p_0 \to 0$, because the Bernoulli-like flow law (8) or (9) then no longer applies.

4. Instrument performance

We can see the implications of these results without great difficulty. Suppose that there is an acoustic pressure p with frequency ω in the mouthpiece cavity. This causes an acoustic flow

$$U_r = p\,Y_r$$

through the reed into the instrument and supplies acoustic energy to it at a rate $p^2\,\mathrm{Re}(-Y_r)$ where p^2 is the real mean-square value of p. At the same time p causes an acoustic flow $U_p = p\,Y_p$ in the pipe which dissipates energy at the rate $p^2\,\mathrm{Re}(Y_p)$. Clearly if more energy is supplied than is dissipated, that is if

$$\mathrm{Re}\,(Y_r + Y_p) < 0 \tag{26}$$

then the oscillations will grow and the instrument will sound, the loudness of the sound being determined, to a first approximation, by the extent to which the inequality is satisfied. Thus Y_r must have a negative real part which is as large as possible and Y_p a positive real part which is as small as possible, implying that the instrument should operate near a pipe impedance maximum.

Of course for a steady sound U_r must equal U_p in magnitude and phase and this can, in general, be achieved only through the agency of non-linear effects which reduce Y_r at large amplitudes. In the steady state then, we must have

$$\mathrm{Re}\,(Y_r + Y_p) = 0 \tag{27}$$

$$\mathrm{Im}\,(Y_r + Y_p) = 0 \tag{28}$$

the first condition being reached largely through adjustment of the amplitude of the oscillation and the second through adjustment of its frequency.

The behaviour of Y_r is given by eqs. (21) and (25) and is most easily appreciated if we neglect the small terms describing the inductive loading contributed by the small mass of air in the reed or lip channel. These are the final terms in the numerator and denominator of eq. (21) and in the denominator of eq. (25). Clearly the sign of the real part of Y_r is determined by the sign of $(1-B)$ and, if this is to be negative, we must have, from eq. (23) either

$$\xi_0 > 0,\ \omega > \omega_r,\ p_0^c > p_0 > p_0^* \tag{29}$$

or

$$\xi_0 < 0,\ \omega < \omega_r,\ p_0 > p_0^*. \tag{30}$$

These are exactly the frequency conditions found before but it is now required in addition that the blowing pressure p_0 must be greater than a critical value p_0^* given by

$$p_0^* = (m_r/s_r)(\beta\,\xi_0/\alpha)\,[(\omega_r^2 - \omega^2)^2 + (\omega\,\omega_r\varkappa_r)^2]/(\omega_r^2 - \omega^2). \tag{31}$$

The existence of this critical pressure has been well proven in the case of woodwind instruments for which the reed strikes inwards. It is less well known for the case of lip-driven brass instruments. We also see that, if we start from a condition of fixed unblown lip or reed opening ξ_0' then, from eq. (16), there is an upper limit

$$p_0^c = \xi_0'\,m_r\,\omega_r^2/s_r \tag{32}$$

in the case of the inward-striking reed, above which it is completely closed and no sound generation can take place. There is no such upper limit for lip-driven brass instruments.

It is useful now to plot polar diagrams for the reed admittance Y_r as a function of blowing pressure p_0. This is done in Fig. 2, using the physical parameters given in Table I and restoring the small terms neglected in the discussion immediately above. It is clear that a lip driven tube can always be made to sound, irrespective of the quality of its resonances, provided a sufficiently high blowing pressure is used.

Table I.
Assumed values of parameters.

Unblown reed opening	ξ_0'	$= \pm 0.5$ mm
Effective reed mass	m_r	$= 0.1$ g
Effective reed area	s_r	$= 0.5$ cm^2
Reed resonance frequency	ω_r	$= 3140$ rad s$^{-1} \to 500$ Hz
Reed parameters	α	$= 1,\ \beta = 0.5$
Effective reed channel width	b	$= 5$ mm
Effective reed channel length	a	$= 2$ mm
Reed damping parameter	$\varkappa_r$	$= 0.1$
Blowing pressure	p_0	$= 5$ kPa
Closing pressure ($\xi_0' > 0$)	p_0^c	$= 10$ kPa

Pressure Conversion: 1 kPa = 10 cm water gauge.

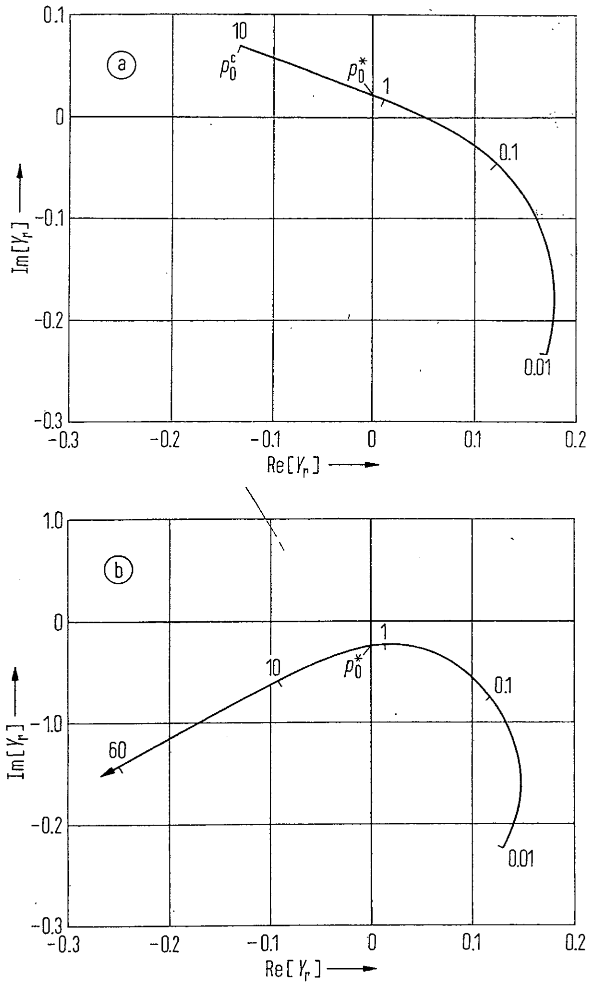

Fig. 2. The real part (Re) and imaginary part (Im) of the reed admittance Y_r, with blowing pressure p_0 in kilopascals (1 kPa = 10 cm water gauge) as parameter, for the cases
(a) $\xi_0 > 0$, $\omega = 0.9\,\omega_r$ and
(b) $\xi_0 < 0$, $\omega = 1.1\,\omega_r$.
In case (a) the reed closes for $p_0 \geqq p_0^c$ and in each case p_0^* is the critical blowing pressure for generator action. Assumed values of other parameters are given in Table I. The units for Y_r are reciprocal S. I. acoustic ohms (i.e. $m^3\,Pa^{-1}\,s^{-1}) \times \times 10^{-6}$.

For a normal reed driven tube with $\xi_0 > 0$, the imaginary part of Y_r is positive so that, from eq. (28), the imaginary part of Y_p must be negative. From eqs. (4) and (5) this implies that θ and $\sin 2kl$ must be positive which in turn implies that the oscillation frequency ω must be less than the neighbouring pipe resonance frequency ω_p. The opposite is true for an outward-striking lip reed.

We can go a little further and plot the way in which the real part of the admittance Y_r varies with frequency for a blowing pressure greater than the critical value p_0^*. This is done in Fig. 3 for several different values of the reed damping $\varkappa_r$. Remembering that we are still treating the system in the linear approximation only, we can now make some additional assertions.

Considering first the normal reed drive for which $\xi_0 > 0$ we see from Fig. 3a confirmation that ω must be less than the resonant frequency ω_r if Re (Y_r) is to be negative. If the reed damping is small then Re (Y_r) has its greatest negative magnitude just below the resonance so that, if the tube has impedance peaks of reasonable height near this frequency, this near-resonant mode will be favoured. This is the situation in the reed pipes of pipe organs. It is not, however, the playing mode desired in woodwind instruments, for which ω_r is generally chosen to be well above the frequency of the first few tube modes. If the reed is highly damped by the players lips, the negative reed admittance is nearly constant

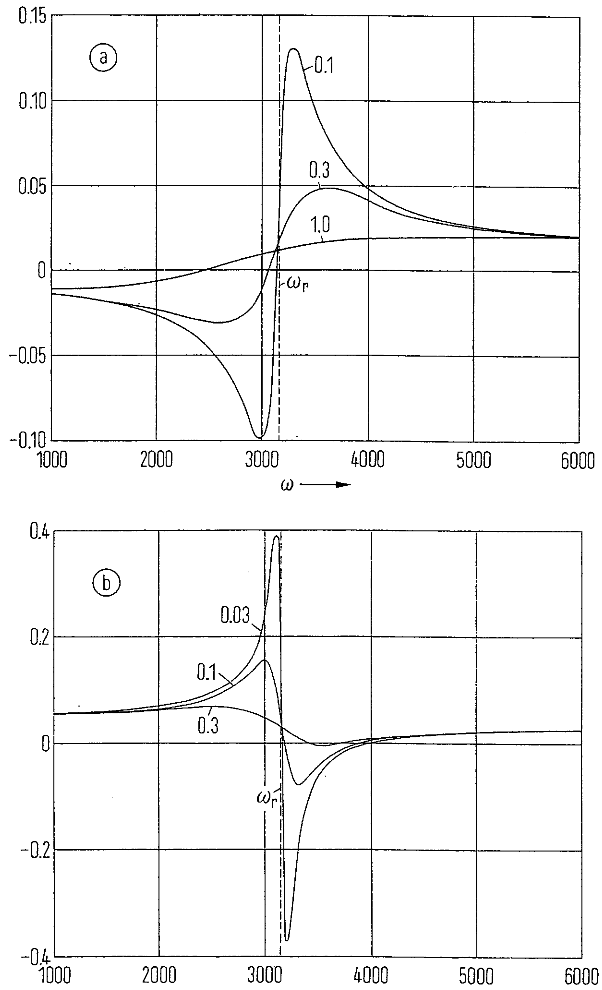

Fig. 3. The real part (Re) of the reed admittance Y_r as a function of frequency ω for $p_0 = 5$ kPa (about $3\,p_0^*$) and
(a) $\xi_0 > 0$,
(b) $\xi_0 < 0$.
The parameter is $\varkappa_r$, the damping coefficient of the reed. Other parameters have the values given in Table I. Units are as in Fig. 2.

for $\omega < \omega_r$ and it is the lowest pipe modes which are excited, as is desired. Little adjustment of lip position on the reed is then required to produce different notes, these being determined by the frequency of the most prominent pipe resonance for each fingering.

In the case of the lip-driven instruments the situation is quite different, as shown in Fig. 3b. The playing frequency ω is now greater than ω_r, the frequency shift being close to $\frac{1}{2}\varkappa_r \omega_r$. There is thus a strong tendency for any mode lying about $\frac{1}{2}\varkappa_r \omega_r$ above the frequency of the lip resonance to be preferentially excited. If $\varkappa_r$ is of order 0.1, the small width of this response region gives adequate discrimination between pipe resonances only one semitone apart in pitch (6 percent different in frequency) in the case of a skilled player, while not being unduly restrictive to the execution of rapid passages in the low register of modern instruments using valves.

The $\varkappa_r$ value of dry cane material from which woodwind reeds are made is probably less than 0.1, but it is likely that the method by which two canes are tied together in instruments like the crumhorn and bagpipes substantially increases this value. When the reed is wet and further loaded by the soft tissue of the players lips, as in modern woodwind instruments, it is easy to see that $\varkappa_r$ may lie in the range 0.3 to 1 as suggested by our discussion.

For the lip drive of brass instruments the situation is very different, for the $\varkappa_r$ value for the players lips, even under muscular tension, may well exceed the value 0.1 required for proper performance on instruments like the French Horn. The explanation of this apparent difficulty seems to lie in some of the terms omitted from our simple theory, in particular the terms representing the finite volume and flow impedance of the airway through the player's mouth. If the mouth cavity is regarded as an acoustic capacitance fed through an air channel of finite resistance, then the air pressure p_0 within it will vary slightly because of flow through the lips. This can be accounted for by adding a term proportional to $\int U \, dt$ to the right hand side of eq. (15). Since U is nearly π out of phase with ξ for a lip reed ($\xi < 0$), $\int U \, dt$ is nearly in phase with $d\xi/dt$ and so reduces the effective value of $\varkappa_r$ on the left hand side. This also accounts for the fact that the lips can be made to "buzz" in the absence of any musical instrument resonator.

5. Non-linearities

Provided the excitation level is not too large, so that expansions like (12), (13) and (14) are fairly rapidly convergent, the system behaviour can be described by the set of eqs. (19), carried if necessary to a larger number of terms. This small-signal approach is the one usually employed and its implications have been carefully explored by Benade and Gans (9], Worman [7], Benade [1] and Fletcher [8].

Because the resonances of a musical horn are never exactly harmonic, a strictly linear excitation mechanism would lead to a sound the frequencies of whose partials were not exact multiples of a common fundamental frequency, giving at best a quaint and at worst a musically unpleasant sound. If the driving mechanism is sufficiently non-linear, however, all the partials of the sound become locked into exact harmonic relationship [8], [10], as is known to be the case for virtually all non-electronic musical instruments producing steady sounds. The expansions leading to eq. (19) have already assumed this to be so.

Investigating this situation, Benade [1], [9] has shown that, rather than considering the regenerative behaviour only at the fundamental frequency, as we have done, we should really take into account the weighted impedance of the drive mechanism, and particularly of the musical horn, over all harmonic components of the sound being produced. We have, at present, nothing to add to Benade's discussion.

Worman [7] has shown that, in the small-signal regime in which the fundamental is dominant, the amplitude of the nth harmonic varies as the nth power of the amplitude of the fundamental. This conclusion is also implied by our eq. (19), as follows. From eqs. (18) and (20), if the fundamental frequency ω is fixed, the quantities p_n, U_n and ξ_n for the nth harmonic are simply proportional to each other. Using this conclusion to determine successively the scond, third, ... harmonic amplitudes p_n from eq. (19), on the assumption that $p_n \gg p_{n+1}$ for all n, leads immediately to the conclusion that $p_n \propto p_1{}^n$. We must remember, however, that this conclusion applies to the pressure amplitudes within the instrument mouthpiece, rather than to the radiated sound. Benade [1] has discussed, and indeed in simple cases it follows from standard acoustical theory, that each musical instrument horn has a characteristic cut-off frequency ω_c above which the radiated acoustic pressure amplitude is simply proportional to the internal pressure amplitude p_n but below which it is proportional to $p_n(\omega/\omega_c)^2$. Since ω_c is typically 10^4 rad s^{-1}, corresponding to about 1.5 kHz, there is thus a very large amount of high-frequency emphasis in the radiated sound spectrum compared with the internal spectrum.

Finally we should note that, even in this low-excitation regime, it is the non-linearity of the excitation mechanism which determines the total amplitude and power of the oscillation. This limitation depends primarily upon the relative magnitudes of the linear and cubic terms in eq. (19), as has been discussed elsewhere for the related case of organ pipe excitation by an air jet [10]. Since the cubic terms vary as ξ^{-3} and the linear terms only as ξ^{-1}, large oscillation amplitude is favoured by a large value ξ_0', and hence of ξ_0, provided p_0 is constant and $p_0 > p_0^*$.

When we come to consider the behaviour at high excitation levels we find that non-linearity plays a dominant part in determining the shape of the internal pressure waveform. We define a high excitation level in this connection to be either one giving sufficient amplitude ξ to the reed vibration so that $\xi \geqq \xi_0$ and the reed closes, or else one giving sufficient amplitude p to the mouthpiece pressure so that $p \geqq p_0 - p_0^*$ and the instantaneous operating point for the reed generator is forced back into the dissipative region, giving a reversal in the sign of the real part of the flow U at the frequency considered. These two non-linearities are to some extent independent and under the control of the player but they are also partly determined by the nature of the reed system and of the instrument horn.

The clarinet has been extensively studied [1], [6], [11] and shows this behaviour well. The optimum blowing pressure for the clarinet is midway between the threshold pressure p_0^* and the pressure p_0^c which closes the reed. Not only does this maximize the dynamic range below the threshold of extreme non-linearity but also it ensures that, when non-linear behaviour begins, it does so nearly symmetrically, giving a flow waveform which approximates a square wave. This is important since the clarinet horn, being nearly cylindrical, has impedance maxima near ω_1, $3\omega_1$, ... and impedance minima near $2\omega_1$, $4\omega_1$, ... If the pressure waveform contains significant even-harmonic components, then the weighted impedance of the horn will be lower than optimal and it will not couple well to the reed generator.

Assuming operation in this symmetrical fashion, the horn has a linear resonant response so that the mouthpiece pressure is also of square waveform. Since the reed is a compliant system of high resonant frequency, its displacement follows the pressure waveform quite closely and indeed this is what was observed in the pioneering work of Backus [11].

Instruments such as the oboe, bassoon and saxophone have nearly conical horns which show impedance maxima at the throat at frequencies ω_1, $2\omega_1$, $3\omega_1$, ... so that it is no longer a condition for efficient operation that the non-linear clipping be symmetrical. Indeed the relative heights of the second and third resonance peaks suggest possible advantages from asymmetric clipping. In the case of the oboe, the smaller initial reed opening (<0.5) mm and higher blowing pressure ($\approx$4.5 kPa) compared with the clarinet ($\approx$1 mm and 3.5 kPa) suggest that the major non-linearity may be that associated with closing of the reed. This may also apply to the bassoon; the situation with the saxophone is less clear. These conjectures, however, require experimental confirmation.

The situation with the lip-driven brass instruments is rather different because of the resonant behavior of the lip reed and the fact that the dynamic level can be increased by a parallel increase of both p_0 and ξ_0'. The resonant behaviour of the lip reed means that it responds only weakly to driving pressures at upper-harmonic frequencies and tends to adjust its motion so as to just close during each cycle. This behaviour was recognised long ago through the elegant photographic studies of Martin [12].

The horns of brass instruments are designed to produce a harmonic series which is well tuned except for the horn fundamental (typically $0.8\,\omega_1$, $2\omega_1$, $3\omega_1$, ...) and only modes based on the higher horn resonances are used in normal playing. For each of these modes the upper resonances provide a well-aligned complete harmonic series so that there is no obstacle to prevent, and perhaps some advantage to be gained from, asymmetric clipping. The design of a typical brass instrument mouth cup, together with the high dynamic level possible, leads to a high mouthpiece pressure and thus to waveform clipping when the lip opening is wide. The asymmetric flow waveform contains harmonics of all orders, which may be further intensified by the non-linearity of the flow through the constriction joining the mouth cup to the main horn. There is, as we have already discussed, a further emphasis of the upper harmonics in the radiated sound.

This discussion of harmonic generation in brass instruments follows that given by Backus and Hundley [13], who showed that mouthpiece pressure waveforms calculated on this basis for a trumpet are in good agreement with measured waveforms produced using a mechanically operated valve to simulate the lip opening. In each case the pressure waveform approached the shape of a half-wave-rectified sinusoid.

Finally we can use this discussion and the previous analysis to estimate the maximum peak-to-peak value of the mouthpiece pressure waveform that can be produced in a wind instrument. We assume that the wall and radiation damping in the horn is sufficiently small that the mouthpiece pressure is driven to the clipping point in each direction and that the playing conditions are those previously discussed: $\xi_0' > 0$ and $\omega \ll \omega_r$ for a woodwind instrument and $\xi_0' < 0$, $\omega \approx \omega_r(1 + \frac{1}{2}\varkappa_r)$ for a brass instrument.

For a woodwind instrument, from eq. (31), one cut-off point is essentially given by

$$(p_0 - p)^* = (m_r \omega_r^2/s_r)(\beta/\alpha)\xi_0 \qquad (33)$$

where, since we are considering large signal conditions for the mouthpiece pressure p, eq. (16) must be written

$$\xi_0 = \xi_0' - (s_r/m_r \omega_r^2)(p_0 - p)^*. \qquad (34)$$

Also from eq. (16), when $\xi_0 = 0$ and the reed closes, we have

$$(p_0 - p)^c = (m_r \omega_r^2/s_r)\xi_0'. \qquad (35)$$

Combining these equations, we find for the maximum peak-to-peak excursion P of the mouthpiece pressure

$$P = p^* - p^c = (m_r \omega_r^2/s_r)[\alpha/(\alpha + \beta)]\xi_0'. \qquad (36)$$

The factor $\alpha/(\alpha + \beta)$ is about 2/3 and the value of P is simply proportional to the product of the effective reed stiffness $m_r \omega_r^2/s_r$ and the unblown reed opening ξ_0', and is independent of blowing pressure provided $p_0^* < p_0 < p_0^c$. Reed instrument players generally control loudness by altering lip tension to change the value of ξ_0'.

In the case of brass instruments the resonant behaviour of the lips makes the situation rather different. The lip motion does not respond instantaneously to pressure variations as in a woodwind reed but executes a sinusoidal motion, the amplitude of which is probably equal to the equilibrium blown lip separation ξ_0 given by eq. (16) with $\xi_0' < < 0$. The lip oscillation is substantially out of phase with the mouthpiece pressure as we have already discussed. Provided the lip damping $\varkappa_r$ is small enough that the oscillation amplitude ξ given by eq. (18) reaches the value ξ_0, then the maximum pressure excursion P will approximate the blowing pressure p_0, the amount of clipping of the waveform depending on the unblown lip opening ξ_0'. Again both ξ_0' and p_0 are under the control of the player.

6. Conclusion

This analysis has served to identify those features which are of primary importance to the understanding of the excitation mechanism in woodwind and brass instruments. Such an analysis should provide the necessary background for more sophisticated studies of particular instruments or, perhaps even more interestingly, of performance techniques upon these instruments.

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Appendix

Standard theory for wave propagation in finite cylindrical tubes shows that for the dimensions and operating frequencies of most musical wind instruments the dominant loss mechanisms are those to the walls. Radiation losses from the open end are not significant for the first few pipe modes and the open end impedance behaves, to a good approximation, like a small inductance giving an "end correction" addition to the tube length which is just 0.6 times the radius. If l is the length of the tube including this correction, then the input impedance (defined as the ratio of acoustic pressure to acoustic volume flow) at its throat end is just

$$Z_p = j(\varrho c/S)\tan[(k - jk')l] \qquad (A1)$$

where ϱ is the density and c the velocity of sound in air, S is the cross-sectional area of the tube, $k = \omega/c$ is the real part of the propagation number and jk' an imaginary component to take account of wall losses.

The expression (A1) can easily be re-expressed in the form

$$Z_p = j(\varrho c/S)[\tan kl - j\tanh k'l]/[1 + j\tanh k'l \tan kl] \qquad (A2)$$

and we see that, at the resonances corresponding to impedance maxima, where $kl = (2n + 1)\pi/2$,

$$Z_p^{(res)} = (\varrho c/S)\coth k'l. \qquad (A3)$$

If we suppose the height of these impedance peaks to be H times the reference value $\varrho c/S$, where H is a slowly-varying function of frequency, since this is also true of k' and $k'l \ll 1$ in general, then we can write eq. (A2) as

$$Z_p = (\varrho c/S)(1 + jH\tan kl)/(H + j\tan kl). \qquad (A4)$$

This result will also be a reasonable approximation for the horns of brass instruments for which the effective length l increases with increasing frequency.

The phase θ of Z_p is given by

$$\tan\theta = \left(\frac{H^2 - 1}{H}\right)\left(\frac{\tan kl}{1 + \tan^2 kl}\right) = \left(\frac{H^2 - 1}{2H}\right)\sin 2kl. \qquad (A5)$$

The real and imaginary parts of Z_p can easily be found from eqs. (A4) and (A5).

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