Autonomous vibration of simple pressure-controlled valves in gas flows

N. H. Fletcher
CSIRO Australia, and Research School of Physical Sciences and Engineering, Australian National University, Canberra 2601, Australia

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Pressure-controlled gas-flow valves are responsible for sound generation in woodwind and brass instruments, and in the vocalization of many animals. When only a single degree of freedom is allowed for the valve motion, four simple configurations are possible, depending upon the effect upon the valve opening of static overpressures applied at either inlet or outlet ports in the presence of the flow. It is shown that, depending upon the valve configuration, there exist particular ranges of acoustic impedance for the inlet and outlet ducts to the valve within which self-sustained valve oscillation is possible. The results are particularly simple when the lengths of these ducts are less than one-quarter of a wavelength at the resonant frequency of the valve, in which circumstance oscillation takes place close to that resonance frequency. The analysis treats only the initiation of oscillations of small amplitude, as a precursor to the maintenance of large-amplitude nonlinear oscillations.

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INTRODUCTION

Pressure-controlled valves are responsible for sound production in musical instruments of the woodwind and brass families, and in animal vocalization. In musical instruments the operation of the valve is usually controlled by feedback from a passive resonator, the instrument horn, while in vocal systems the valve itself is the controlling entity and the horn resonator serves primarily as an acoustic filter. There have been many detailed analyses of the operation of reed- and lip-driven woodwind instruments,[1–8] of human vocal systems,[9–14] for which quite sophisticated models have been developed, and of avian vocal systems,[15–16] these references being only some of many in the literature. There have, however, been only a few general treatments of the oscillation problem[5,7,17] and these have mostly been particularized to the case of musical instruments. There does not seem to have been published a simple unified discussion of the general conditions under which oscillations of the various simple types of pressure-controlled valve can be initiated and maintained. It is the purpose of the present paper to remedy this omission by analyzing the behavior of simple models with all possible configurations.

The simple valves with which we are concerned are of three types, as indicated schematically in Fig. 1. In each case, the motion of the valve flaps is defined by a single geometric parameter. Our classification is also simply geometric and is based on the effect upon the valve of an additional small steady overpressure applied from one or other of its two sides, in the presence of a pre-existing flow but omitting from consideration any forces caused by Bernoulli pressure. If a small overpressure applied from the upstream side of the valve causes it to open further, then we define a parameter σ_1 to have the value $+1$, while if this overpressure tends to close the valve then $\sigma_1 = -1$. Similarly, if a steady overpressure applied from downstream of the valve causes it to open further, then a parameter σ_2 is given the value $+1$, while if it tends to close the valve then $\sigma_2 = -1$. The configuration of a valve is thus defined by the couplet (σ_1, σ_2). By an obvious contraction of the notation, woodwind-type reed valves as shown in Fig. 1(a) are of type $(-,+)$, simple models of the lip-valve driving brass-family musical instruments and the valve in the human larynx, as shown in Fig. 1(b), are of type $(+,-)$ (though we see later that such models are of questionable validity), and the valve in the avian syrinx, as in Fig. 1(c), is of type $(+,+)$. The reed pipes of the pipe organs are of type $(-,+)$ like woodwind reeds. Harmoniums and harmonicas generally use sets of reeds of types $(+,-)$ and $(-,+)$ in an apparently "free" configuration without any attached resonators, one of the sets being activated by blowing and the other by sucking.[4] There does not appear to be a practically useful example of a valve of type $(-,-)$, but such a valve could perhaps be constructed. This classification is an extension of that used by Helmholtz,[17] who described woodwind-type reeds $(-,+)$ as "striking inwards" and lip-type valves $(+,-)$ as "striking outwards." By analogy, we might describe syrinx-type valves $(+,+)$ as "striking sideways."

It is important to emphasize that, in reality, pressure controlled biological valves, such as brass instrument player's lips, the human larynx, and the avian syrinx, are much more complex than suggested in Fig. 1 and require more than one geometric parameter to specify their configuration. This may mean that they cannot be unambiguously classified into our simple scheme. We return to comment upon this briefly later.

Whether or not a valve has an attached resonator, it must always have some form of pipe connection to at least

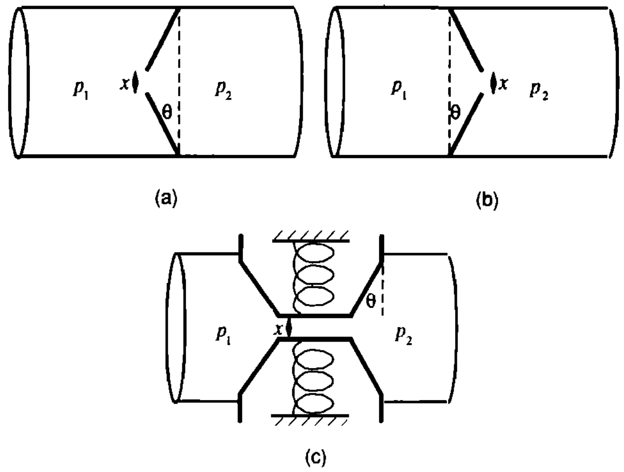

FIG. 1. Simplified models of three commonly occurring configurations of valves, described by the couplet (σ_1,σ_2). (a) A valve of configuration $(-,+)$, corresponding to a woodwind or organ-pipe reed valve; (b) a valve of configuration $(+,-)$, corresponding to a brasswind instrument lip valve or to a human vocal valve according to one possible model; and (c) a valve of configuration $(+,+)$, corresponding to avian syrinx valve, or to a lip or vocal valve according to an alternative model.

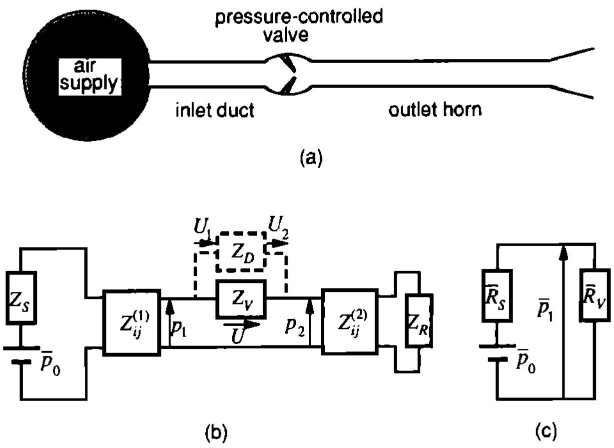

FIG. 2. (a) Schematic diagram of a pressure-controlled valve connected to appropriate inlet and outlet ducts. (b) Analog network describing a pressure-controlled valve fed from a static pressure generator $\bar{p}_0$ of internal impedance Z_S. A pipe with impedance coefficients $Z_{ij}^{(1)}$ connects the static pressure generator to the valve and a horn with impedance coefficients $Z_{ij}^{(2)}$ exhausts to the air, where it is terminated by a radiation impedance Z_R. The shunt impedance across the valve allows for oscillating flow caused by displacement of the valve tongue. (c) The steady-flow version of the same circuit, assuming the ducts present no flow resistance at zero frequency.

one of its ports. The acoustic impedance of these connecting pipes is, as we see later, vitally important to the vibration behavior of the valve. Indeed, our purpose is to determine the conditions under which valves of the three types mentioned can sustain spontaneous oscillations, once the connections to the two valve ports have been specified. A linearized theory is adequate to explore this problem, but does not suffice to define the subsequent large-amplitude vibrations of the valve. To treat these we must resort to numerical solution of the full equations defining the system,[18–20] either in the time domain or in the frequency domain.

Before we begin, it is important to make one qualification to our discussion. We have referred to valves of configuration $(-,+)$ as being of woodwind type, and this carries the incorrect implication that we are discussing the operation of these reeds as in complete musical instruments. This is, however, not the situation that we wish to consider. In a woodwind instrument, the natural frequency of the reed itself is high compared with the playing frequencies, and the method of analysis to be presented is unduly cumbersome when applied to this case. Rather, we wish to consider situations more closely analogous to reeds of the organ-pipe variety, in which the natural frequency of the reed is lower than, or comparable to, the frequency of the first mode of the resonator to which it is attached. Operation of the reed valve then takes place near the frequency of the peak in the negative acoustic conductance that has been identified[5] just below the resonance frequency of the reed in this case. To avoid confusion we shall therefore refer to the configuration $(-,+)$ as being that of an organ-pipe reed. Valves with other configurations similarly operate near their negative conductance peak, and we should remark that this is necessarily true, even for close coupling to a resonator, for valves of configuration $(+,-)$, for which the conductance is negative only in a small frequency range just above the valve resonance frequency.[5]

I. THEORY

Suppose that the physical system has the form shown in Fig. 2(a). An air supply, with internal acoustic impedance Z_S, creates a steady overpressure $\bar{p}_0$. In most of the systems with which we are concerned, this pressure source is the lungs, and Z_S is large compared with the impedances of the connecting air ducts. Some sort of duct or horn connects the source to the valve, and we can characterize this by a set of impedance coefficients $Z_{ij}^{(1)}$ as follows. Suppose that p_i is the acoustic pressure at one end of the duct, denoted by i, and U_i the acoustic volume flow into that end, with p_j and U_j being similar quantities for the other end j. Then the relationship between these quantities can be written

$$p_i = Z_{ii}U_i + Z_{ij}U_j,$$
$$p_j = Z_{ji}U_i + Z_{jj}U_j, \tag{1}$$

and it can be shown that $Z_{ji}=Z_{ij}$. These equations then define the impedance coefficients Z_{ij}, which can be readily calculated for ducts of simple geometry and are given a superscript (1) or (2) to indicate the duct to which they refer.

The exit side of the valve is connected, in general, to another horn with impedance coefficients $Z_{ij}^{(2)}$ which is terminated by a radiation impedance Z_R. We should really know the values of these impedance coefficients both near the operating frequency of the valve generator and also in the steady flow domain, but it will be adequate for our discussion to assume zero resistance to steady flow through both horns. It is useful, in our subsequent development, to specify also the pressures p_1 and p_2 immediately upstream and downstream from the valve. These will be time-varying quantities when the valve is oscillating.

In analyzing this system, it is helpful to use the analog network shown in Fig. 2(b) and the steady-flow version of Fig. 2(c). We specify steady pressures and steady volume flow by barred letter $\bar{p}_i$ and $\bar{U}$ and use $\hat{p}_i$ and $\hat{U}$ to indicate the oscillating parts of these quantities. If $\bar{R}_S$ is the resistive part of the lung impedance Z_S and $\bar{R}_V$ the resistive part of the valve impedance Z_V, both at zero frequency, then

$$\bar{p}_1=\frac{\bar{p}_0\bar{R}_V}{\bar{R}_S+\bar{R}_V},\quad \bar{p}_2=0. \tag{2}$$

It turns out to be much simpler to use the value of the steady pressure $\bar{p}_1$ just upstream of the valve to characterize the system rather than using the generator pressure $\bar{p}_0$, so we shall make this change of viewpoint. It is easy to use (2) to relate this to the source pressure if we wish.

Now, we need an equation to quantify the steady flow through the valve and thus to give a value for valve resistance $\bar{R}_V$ at zero frequency, defined to be the ratio of the steady pressure drop to the steady volume flow. For this we need two equations. The first specifies the flow $\bar{U}$ through the valve when the opening between its faces is $\bar{x}$ and the steady pressures on its two sides are $\bar{p}_1$ and $\bar{p}_2$. Provided the channel length through the valve is small and the flow velocity high, this is given by Bernoulli's equation as

$$\bar{U}=(2/\rho)^{1/2}W(\bar{p}_1-\bar{p}_2)^{1/2}\bar{x}, \tag{3}$$

where ρ is the density of air and W is the width of the valve. Equation (3) is, of course, only an approximation, and we should not expect it to hold exactly when the valve is nearly closed or when the flow is not simply two-dimensional. [A somewhat more general empirical form of this equation involving $(\bar{p}_1-\bar{p}_2)^{\alpha}\bar{x}^{\beta}$ with $\alpha\approx 2/3$ and $\beta\approx 4/3$, was reported by Backus[3] for the clarinet, but this has not been substantiated by later experiments.] Equation (3) leads to a value for the valve resistance at zero frequency, $\bar{R}_V=(\bar{p}_1-\bar{p}_2)/\bar{U}$, once we have evaluated $\bar{x}$ in terms of $\bar{p}_1-\bar{p}_2$.

Equation (3) can also be applied as a quasistatic approximation for the time-varying flow of air through the valve, provided the frequency is much less than the frequency of the valve response, simply by removing the bars from all the quantities involved. Before we resort to this approximation, however, it is necessary to examine the behavior in rather more detail.

A second equation is needed to describe the motion of the valve itself. This could be quite complicated, since even a simple system like a clarinet reed has many possible vibrational modes. To be general, we should consider the behavior of each mode and sum their displacements in the throat of the valve to find the clear opening area. It is enough for our present discussion, however, to consider the valve tongue to behave in a simple manner with a single resonance frequency ω_0, damping k, and effective mass m. If the valve presents an effective area S_1 to the upstream pressure p_1 and an effective area S_2 to the downstream pressure p_2, as shown in Fig. 3(a), then its motion can be represented by the equation

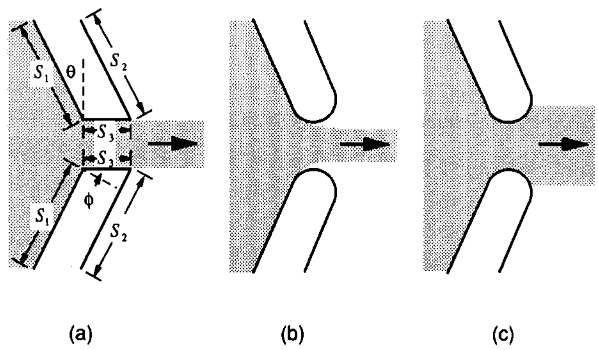

FIG. 3. (a) Development of a jet in air flow thorugh a simple valve, with corresponding definitions of the geometrical areas S_i and angles θ and ϕ. For a valve with two flaps, as shown, the areas S_i for the two flaps must be added together. (b) More realistic geometry for a biological valve, showing flow separation and a vena contracta effect. (c) Possible alternative flow geometry with delayed detachment of the jet flow.

$$\frac{d^2x}{dt^2}+2k\frac{dx}{dt}+\omega_0^2(x-x_0)=\frac{1}{m}(\sigma_1 p_1 S_1+\sigma_2 p_2 S_2)+\frac{1}{m}p_2 S_3, \tag{4}$$

where x_0 is the valve opening in the absence of any applied pressure. The final term in this equation represents the influence of the Bernoulli pressure produced by flow through the throat of the valve—the flow velocity is $v=[2(p_1-p_2)/\rho]^{1/2}$, the pressure in the throat is $p_1-\frac{1}{2}\rho v^2$, and we have assumed an effective area S_3 of the valve flaps over which this force acts. For simplicity we rewrite (4) in the form

$$\frac{d^2x}{dt^2}+2k\frac{dx}{dt}+\omega_0^2(x-x_0)=(\sigma_1\mu_1 p_1+\sigma_2\mu_2 p_2), \tag{5}$$

where $\mu_1=S_1/m$ and $\mu_2=(S_2+\sigma_2 S_3)/m$. Omitting the time variation, we see that the steady opening of the valve is

$$\bar{x}=x_0+(\sigma_1\mu_1\bar{p}_1+\sigma_2\mu_2\bar{p}_2)/\omega_0^2. \tag{6}$$

The division of valve flap areas into the three classes S_1, S_2, and S_3 is somewhat artificial in a general sense, though justified for the simple valve configurations we are considering. The total force acting on the valve flaps is properly obtained by evaluating the pressure at all points in the fluid and then integrating over the valve flap area, taking account the moments if the valve motion is essentially angular. While the Bernoulli force is significant only in the narrow part of the valve channel, the extent to which the downstream surface of the valve flap is acted upon by a downstream pressure p_2 depends upon separation of the flow from the surface and the formation of a jet,[21] as shown in Fig. 3. The location of the separation point depends upon the shape of the valve flap and the speed of the flow. While the separation point is usually well defined for a flap with a sharp edge, as in Fig. 3(a), the same is not true for a curved flap, as in Fig. 3(b) or (c). This is one reason for uncertainty about the classification of vocal and lip valves, particularly when the valve deflection itself involves several geometrical parameters.

We have used the terms effective mass and effective area in this development, since only in particularly simple cases are these the true mass and geometric area. The conversion from true to effective quantities essentially consists of relating the displacement x to the shape of the vibration mode involved, whether this be rotation about a spring-loaded axis or bending of a cantilever beam, and similarly averaging the pressures over the mode shape. For a valve consisting of two stiff flaps mounted on spring hinges and making an angle θ with the opening direction x, as shown in Fig. 1, and more specifically in Fig. 3(a), the opening x of the valve is related to the deflection ξ of the valve tip by

$$x=x_0+\xi \sin \theta \tag{7}$$

and we must consider the valve motion as rotation about a fixed axis. If we do this, and then convert from changes in the angular coordinate θ to changes in the valve-opening coordinate x, we find that

$$m\approx m^{\text{true}}/3 \sin \theta; \quad S_{1,2}\approx S_{1,2}^{\text{geom}}/2; \quad S_3\approx S_3^{\text{geom}} \sin \phi. \tag{8}$$

The effective valve of S_3, in particular depends upon the exact geometry of the valve channel, as defined for the simple case by the angle ϕ of Fig. 3(a). The way in which the conversion factors are apportioned between m and the S_i is arbitrary to some extent, since only quotients S_i/m enter the final result. It is clear, in this case, that a valve geometry with very small θ is very inefficient as an acoustic generator, if its displacement mode is as described by this model, since a high blowing pressure is required to increase θ to a sufficient extent that there is appreciable flow.

For valves with different geometries or different displacement modes, the conversions are, of course, rather different. If one of the tongues of Fig. 1 is replaced by a rigid stop, for example, as in an organ-pipe or clarinet reed, then the associated redefinition of the relation (7) between x and θ means that $\theta\approx\pi/2$.

For a valve of configuration $(+,+)$, as in Fig. 1(c), the relations are

$$m\approx m^{\text{true}}; \quad S_{1,2}\approx S_{1,2}^{\text{geom}} \sin \theta; \quad S_3\approx S_3^{\text{geom}}, \tag{9}$$

where the definition of θ is now as in Fig. 1(c). It is interesting to note that, if we take $\theta=0$ in this geometry so that $S_1=S_2=0$, then the configuration (σ_1,σ_2) is formally undefined, and the operation of the valve depends entirely upon the Bernoulli pressure. The effective configuration resulting from this depends upon the position of separation of the flow, as in Fig. 3, and this may itself depend upon the flow speed.

Now suppose that the valve oscillates with very small amplitude at a frequency ω so that we can replace U by $\bar{U}+\hat{U}(\omega)$, p_i by $\bar{p}_i+\hat{p}_i(\omega)$, and x by $\bar{x}+\hat{x}(\omega)$ in (3) and (5). The quantity $\hat{U}(\omega)$ is of the form $\hat{U}e^{j\omega t}$, where $\hat{U}$ is a complex amplitude incorporating a phase factor $e^{j\phi}$, and similarly for $\hat{p}(\omega)$ and $\hat{x}(\omega)$. The network in Fig. 2(b) can be solved explicitly for components at this assumed oscillation frequency, and the important result comes from the central mesh of the network. If $\hat{U}$ is the oscillating volume flow through the valve itself, and $\hat{U}_1$ and $\hat{U}_2$ the oscillating flows produced by physical displacement of the valve flaps, then we see that

$$\hat{p}_1=-Z_1(\hat{U}+\hat{U}_1), \quad \hat{p}_2=Z_2(\hat{U}+\hat{U}_2), \tag{10}$$

where Z_1 is the input impedance of the supply pipe, terminated by the supply impedance Z_S, as seen from the valve, and Z_2 is similarly the input impedance of the exhaust horn, terminated by its radiation impedance Z_R. In (10),

$$Z_1=Z_{11}^{(1)}-\frac{(Z_{12}^{(1)})^2}{Z_{22}^{(1)}+Z_0}, \quad Z_2=Z_{11}^{(2)}-\frac{(Z_{12}^{(2)})^2}{Z_{22}^{(2)}+Z_R}, \tag{11}$$

where the end of each horn connected to the valve is given the subscript 1 in the coefficients Z_{ij}. The displacement flows $\hat{U}_1$ and $\hat{U}_2$ are given by

$$\hat{U}_1=j\omega\sigma_1 S_1\hat{x}, \quad \hat{U}_2=-j\omega\sigma_2 S_2\hat{x}, \tag{12}$$

with the same effective areas S_1 and S_2 as before. In a valve such as that formed by a clarinet reed or our simple model for the human vocal folds, $\hat{U}_1\approx\hat{U}_2$, while for a symmetric valve such as the avian syrinx, shown in Fig. 1(c), $\hat{U}_1\approx-\hat{U}_2$, which is allowable since oscillation of the valve gates causes a net volume change within the valve. The possibility of a net valve current of this type appears to have been first recognized by Saneyoshi *et al.*[7] While it is straightforward to include the displacement flows through the valve in this way, it complicates the algebra and obscures the argument. For this reason, and since the displacement flow is generally at least an order of magnitude smaller than the fluid flow through the valve, we shall omit these correction terms in the analysis that follows.

Before substituting the expansions for U, p_1, and p_2 in (3), we must recognize that this equation contains only the resistive part of the acoustic impedance, so that we need to include a further term to account for the acoustic inertance of the air in the reed opening. If we take δ to be the length of the channel through the opening, then its area is Wx and, in the simple-jet case of Fig. 3(a), its acoustic inertance is $\rho\delta/Wx$. The effective pressure driving the oscillatory flow is therefore not p_1-p_2 as in (3), but rather

$$(\hat{p}_1-\hat{p}_2)-\frac{\rho\delta}{Wx}\frac{\partial U}{\partial t}=(\hat{p}_1-\hat{p}_2)-j\omega\delta\left(\frac{\rho}{2}\right)^{1/2}\times(\bar{p}_1-\bar{p}_2)^{1/2}\left(\frac{2\hat{x}}{\bar{x}}+\frac{\hat{p}_1-\hat{p}_2}{\bar{p}_1-\bar{p}_2}\right). \tag{13}$$

Once again, in the interest of algebraic simplicity, and because the correction term is small compared to p_1-p_2, we shall omit this refinement in the following analysis.

Now, setting $\bar{p}_2=0$, in accord with our assumption that the resistance of the exit horn to steady flow is very small (or equivalently by measuring $\bar{p}_1$ relative to $\bar{p}_2$), substituting the small-signal expansions for U and the p_i in the quasisteady flow equation (3), expanding, and collecting together the quantities with frequency ω, we see that

$$\hat{U}=(2\bar{p}_1/\rho)^{1/2}W\{[\bar{x}(\hat{p}_1-\hat{p}_2)/2\bar{p}_1]+\hat{x}\} \tag{14}$$

to first order. The steady flow, in this approximation, is given by (3) with $x=\bar{x}$ as given by (6) and $\bar{p}_2=0$. We can substitute in (14) the simplified form of (10), omitting $\hat{U}_1$ and $\hat{U}_2$, to obtain

$$\hat{U}=2W\bar{p}_1\hat{x}/[(2\rho\bar{p}_1)^{1/2}+W\bar{x}(Z_1+Z_2)]\,. \qquad (15)$$

This equation, used in the simplified form of (10), gives expressions for $\hat{p}_1$ and $\hat{p}_2$ in terms of $\hat{x}$ that can be substituted in (5) to give

$$-\omega^2\hat{x}+2j\omega k\hat{x}+\omega_0^2\hat{x}=\left(\frac{2W\bar{p}_1(\sigma_2\mu_2Z_2-\sigma_1\mu_1Z_1)}{(2\rho\bar{p}_1)^{1/2}+W\bar{x}(Z_1+Z_2)}\right)\hat{x}. \qquad (16)$$

Equation (16) is the key equation of our analysis. The left side is simply what we expect for the damped free vibration of the reed, while the right side contains the influence of the blowing pressure $\bar{p}_1$ and the upstream and downstream impedances Z_1 and Z_2. These impedances are generally complex quantities, so that the term on the right is complex, independently of the effect of the small correcting terms for valve channel inertance and valve flap displacement. Its real part can be combined with the term $\omega_0^2\hat{x}$ on the left side to affect the resonance frequency, while the imaginary part can be combined with the term $2jk\omega\hat{x}$ to affect the damping. The oscillation frequency ω is given by

$$\omega^2=\omega_0^2-\mathrm{Re}\left(\frac{2W\bar{p}_1(\sigma_2\mu_2Z_2-\sigma_1\mu_1Z_1)}{(2\rho\bar{p}_1)^{1/2}+W\bar{x}(Z_1+Z_2)}\right), \qquad (17)$$

where Re implies the real part of the following expression. We are not particularly concerned here with this result, though clearly we must have a real vale for ω, which implies that the correction term on the right must be greater than $-\omega_0^2$. Our main interest is rather with the conditions under which self-maintained oscillation can commence. For this to happen, we must have

$$\mathrm{Im}\left(\frac{2W\bar{p}_1(\sigma_2\mu_2Z_2-\sigma_1\mu_1Z_1)}{(2\rho\bar{p}_1)^{1/2}+W\bar{x}(Z_1+Z_2)}\right)>2k\omega, \qquad (18)$$

where Im implies the imaginary part. In both (17) and (18), we must use the expression (6) giving $\bar{x}$ in terms of the static pressure $\bar{p}_1$. We must also use the value of ω given by (17) both in the right-hand side of (18) and also in evaluating the impedances Z_1 and Z_2 in both (17) and (18). This implies the necessity for a recursive calculation, but fortunately, as we see below, it is an adequate approximation for the problem that we are addressing here to take $\omega=\omega_0$. This is clearly true for small values of $\bar{p}_1$, since the correction term in (17) vanishes as $\bar{p}_1\to 0$, and substitution of typical values for the parameters involved suggests that it remains a valid approximation as long as there is not an impedance maximum for Z_1 or Z_2 within the possible operating frequency range.

Equations (17) and (18), as separated forms of (16), form the basis of our subsequent discussion. We should recall that we have made simplifications in these two equations by omitting the displacement flows $\hat{U}_1$ and $\hat{U}_2$ described in (10) and (12) and the valve aperture inertance correction given by (13). Both these contributions could be included at the expense of complication to the algebra. Because both these corrections are small in most cases of interest, however, we can proceed to draw general conclusions based upon the simpler analysis.

II. GENERAL CONCLUSIONS

The general case clearly has many variations, so we examine here some particular cases in order to show the behavior of different sorts of valves with rather simple loading impedances. The first simplification we can make, which has already been applied in (16)–(18), is to neglect both the valve channel inertance, given by (13), and the displacement flows given by (12). Insertion of typical numerical values shows that these are both an order of magnitude less than the terms to which they are corrections, provided only that the operating pressure is more than a few hundred pascals, equivalent to a few centimeters water gauge pressure. While it is straightforward to include the effect of the resistive parts of the impedances Z_{ij} in a numerical calculation, we can deduce important general principles of valve behavior by neglecting this refinement. With this in view, we neglect wall losses in the two horns, neglect the resistive part of the radiation impedance Z_R, and take the source impedance Z_S to be very much larger than $Z_{11}^{(1)}$. The two loading impedances Z_1 and Z_2 of (11) are then both purely imaginary and we shall denote them by jX_1 and jX_2, respectively. Under these simplifying assumptions, (17) becomes

$$\omega^2=\omega_0^2-\frac{W^2(x_0+\sigma_1\mu_1\bar{p}_1/\omega_0^2)(\sigma_2\mu_2X_2-\sigma_1\mu_1X_1)(X_1+X_2)}{\rho+(W^2/2\bar{p}_1)(x_0+\sigma_1\mu_1\bar{p}_1/\omega_0^2)^2(X_1+X_2)^2}, \qquad (19)$$

while the inequality (18) becomes

$$\frac{(2\rho\bar{p}_1)^{1/2}W(\sigma_2\mu_2X_2-\sigma_1\mu_1X_1)}{\rho+(W^2/2\bar{p}_1)(x_0+\sigma_1\mu_1\bar{p}_1/\omega_0^2)^2(X_1+X_2)^2}>2k\omega_0, \qquad (20)$$

where we have substituted from (6) for $\bar{x}$.

Equations (19) and (20) are in a form that lets us make some simple explicit statements about the behavior of different valve configurations. For simplicity, we shall also make the assumption that $\mu_1=\mu_2$, which is true for many practical systems, although it is easy to treat the more general case if we wish.

First, consider the configuration $(-,+)$ of Fig. 1(a). If the inequality in (20) is to be satisfied, then certainly we must have

$$(-,+)\Rightarrow X_1+X_2>0. \qquad (21)$$

This means that the sum of the termination impedances must be inertive, and reflects the observation that, when tuning a reed pipe in an organ by altering the resonance frequency of the reed, oscillation can be maintained over a large frequency range up to the pipe resonance, but not above it. If the condition (21) is satisfied, then (19) immediately tells us that

$$(-,+)\Rightarrow\omega<\omega_0. \tag{22}$$

This conclusion is in agreement with earlier analysis.[5]

Conversely, for the configuration $(+,-)$ of Fig. 1(b), we must have

$$(+,-)\Rightarrow X_1+X_2<0. \tag{23}$$

This means that the sum of the termination impedances in this case must be compliant if the valve is to oscillate. An oboe reed blown from the staple end refuses to crow because the inertance of the staple tube is dominant. The fact that a trumpet player can easily buzz his lips in the absence of the instrument can be explained by the assumptions that the lip valve is of type $(+,-)$, because the small volume enclosed by the player's mouth gives a capacitive impedance. As we see below, however, this can also be explained by the assumption that the lip valve is of type $(+,+)$. From (19), for the $(+,-)$ case,

$$(+,-)\Rightarrow\omega>\omega_0. \tag{24}$$

For a valve of configuration $(+,+)$ as in Fig. 1(c), we must by the same argument have

$$(+,+)\Rightarrow X_1-X_2<0. \tag{25}$$

In this system, a capacitive up-stream impedance and an inertive down-stream impedance both favor oscillation. An assumption that this is the configuration of a brasswind player's lips or of the human larynx is thus also in accord with the observed autonomous oscillation. From (19), the frequency shift in the $(+,+)$ case has the same sign as $(X_1^2-X_2^2)$.

A valve with configuration $(-,-)$, if one existed, would require

$$(-,-)\Rightarrow X_1-X_2>0. \tag{26}$$

This requires either a narrow tube leading from the source to the valve, to give an inertive impedance, or an outlet duct in the form of some sort of open horn operating just above one of its resonance frequencies to give an acoustic compliance. The sign of the frequency shift away from the valve resonance will be that of $(X_2^2-X_1^2)$.

Suppose now that the auxiliary impedances are arranged as described above for each valve type, so that autonomous oscillation is possible in principle. We wish to determine the conditions under which it will occur in practice. Once the physical parameters are given, we can plot the left side of the inequality (20) as a function of the valve inlet pressure $\bar{p}_1$ and compare it with the damping expression on the right side. This is done qualitatively in Fig. 4. For the two cases having $\sigma_1=+1$, in which the blowing pressure tends to open the valve, there is no limit to the blowing pressure or flow, except that in reality our simple geometric model for the valve will cease to apply for very high pressures and consequent large valve openings. If $\sigma_1=+1$, the left side of the inequality goes from zero through a maximum value and then declines again toward zero as $\bar{p}_1$ is increased. Depending upon the damping k, this curve will either not reach the line representing the right side of the inequality, and thus no oscillation can

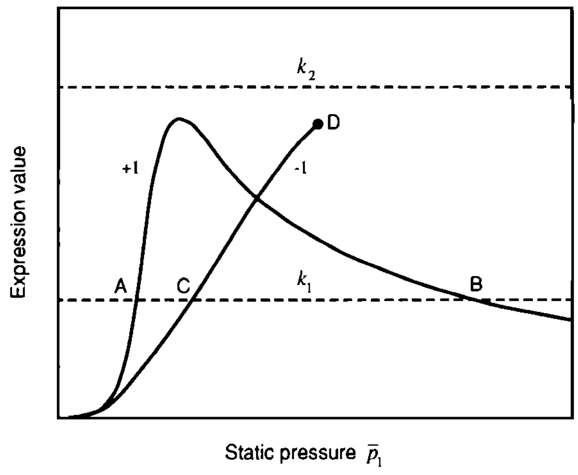

FIG. 4. Qualitative behavior of the two sides of the inequality (20) as functions of the blowing pressure $\bar{p}_1$ for the cases $\sigma_1=+1$, corresponding to a valve that is blown open by the applied static pressure, and $\sigma_1=-1$, corresponding to a valve that is blown closed by applied static pressure. For low damping k_1, the $+1$ curve lies above the damping curve over the pressure range from A to B so that oscillation can occur within this range. Similarly, oscillation can occur for the -1 case from C up to point D at which the valve becomes completely closed. For higher damping k_2, autonomous oscillation is unable to occur in either case.

occur, or else it will cut it twice, giving a limited pressure regime in which autonomous oscillation is possible. In practice the upper pressure limit for oscillation is usually so large that nonlinear effects obscure its existence. For the two cases having $\sigma_1=-1$, in which the blowing pressure tends to close the valve, the value of the left side of the equation rises steadily from zero and then terminates abruptly as the valve is blown completely closed. Depending upon the damping, again, we either have no oscillation or else autonomous oscillation between a threshold blowing pressure and the pressure at which the valve closes.

III. PARTICULAR CASES

Let us examine three particular cases in greater detail. Suppose that a valve of specified configuration is connected to a steady pressure source of high internal impedance by a pipe of length l_1 and cross-sectional area A_1 and exhausts to the free air through another pipe of length l_2 and area A_2. It is often an adequate approximation to assume that pipe 1 is acoustically closed at the end remote from the valve and that pipe 2 is acoustically open, and to neglect dissipation in these ducts. Their input impedances are then

$$Z_1=-j\frac{\rho c}{A_1}\cot\left(\frac{\omega l_1}{c}\right),\quad Z_2=j\frac{\rho c}{A_2}\tan\left(\frac{\omega l_2}{c}\right), \tag{27}$$

where c is the velocity of sound in air. If we assign numerical values to the physical quantities involved, we can now calculate the range of pipe lengths and diameters over which the valve can oscillate. As an example, let us choose a valve with the basic parameters

$$m=100\text{ mg};\quad S_1=S_2=1\text{ cm}^2;\quad S_3=0;\quad x_0=1\text{ mm};\quad \omega_0=1000\text{ s}^{-1};\quad l_1=l_2=100\text{ mm}. \tag{28}$$

The valve thus has size and other physical parameters roughly comparable with those of the reed of an organ pipe, the vocal flaps of a human larynx, or the syringeal membranes of a large bird. We leave the valve damping k as a free parameter.

A. Nonresonant case

Let us assume that the lengths of both supply and exhaust tubes are small compared with a quarter-wavelength at the valve resonance frequency ω_0. The situation then corresponds to the practically important cases of sound generation in the human larynx or the avian syrinx, as well as to more trivial examples such as buzzing the lips in the absence of a brass instrument or causing the reed of an oboe or bassoon, removed from the instrument, to crow.

To an adequate approximation, in this case, we can write (27) in the form

$$Z_1 \approx -j(\rho c^2/V\omega), \quad Z_2 \approx j\,(\rho l_2\omega/A_2), \tag{29}$$

where V is the volume of the entry tube. The volume of the entry tube and the diameter of the exhaust tube therefore make convenient parameters in terms of which to describe valve behavior.

Because the system is operating below the first resonance of either of the ducts, the reactance X_2 of the open outlet is certainly small, and the reactance X_1 of the inlet duct is small provided that its volume is not very small. Under these conditions, from (19), we see that

$$\omega \approx \omega_0 \tag{30}$$

the direction of the deviation from the valve resonance frequency being determined by the configuration of the valve, as discussed in Sec. II.

The first case to be considered is the configuration $(-,+)$ of Fig. 1(a). For convenience in this example, we choose the volume of the inlet chamber to be very large, so that it does not affect the result, and vary the diameter of the exit tube. The calculated threshold behavior for three values of the valve damping parameter k, corresponding to quality factors Q of 10, 5, and 2.5, respectively, is shown in Fig. 5. There is a range of outlet tube diameters, extending downward from about 20 mm, over which the valve oscillates readily, but for larger diameters oscillation cannot be initiated. The results for tube diameters less than a few millimeters cannot be regarded as physically reliable, since we have neglected the effect of wall losses in the calculation of Z_2. For the parameters chosen, the reed actually closes for pressures in excess of 1000 Pa, so that the curves terminate at the upper border of the graph.

The next case is that of a valve with configuration $(+,-)$, as shown in Fig. 1(b). For convenience, we now take the diameter of the exit tube to be large, so that the valve effectively exhausts into free air, and vary the volume of the inlet chamber. The thresholds for oscillation, for the same three values of the damping parameter k, are then as shown in Fig. 6. It is clear that the threshold pressure for valve oscillation is a strong function of inlet volume, and

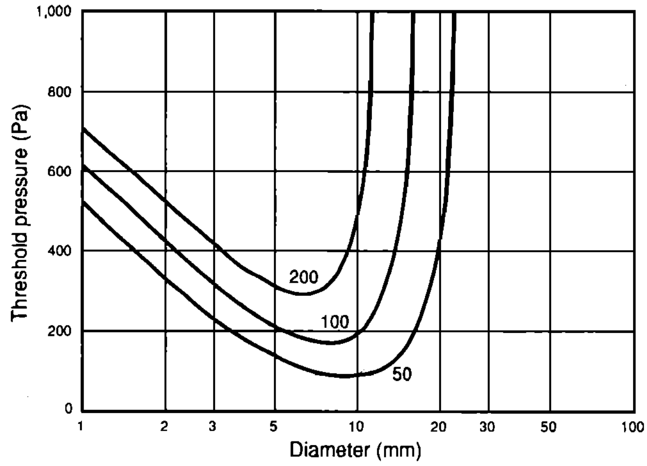

FIG. 5. Threshold blowing pressure for oscillation of a valve of configuration $(-,+)$, as in an organ reed pipe, as a function of the diameter of the exhaust tube, assumed 100 mm in length, when supplied from a reservoir of large volume. The parameter on the curves is the damping coefficient k; the values selected correspond to quality factors 10, 5, and 2.5, respectively, for the reed resonance.

that this volume must in fact lie between about 10 and 500 cm^3 for efficient operation with the particular valve parameters chosen.

Finally, we consider a valve with configuration $(+,+)$ as in Fig. 1(c). In this case, we show in Fig. 7 the behavior of a $(+,+)$ valve when supplied from a reservoir of volume 100 cm^3. The added impedance of the reservoir enhances the instability of the valve, when the exhaust tube diameter is large, and oscillation is maintained even when the exhaust tube is effectively removed, as for a $(+,-)$ valve. If a reservoir of similar volume had been connected to the $(-,+)$ valve of Fig. 5, then oscillation would have been inhibited for exhaust tube diameters greater than about 10 mm instead of 20 mm.

In relation to these three figures, we should remark that the damping coefficient k is not simply the one that

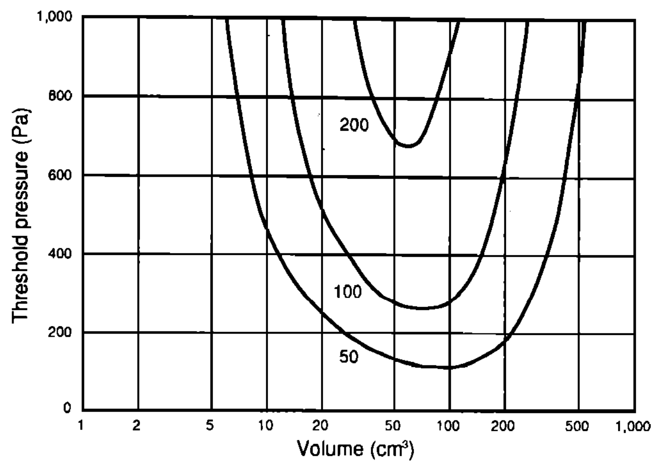

FIG. 6. Threshold blowing pressure for oscillation of a valve of configuration $(+,-)$, as in one possible model for a brasswind player's lips or for the human larynx, as a function of the volume from which it is supplied, assuming exhaust to the open air. The parameter on the curves is the damping coefficient k, as in Fig. 5.

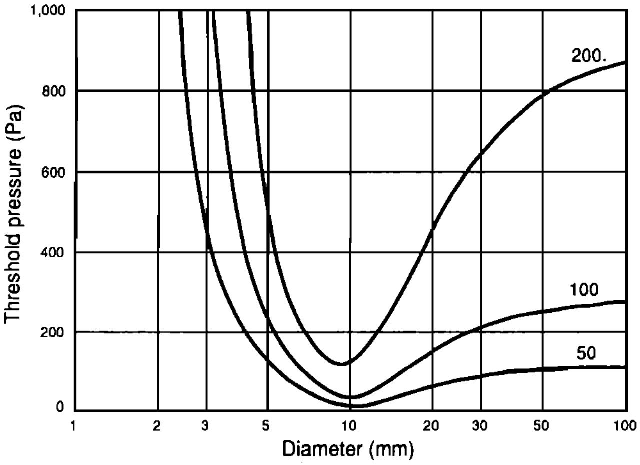

FIG. 7. Threshold blowing pressure for oscillation of a valve of configuration $(+,+)$, as in an avian syrinx or in an alternative model of the brasswind player's lips or the human larynx, as a function of the diameter of the exhaust tube, assumed 100 mm in length, when supplied from a reservoir of volume 100 cm^3.

would be measured from the damped decay of vibration of the reed tongue in the absence of any blowing pressure. Aerodynamic flow through the valve may well contribute further damping and, since this depends upon vortex generation, the damping may depend upon the direction of the flow through a given valve, rather than simply upon its magnitude. There has been relatively little investigation of these effects.[4]

B. Resonant case

In musical wind instruments, the outlet duct takes the form of a horn of some sort, and the resonances of this horn determine the oscillation frequency of the pressure-controlled valve generator. This can happen because the impedance Z_2 given by (27) can be reactive, very large, and of either sign, close to the horn resonance. If we assume that X_2 is very large in (19) then we see that the difference between ω^2 and ω_0^2 is of the same order as ω_0^2, while its sign is opposite to that of σ_2.

This means that, for a woodwindlike reed of configuration $(-,+)$, the operating frequency can be close to that of a horn resonance, provided the frequency of that resonance is less than the resonance frequency ω_0 of the reed. For an organ reed pipe, the situation is rather different, since the reed is tuned to nearly match the pipe resonance, so that both X_2 and the correction to ω_0 are small. If a brass-instrument player's lips are adequately represented by a valve of type $(+,-)$, as assumed both by Helmholtz[17] and by Fletcher,[5] then we expect that the playing frequency ω should be higher than the lip resonance frequency ω_0. The careful measurements of Saneyoshi *et al.*,[7] which included an evaluation of reed and lip resonance frequencies under playing conditions, confirm the prediction above for clarinet and bassoon players, but show that, contrary to expectation, the resonance frequency of a euphonium player's lips is somewhat higher than the frequency of the note being played. These authors take this to imply that the lip-valve has an effective configuration $(-,+)$, but this seems physically unlikely. The observations are equally explicable if the lip-valve actually functions as though having configuration $(+,+)$, like an avian syrinx, the lip motion being essentially at right angles to the air flow direction. For a system as mechanically complex as a vibrating lip or larynx, however, such a single-parameter model can at best provide a rough approximation to the true behavior.

IV. CONCLUSION

This brief analysis has provided a coordinated and comparative view of the autonomous oscillation behavior of four configurations of simple valve. Together they encompass the valve types met with in organ pipes and brass musical instruments, in the human vocal system, and in the avian syrinx. The treatment is concerned only with the conditions under which autonomous oscillation can be maintained in these valve systems, and shows the crucial importance of the acoustic impedances of the inlet and outlet ducts to the valve. Several refinements to the treatment that were neglected in the interests of simplicity can be included in a more careful analysis, but do not affect the general conclusions.

The treatment has the advantage of great analytical simplicity, so that it is possible to examine with great ease the effect of changing particular parameters. We have displayed such variations only in relation to the supply and exhaust ducts individually, but this can easily be extended to the ducts in combination, as in Fig. 7, or to variation of parameters such as initial valve opening x_0, valve resonance frequency, valve mass, and effective area.

Because this analysis has concerned itself only with the initiation of oscillations, the underlying theory has been a linearized approximation. To treat the subsequent development of oscillations to their full amplitude, a genuine nonlinear treatment is necessary, either in the time domain or in the frequency domain.[18–20] Such nonlinear theories almost inevitably deal with the detailed behavior of a single specific case, and necessarily involve detailed numerical computation. The present discussion, taken together with more specific analyses in the literature, provides a suitable starting point.

ACKNOWLEDGMENT

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