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ACOUSTIC AND AERODYNAMIC DETERMINANTS OF THE SOUND QUALITY OF FLUTES

Neville H. Fletcher
CSIRO Australia, and also
Research School of Physical Sciences and Engineering
Australian National University, Canberra 0200, Australia

ABSTRACT

A theoretical description of the interaction of a plane jet with an acoustic flow field is presented, along with a discussion of the interaction of this jet with the lip of a tube resonator to produce a self-sustained oscillation. This description is related to the sound production mechanism in flutes of various families, and the factors controlling pitch, overblowing, and harmonic development of the sound are discussed. Performance techniques are also briefly described. Despite the reasonable success of this description, it is pointed out that the concept of a mixing region conceals our ignorance of the aerodynamic processes taking place, and in particular the role of vorticity. It is surmised that invocation of these processes will prove necessary to a proper understanding of details of the art of embouchure-hole and head-joint design.

Introduction

Flutes are among the oldest instruments in the common musical heritage of mankind. They come in several different varieties which can be classified in various ways. One division is into those flutes with a mechanically defined windway, such as whistles, recorders and organ pipes, and those in which the windway is defined by the players lips. I shall be concerned here primarily with this second class of instruments. Within this class we can then recognise several sub-classes. An obvious division is into the stopped-pipe flutes, of which the Panpipes or nai is a common example, and the open-pipe flutes which constitute the rather more common instruments. Within the class of open-pipe flutes we can then recognise those that are end-blown, such as the Japanese shakuhachi, and those that are blown transversely, such as Western orchestral flutes.

There are many things that could be discussed about flutes, and related instruments such as the shakuhachi and the recorder. Some of these have been treated in detail in books,[1-5] but here I would like to concentrate on our present understanding of things that affect the sound quality of the instruments. This includes, of course, the nature of the sound generation mechanism itself, as well as more subtle physical phenomena and performance techniques that determine the loudness and harmonic development of the sound. I will not have time to say anything about other practically important matters such as the arrangement of finger holes, the mechanisms by which the keys are operated, and the resulting tuning of the scale of the instrument.

Before beginning on this survey I would like to pay tribute to some of the people whose studies over the years have brought our knowledge to its present state. Those two giants of acoustics, Helmholtz and Rayleigh, both made notable contributions, though they spent relatively little time on the instrument. More recently, however, we are vastly indebted to John Coltman[6-14] for his masterly series of studies on all aspects of the flute—he should really be giving this invited paper rather than I. Other people have also had valuable input to the subject, and I should mention in particular Sam Elder[15-16] for his work on the jet-drive mechanism and Art Benade for his work on the flute head joint[17], as well as on the acoustics of all the musical instruments[3]. Lastly let me mention those fluid dynamicists, such as Michael Howe[20] and Mico Hirschberg[21,22], who have recognised our ignorance of some of the basic flow mechanisms responsible for sound production from an air jet and are attempting to place this study on a more rigorous basis.

The Unstable Jet

An air jet provides the prime excitation mechanism for all flute-like instruments, and does so because of a basic instability mechanism that was discussed in detail by Rayleigh. A sinusoidal disturbance on a plane jet propagates in the flow direction with a well defined wave speed, and its amplitude grows in time, provided that its wavelength is not too short compared with the thickness of the jet.

In the case of a jet issuing from a flue slit into an acoustic field with a velocity component normal to the plane of the jet, as we find in an organ pipe, we can understand the behavior as follows. The jet as a whole is moved back and forth by the field, except at the slit exit where it is held still. The flue slit thus acts as the source of a perturbing wave which propagates along the jet and grows in amplitude as it does so. If the acoustic field has velocity amplitude u and frequency ω, then the transverse "sinuous" displacement of the jet at distance x from the slit can be written

$$y(x,t) \approx -\frac{ju}{\omega}\{\cos\omega t - e^{\mu x}\cos[\omega(t - x/v)]\} \qquad (1)$$

where μ is the spatial growth rate and v the speed of the wave on the jet. This expression is not exact near the origin, but is a good approximation provided $\mu \gg \omega/v$. Expression (1) applies, however, only while the amplitude $y(x)$ of the wave is less than the thickness of the jet. For greater amplitudes, as found further from the flue slit, the growth rate becomes approximately linear, so that

$$y(x,t) \to 0.2(x - x_0)e^{j\omega(t - x/v)} \qquad (2)$$

where x_0 is a positive constant. In the regime to which (2) applies the behavior is really more complex than this, however, and the jet breaks up into a linearly growing vortex street. The form of an observed jet instability wave is illustrated in Fig. 1, which confirms this analysis.

The behavior of the wave velocity v and growth factor μ depends upon the thickness and velocity profile of the jet and upon whether it is laminar or turbulent. These relations are best expressed in terms of the Strouhal number kb where b is the half-width of the jet and $k = \omega/v$ is the disturbance wave number on the jet. Generally speaking, the velocity v rises from a rather small value as $k \to 0$ to a little more than half the flow speed V_J on the centerline of the jet for $kb \approx 2$, in which condition the wavelength of the disturbance is a little more than the full width $2b$ of the jet. The amplification factor μ similarly rises from a small value as $kb \to 0$, goes through a broad maximum for $kb \approx 0.6$, and falls to become negative, representing attenuation of the wave as it propagates, for $kb > 2$. The maximum value of μ is expressed nondimensionally as $\mu b \approx 0.4$. The general form of this behavior, shown in idealized form in Fig. 2, has been confirmed by experimental studies on both laminar and turbulent jets[23-25], though details are a little different from this idealization.

For the sort of jets that we meet in flutes, with a blowing pressure of about 1 kPa and a width of about 1 mm, we find that $v \approx 20\ \mathrm{m\,s^{-1}}$ and $\mu \approx 100\ \mathrm{m^{-1}}$ at typical sounding frequencies. Since a typical jet length is about 10 mm, the wave propagation time from the flue to the lip of the pipe is about 0.5 ms and the amplification about $e^{10} \approx 10^4$. Both the time delay and the wave growth are of great significance for the operation of the jet-drive mechanism.

Jet-Pipe Interaction

While the behavior of the wave on the jet is important, as we see presently, the interaction of the jet with the pipe lip is even more important and much less well understood. It has recently become clear that, even in the simplest approximation, there are two aspects to consider. These are illustrated in Fig. 3. When a shallow cavity is excited into oscillation by an airstream flowing over its mouth, the flow generally re-attaches at the downstream wall[16]. The oscillation is then driven by the displacement associated with the streamline dividing surface, and resonance occurs when there is some sort of match between the wavelength of the disturbance on the jet and the natural frequency of the cavity[16,26]. In an open-pipe resonator, however, and probably also in a sufficiently long closed-pipe resonator, the jet actually flows into the pipe for part of each cycle and we must consider this flow as an additional, and generally dominant, contribution to the driving mechanism. The situation is, indeed, almost exactly analogous to the mechanism driving a reed instrument such as the clarinet. In what follows I shall ignore the first contribution and concentrate on the second.

The simplest case to analyse is the end-blown flute or organ pipe, as shown in Fig. 4. The jet, assumed to have velocity V_J enters the pipe and contributes a volume flow U_J which varies sinusoidally as the deflection of the jet oscillates with an amplitude that is small compared with its thickness. We assume that the jet flow mixes with the total pipe flow U_P within the shaded region, which is assumed to be short compared with the wavelength of sound so that compression effects can be neglected. Simple considerations of conservation of volume and momentum fluxes[15,27] then lead to the result

$$U_P = \left[\frac{\rho(V_J + j\omega\,\delta)}{(Z_P + Z_M)S_P} \right] U_J \tag{3}$$

where δ is the end-correction of the pipe at its mouth, S_P is the cross-sectional area of the pipe, and the flows are written as complex functions. Z_P is the acoustic impedance of the pipe body and Z_M that of the pipe mouth, so that $Z_P + Z_M$ is the series impedance of these two quantities and goes through a minimum value at the natural resonance frequency of the complete pipe.

In a real situation the amplitude of jet displacement is not small compared with the jet width, and we must also consider the velocity profile across the jet[23-25], which generally has a bell-like shape tending towards

$$V_J(x,y) = V_0(x)\,\mathrm{sech}^2[(y - y_0)/b] \tag{4}$$

where b is a measure of the half-width of the jet and y_0 is the offset of the undisturbed jet relative to the pipe lip. The jet half-width b grows with distance, the details depending upon whether the jet is laminar or turbulent, and the central velocity decreases in such a way that the total momentum flux, proportional to $V_0^2 b$, is conserved. These circumstances combine to produce saturation in the power input to the pipe from the jet and also generate harmonics of the basic motion, as we discuss later.

At this stage we should introduce the modification necessary to make the analysis apply to transversely blown flutes[12]. If we assume simple transverse blowing, which we see later is not a very good assumption in actual flutes, then the only modification is to the first term in (3). If the jet enters the pipe at an angle θ to the pipe axis, then the expression analogous to (3) becomes

$$U_P = \left[\frac{\rho(V_J \cos\theta + j\omega\,\delta)}{(Z_P + Z_M)S_P} \right] U_J\,. \tag{5}$$

For a truly transverse jet excitation $\theta = 90°$, so that the first term in the numerator vanishes. For a real flute with a significant wall thickness, or a chimney under the embouchure hole, part at least of the mixing region follows the curve of the walls so that some average between (3) and (5) must be taken, essentially just reducing the contribution of the first term. This analysis suggests that an end-blown flute, such as the shakuhachi[28], makes a more efficient use of the jet coupling than does a transverse flute, and this also has some implications for the harmonic development of the sound.

Jet Drive and Sound Production

Expressions (1)–(4), which have been around for a long time, give a reasonably good account of many aspects of jet-excited pipe behavior in the case of organ pipes or end-blown flutes. The jet excitation mechanism can be summarised[29] in terms of an acoustic admittance associated with the jet as illustrated in Fig. 5, which is just one of a variety of possible representations. The jet admittance Y_J has a spiral form in the complex plane when considered as a function of either the blowing pressure p_0, the frequency ω or the jet length l. The spiral is different in each case, but Fig. 5 shows the general behavior in schematic form. There is a particular blowing pressure for any given jet length and pipe resonance for which the jet admittance is real and negative, corresponding to the point A on the curve, which corresponds to about half a wavelength of the transverse jet disturbance between the flue slit and the far edge of the embouchure hole. At this point, provided the magnitude of the jet admittance is greater than the conductance of the pipe, the oscillation is at the natural resonance of the pipe and becomes self-sustained and grows until a limit is reached. For a rather lower blowing pressure the admittance becomes capacitive, which effectively increases the end correction and lowers the sounding frequency, while for a higher blowing pressure the jet admittance is inertive and the sounding frequency is above the pipe resonance. If the blowing pressure is too far removed from the point A then the operating point on the spiral moves into the right half-plane, the jet conductance become positive, and oscillation ceases. If the blowing pressure is increased well above that necessary to sustain the fundamental mode of the pipe, then the conditions for oscillation may be satisfied for one of the higher modes, and the pipe will "overblow" to sound this mode.[7,30]

The adjustment of blowing pressure and jet length are vital features of flute performance technique, both adjustments being necessary if the high notes are to be produced cleanly and in tune. This is illustrated in Fig. 6, which shows the ranges of relative blowing pressure over which each of the first three pipe modes can be made to sound, as derived from equations (1) and (3). The broken lines indicate regions in which the jet conductance, though negative, may be less than the pipe conductance. The square symbols on the range lines suggest blowing pressures at which the individual modes can be made to sound without contamination from other modes, leading to a reliable oscillation regime. The blowing pressures indicated are seen to increase approximately in proportion to mode frequency.

Measurements[7,31] in fact show that players of the orchestral flute have a common procedure which involves using a blowing pressure that is closely proportional to the mode frequency to be produced, while at the same time shortening the jet length, as indicated in Figs 7 and 8. These two adjustments, both of which are maintained nearly independently of dynamic level, have the effect of keeping the operating point on the jet admittance spiral of Fig. 5 in approximately the same position for all notes. For an instrument with a fixed windway, such as the recorder, the second adjustment is not available to the player, but we still find approximately the same increase of blowing pressure with pitch.[5]

The loudness of the sound produced by a flute depends on the total driving force that can be produced when the jet is completely switched into and out of the pipe mouth in each cycle of the oscillation. Since the jet center velocity V_J varies as $p_0^{1/2}$ in Bernoulli flow, the jet volume flow U_J in (3) or (5) is linearly proportional to the cross-sectional area of the jet and to the square root of the blowing pressure p_0, while the first term has an additional factor of $p_0^{1/2}$. To retain the operating point in the left half of the admittance plane of Fig. 5, as is necessary for any sounding at all, the extreme range of jet velocity is about a factor 3, corresponding to a factor 9 in blowing pressure, but the allowable range in actual playing is actually smaller than this—perhaps a factor of about 3 only in p_0—because of the need both to maintain an adequately large negative real part to Y_J and to keep the sounding frequency close to its nominal value. When using one of the upper modes in playing, as is necessary in all but the lowest register of the instrument, the player must also choose a blowing pressure that allows this mode to sound as cleanly as possible, without contamination from higher or lower modes, as discussed in relation to Fig. 6.

On the other hand, the jet cross section can be varied by at least a factor 10 by increasing the muscular tension in the player's lips, so that this represents the most important means of loudness control in flutes. Measurements of performance techniques of many flute players confirm these conclusions[31], as shown in Fig.9. While there should be no lower limit to the sound power produced, the low end of the dynamic range is unstable against minor lip fluctuations, and the dynamic range of an orchestral flute is only about 10 dB in sound power. The apparent loudness, however, varies more than this because of variations in the radiated spectrum.

The theory encapsulated in equations (1)–(3) appears to be completely linear, but this conceals the variation of jet velocity across its profile as given by (4). When this is inserted in the equations then not only do they describe the limiting value of the oscillation discussed above, but also the harmonic development[33]. Fairly clearly, at least on this simple theory, if the jet offset y_0 is zero, then the whole oscillation is symmetrical and no even harmonics are generated. In normal playing this condition is not achieved and the strength of even harmonics will depend in detail on the offset, and thus on the way in which the player directs the airstream into the mouth of the instrument. The levels of the odd harmonics also depend upon the precise value of the offset. Such a dependence can be readily calculated if the jet profile is known, and measurements of the radiated spectrum confirm the behavior. This comparison between theory and experiment is shown in Fig. 10. Good players are able to make subtle variations in the harmonic content of their sound by manipulating details of the jet width and direction, as is also the case in the static voicing of organ flue pipes.

The role of the jet profile in determining harmonic development is also important, and this depends upon both the length and thickness of the jet, as well as on the detailed shape of the windway or of the player's lips. In general terms we expect the profile of the jet to have roughly "top-hat" form as it emerges from the flue slit. This profile is then progressively modified to the limiting form given by (4) by diffusion of vorticity in from the faces of the jet. The profile of a long jet will therefore be smoother than that of a short jet, and the profile of a narrow jet will achieve a smooth form after a shorter travel distance than will that of a broad jet. This is illustrated in the case of an organ-pipe jet in Fig. 11. Since vorticity diffusion is a time-dependent process, the smoothing of the jet over a given travel distance will also decrease as the blowing pressure is raised.

Since the harmonic content of the excitation function for an oscillating jet dictates the harmonic content of the radiated sound, we can therefore state that harmonic development is increased by having (a) a short jet length, (b) a high blowing pressure, and (c) a broad flue slit. Some of these conditions are contradictory in terms of performance technique, but they combine to predict (a) an increase in harmonic content for loud playing, and (b) a variation in tone quality from bright to dull as the jet length is varied from rather short to rather long. These predictions also conform to the conclusions drawn from a study of actual performance by different players, as shown in Fig. 12, and to experience in voicing organ flue pipes. The increase in harmonic development with "loud" playing is almost more important to the psychophysical impression of loudness than is the increase in total sound pressure level.

To these considerations should be added the remark that a musical flute sound is not steady but has a controlled vibrato, usually at a frequency of around 5 to 6 Hz. Measurements[31] show that this vibrato is largely derived from a periodic variation in air pressure, and subjective assessment suggests muscular movement of the diaphragm as the control agent, though laryngoscopic studies show that the vocal folds in professional players are partly tensed and participate in this rhythmic oscillation[32]. The main acoustic consequence of vibrato is a periodic variation in tone quality, rather than in pitch or in the amplitude of the fundamental of the sound.

To our knowledge there has been little or no careful investigation of the jet drive mechanism in long closed pipes such as found in the Panpipes. The major difference would appear to be that there can be no steady flow into the pipe in this case, while such a flow is not prohibited and probably occurs in open-pipe flutes. The principal effect of this restriction would appear to be an automatic control of the static offset of the jet at the edge of the embouchure hole to achieve a zero-flow condition. Such feedback should make the instrument easier to play than one with an open pipe, and indeed this does seem to be true. The case of a shallow cavity exposed to an airstream appears to represent an extreme of this situation, in which the control of jet flow takes place in a time that is small compared with the oscillation period, so that a no-flow condition is imposed continuously and the jet is effectively re-attached at the downstream lip of the cavity.

The end-blown shakuhachi appears to represent the opposite extreme to the stopped pipe, since there is almost no impediment at all to free steady flow of the jet down the instrument tube. The player must therefore exert very conscious control over jet direction in order to achieve a steady sound. At the same time, this freedom and the probably greater jet-drive efficiency of end-blown instruments compared with their transversely-blown counterparts combine to give the shakuhachi a very great tonal range in the hands of an expert player.

Details of Jet-Pipe Interaction

Although the approach outlined above gives moderately good agreement with experiment in terms of blowing-pressure range, sound power output, acoustic spectrum, overblowing phenomena, initial transients and performance techniques, it is unsatisfactory in detail because of certain numerical deviations from experiment and, more particularly, because of the ignorance concealed within the concept of a mixing region in which the jet flow becomes merged with the pipe flow. The importance of this region becomes apparent from a simple calculation of energy flux, based on the model above. When the jet is completely switched into and out of the mouth in each oscillation cycle, the energy flux in the jet is $\rho V_J^3 S_J/2$, with half of this energy flowing into the pipe. About half of this power is in the steady flow into the pipe and half in the oscillating flow. If the quality factor of the pipe resonance is Q, then its impedance R at resonance is purely resistive and equal to $\rho c/QS_P$, and the acoustic power flux in the pipe oscillation is $RU_P^2/2$. Using (5) we then find an efficiency of energy conversion for the total jet flow given by

$$E \approx Q\left(\frac{S_J}{S_P}\right)\frac{(V_J^2\cos^2\theta+\omega^2\delta^2)}{cV_J}. \tag{6}$$

Inserting typical figures for a flute jet, even taking $\cos\theta = 1$, gives a conversion efficiency of only a few percent.

This indicates that typically more than 95 percent of the pneumatic energy carried by the jet into the pipe is dissipated in the mixing region. Of the remaining 5 percent or so that is transferred to the acoustic oscillation of the air in the pipe, most is dissipated in viscous and thermal losses to the pipe walls, so that only about 1 percent of the initial pneumatic energy is radiated as sound. Typical values for moderately loud playing are a power input of about 100 mW—a jet flow of about 100 $\mathrm{cm^3 s^{-1}}$ at a pressure of about 1 kPa—and a sound power output of about 1 mW.

The dynamics of the mixing region involves two different processes. In the first place, the jet excites higher evanescent modes of the pipe with nearly the same efficiency as it does the basic plane-wave mode. These higher modes, shown in Fig. 13, do not propagate, because the pipe is generally too narrow to support them, and do not radiate much because of their multipole nature. Their amplitudes, in fact, are significant only within a distance of about one pipe diameter from the pipe mouth, which is just about the dimension of the mixing region. As well as dissipating energy through viscous and thermal processes, these higher modes can also influence details of the jet interaction. In the initial pipe transient they may be partly responsible for the "chiff" or "chonk" that is noticeable in instruments such as recorders and stopped organ pipes.

The second process, which is completely neglected in the acoustic approximation used so far, involves the interaction of vorticity, and particularly of concentrated vortices, with the acoustic flow. That such vortices are present is clear from flow visualisation experiments.[7,21] They accompany the tip of the jet in the initial transient, if it is sharply initiated, and are clearly visible both inside and outside the pipe mouth in steady speech. They do not generally develop, however, on the part of the jet between the flue slit and the further edge of the embouchure hole or upper lip of the pipe. A vortex moving across an acoustic field can give energy to, or take energy away from, the field, and indeed the large dissipation in the mixing region can be treated, at least in principle, in terms of such interactions[21], as well as in terms of simple viscous losses associated with the diffusion of vorticity as the jet mixes with the surrounding air in the pipe. No really satisfactory detailed approach to this problem has yet been formulated.

A major unsolved question is whether a rigorous aerodynamic treatment, including the role of vorticity and of isolated vortices in particular, contributes any new phenomena to the discussion, or whether it simply fills in the details of the processes by which the jet slows and diffuses in the mixing region, without changing the essentials of the outcome. The "standard" theory that we have outlined above takes the second view, but the proponents of the vorticity approach believe that a detailed study of vortex interaction will contribute an radically new understanding and an essentially new interaction mechanism.

Flute Design and Tone Quality

A woodwind instrument can be thought of in two parts—the sound generation mechanism, and the passive horn resonator with its finger holes and keys. Clearly both of these have a great influence on sound quality for rather different reasons, and we consider them in turn. We note that we are indebted to the careful experiment and design work of Theobald Boehm[1] some 150 years ago for the form of the modern orchestral flute. The design principles and detailed dimensions that he established have changed scarcely at all in the intervening years.

The resonator must provide, for each note on the instrument, at least one resonance that is of appropriate frequency to produce that note. Generally the situation is more complex than this, because of the nonlinearity and harmonic generation of the sound-producing mechanism, so that it is desirable to have several resonances that are in very nearly integral frequency relationship to reinforce several harmonics of the fundamental.[19] For flute-like instruments, as we have already seen, the air jet mechanism has a complex acoustic admittance so that it not only supplies energy but also causes the sounding frequency to differ from the frequency of the fundamental resonance on which the note is based. For ordinary flute performance, the blowing pressure is usually high enough to raise the sounding frequency above the frequency of the fundamental resonance but, because the jet amplification is greatly reduced at higher frequencies, the effect on higher resonances is small. A properly tuned resonator should therefore have its resonances stretched above simple integer relationship, particularly for notes in the low register for which harmonic development is specially important. This is achieved in a flute by the tapered design of the head joint and by appropriate adjustment of the cavity above the embouchure hole[17,34], these features being shown in Fig. 14. The cavity length is normally adjusted to about 15 mm so that the end correction at the embouchure hole is approximately constant, as shown in Fig. 15. The total mode frequency correction, as shown in Fig. 16, is then essentially caused by the taper of the head joint.

There is one other important feature of the resonator, and that is its behavior when several finger holes are open. The size and placing of the finger holes influences not only the frequency of the main resonance on which the note is based, but also the frequencies and heights of many other incidental resonances. Many small compromises in hole diameter and placing are possible—the baroque flute necessarily had small finger holes that are covered by the fingers themselves, while the modern orchestral flute has large finger holes that are closed by padded keys. The size of the finger holes in relation to the bore of the tube affects both the radiation efficiency and also the upper frequency at which resonances are found—a sort of finger-hole lattice cutoff frequency that is low if the finger holes are small and higher if they are large[3,18]. This latter effect, however, can be taken as characterising the instrument as a whole, and can not really be altered significantly at the discretion of the instrument maker.

A great deal of attention has been given to the effect of wall material on tone quality, the discussion resting upon the possibility that the mechanical vibration of the walls, though of small amplitude, may nevertheless significantly influence perceived sound quality. It is true that the resonator of a flute does have mode frequencies that lie within the compass of the instrument, and others at frequencies comparable to those of the upper harmonics of its sound. Far more important than wall material in influencing the frequencies and mechanical admittances of these resonances, however, is the simple matter of wall thickness, and there seems to have been little study of this. The currently accepted conclusion is that, within reasonable limits and for normal tubes of circular cross section, wall material has no detectable effect on tone quality.[9]

This conclusion does not mean that wall material is of no importance, however. Workability, mechanical strength, appearance, and freedom from corrosion are all vitally important. Indeed workability and mechanical properties may influence the detailed geometry of components such as flute head joints produced by cold drawing over a mandrel and so indirectly affect tone quality. These considerations have led to the almost universal use of silver or gold as preferred materials for metal flutes.

Apart from the player's lips, the geometry of the embouchure hole, as illustrated in Fig. 17, is probably the most critical determinant of flute tone quality. There are several important dimensions involved, among them the size and shape of the embouchure hole, the height of the chimney—previously determined by the wall thickness in wooden flutes—and the sharpness and degree of undercutting of the edge of the embouchure hole. It is in treating such detailed effects that the next generation of theoretical models involving vorticity will show their importance. It is possible, however, to reach some conclusions on the basis of the simple model we have already discussed.

From an analysis of the mouth impedance Z_M and hence of the end correction δ at the embouchure hole of the flute, we discover that, with the stopper adjusted to give best intonation, the range over which the end correction is approximately constant is decreased if the embouchure hole is made smaller. This has the effect of reducing the harmonic development and hence the brightness of the sound. A small embouchure hole also limits the width of the jet from the player's lips and hence the maximum flow and loudness of the sound. A large embouchure hole has the opposite effect. There is, of course, a limit to the range of size variation that is acceptable to the player, since ease of muscular control of the embouchure and achievable tonal flexibility are almost more important than simple harmonic development.

The height of the chimney also has an important effect on ease of playing. If the chimney is very high, then the jet when deflected by the chimney wall is moving parallel to the acoustic flow and the factor $\cos\theta$ of (5) is unity for this part of the mixing region. Conversely, if the chimney is very low then $\cos\theta \approx 0$ and we must rely upon the second term in (5) to provide the jet drive. This second term is itself proportional to the end correction δ which decreases with decreasing chimney height, and also proportional to the sounding frequency. We are therefore not surprised to find that a flute with an extremely low chimney is hard to sound, especially in the low register. At the other extreme, a very high chimney has much the same effect as a very small embouchure hole, and makes the tone rather dull. Again a compromise must be found by experiment.

Finally we note that the embouchure hole is usually undercut at an angle of about 7° and the edge of the hole is very slightly rounded so that it is not sharp. The reason for the undercut is not clear, though it would appear to direct the jet more smoothly into the head tube itself. We might expect that a rounding of the junction of the chimney with the tube might also have a significant effect on details of the flow. An understanding of just how these adjustments affect processes in the mixing region must, however, await a more detailed aerodynamic study. The slight rounding of the lip of the embouchure hole is thought to reduce high-frequency turbulence noise, which is an undesirable addition to the sound.

The crafting of head joints for flutes is a specialised art in which intuition and experiment still take the place of detailed understanding. It is also perhaps necessarily an individual art, because the ideal embouchure hole shape depends closely upon the shape of the player's lips and upon the performance technique used.

Conclusions

As a result of the work of many people, particularly over the past thirty years, we now have a reasonably complete first-order understanding of the acoustics of sound production in instruments of the flute family. Despite the general success of the theoretical description, however, there are many unanswered questions, and at least one central part of the mechanism is still shrouded in mystery as far as its details are concerned. It remains a challenge to modern acoustics and fluid dynamics to produce as completely satisfactory treatment of sound production in this, one of the oldest of instrument families.

References

1 T. Boehm, *The Flute and Flute Playing* (1871), trans. D.C. Miller (reprinted Dover, New York 1964)

2 P. Bate, *The Flute* (Ernest Benn, London 1975)

3 A.H. Benade, *Fundamentals of Musical Acoustics*, (Oxford University Press, New York 1976; reprinted Dover, New York 1992)

4 N.H. Fletcher and T.D. Rossing, *The Physics of Musical Instruments* (Springer-Verlag, New York 1991)

5 J. Martin, *The Acoustics of the Recorder*, (Moeck Verlag, Celle 1994)

6 J.W. Coltman, "Resonance and sounding frequencies of the flute," J. Acoust. Soc. Am. **40**, 99–107 (1966)

7 J.W. Coltman, "Sounding mechanism of the flute and organ pipe," J. Acoust. Soc. Am. **44**, 983–992 (1968)

8 J.W. Coltman, "Acoustics of the flute," Physics Today **21**(11), 25–32 (1968)

9 J.W. Coltman, "Effects of material on flute tone quality," J. Acoust. Soc. Am. **49**, 520–523 (1971)

10 J.W. Coltman, "Jet drive mechanisms in edge tones and organ pipes," J. Acoust. Soc. Am. **60**, 725–733 (1976)

11 J.W. Coltman, "Acoustical analysis of the Boehm flute," J. Acoust. Soc. Am. **65**, 499–506 (1979)

12 J.W. Coltman, "Momentum transfer in jet excitation of flute-like instruments," J. Acoust. Soc. Am. **69**, 1164–1168 (1981)

13 J.W. Coltman, "Time-domain simulation of the flute," J. Acoust. Soc. Am. **92**, 69–73 (1992)

14 J.W. Coltman, "Jet behavior in the flute," J. Acoust. Soc. Am. **92**, 74–83 (1992)

15 S.A. Elder, "On the mechanism of sound production in organ pipes," J. Acoust. Soc. Am. **54**, 1554–1564 (1973)

16 S.A. Elder, "The mechanism of sound production in organ pipes and cavity resonators," J. Acoust. Soc. Jpn (E) **13**, 11–23 (1992)

17 A H. Benade and J.W. French, "Analysis of the flute head joint," J. Acoust. Soc. Am. **37**, 679–691 (1965)

18 A.H. Benade, "On the mathematical theory of woodwind finger holes," J. Acoust. Soc. Am. **32**, 1591–1608 (1960)

19 A.H. Benade and D.J. Gans, "Sound production in wind instruments," Ann. N.Y. Acad. Sci. **155**, 247–263 (1968)

20 M.S. Howe, "Contributions to the theory of aerodynamic sound, with applications to excess jet noise and the theory of the flute," J. Fluid Mech. **71**, 625–673 (1975)

21 A. Hirschberg, A.P.J. Wijnands, A. van Steenbergen, J.J. de Vries, B. Fabre and J. Gilbert, "Jet drive and edgetone in flue organ pipes," Colloque Modeles Physiques, ACROE, INPG, Grenoble, France, 19–21 September 1990

22 M.P. Verge, B. Fabre, W.E.A. Mahu, A. Hirschberg, R. van Hassel, A.J.P. Wijnands, J.J. de Vries and C.J. Hogendoorn, "Jet formation and jet velocity fluctuations in a flue organ pipe," J. Acoust. Soc. Am. **95**, 1119–1132 (1994)

23 N.H. Fletcher and S. Thwaites, "Wave propagation on an acoustically perturbed jet," Acustica **42**, 323–334 (1979)

24 S. Thwaites and N.H. Fletcher, "Wave propagation on turbulent jets," Acustica **45**, 175–179 (1980)

25 S. Thwaites and N.H. Fletcher, "Wave propagation on turbulent jets. II. Growth," Acustica **51**, 44–49 (1982)

26 S. Yoshikawa and J. Saneyoshi, "Feedback excitation mechanism in organ pipe," J. Acoust. Soc. Jpn (E) **1**, 175–191 (1980)

27 N.H. Fletcher, "Jet-drive mechanism in organ pipes," J. Acoust. Soc. Am. **60**, 481–483 (1976)

28 Y. Ando, "Input admittance of shakuhachis and their resonance characteristics in the playing state," J. Acoust. Soc. Jpn (E) **7**, 99–111 (1986)

29 S. Thwaites and N.H. Fletcher, "Acoustic admittance of organ pipe jets," J. Acoust. Soc. Am. **74**, 400–408 (1983)

30 N.H. Fletcher, "Nonlinear interactions in organ flue pipes," J. Acoust. Soc. Am. **56**, 645–652 (1974)

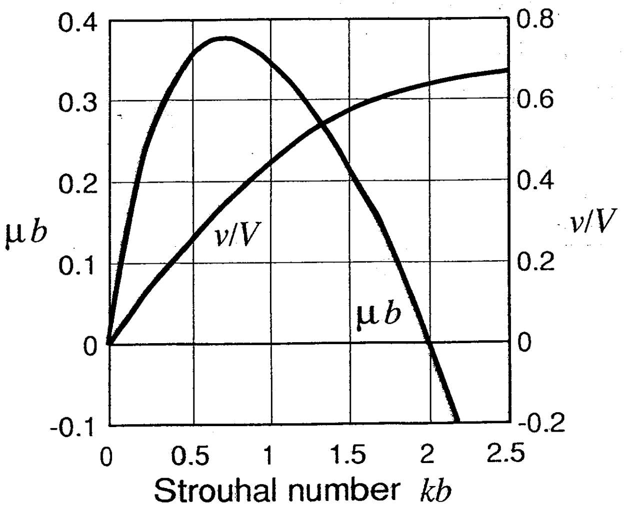
31 N.H. Fletcher, "Acoustical correlates of flute performance technique," J. Acoust. Soc. Am. **57**,
233–237 (1975)

32 S. Mukai, "Laryngeal movement while playing wind instruments," Proc. Int. Symp. Musical Acoust., Tokyo, August-September 1992, pp. 239–242

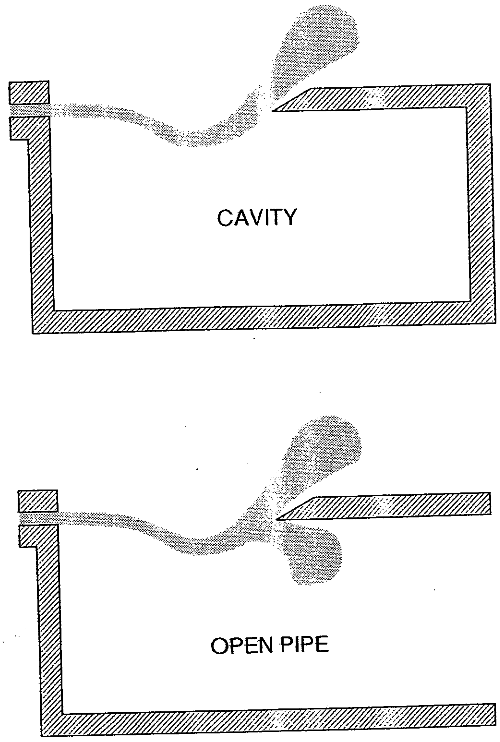
33 N.H. Fletcher and L.M. Douglas, "Harmonic generation in organ pipes, recorders and flutes," J. Acoust. Soc. Am. **68**, 767–771 (1980)

34 N.H. Fletcher, W.J. Strong and R.K. Silk, "Acoustical characterisation of flute head joints," J. Acoust. Soc. Am. **71**, 1255–1260 (1982)

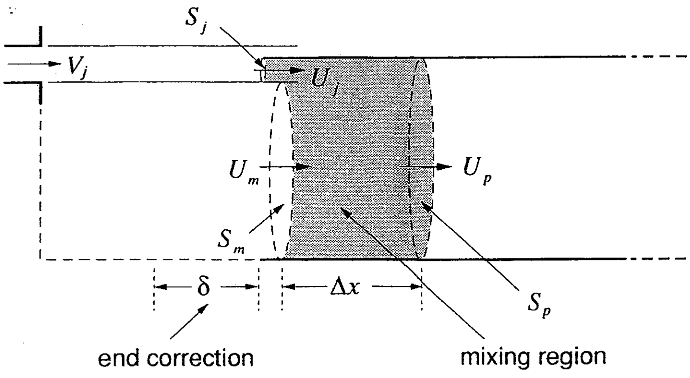
1. Powder-trail photograph of a plane jet entering a transverse acoustic flow field. Physical parameters are appropriate to a flute or organ-pipe jet, and the transverse dimension of the picture is about 30 mm. [Thwaites and Fletcher]



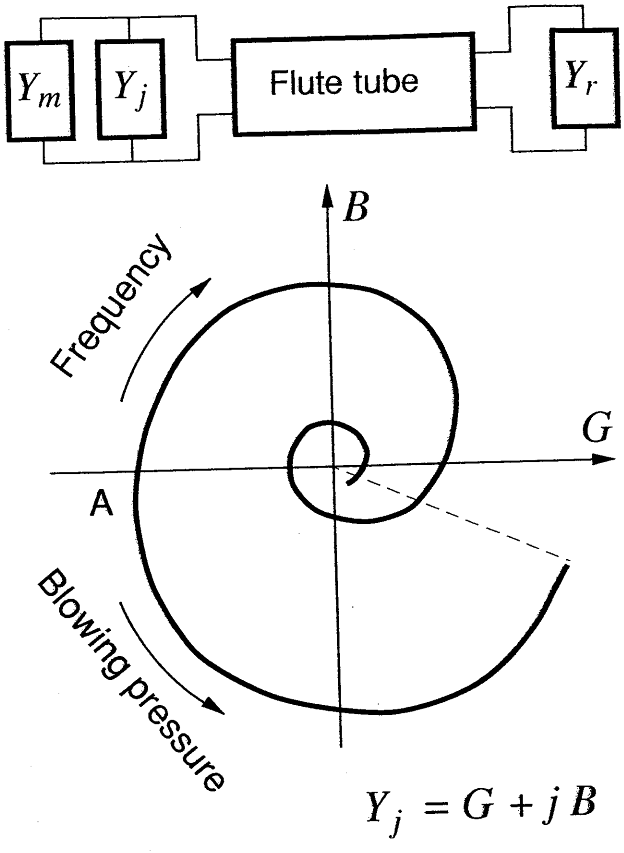
2. Approximate behavior of the amplification factor μ and wave velocity v on a jet of half-width b and central velocity V. [Thwaites and Fletcher]



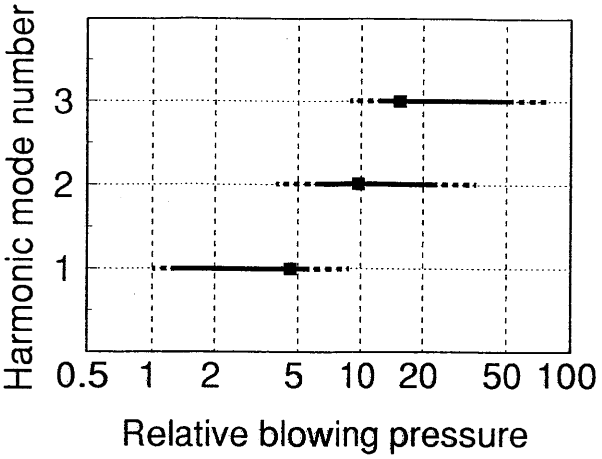
3. Contrasting behavior of an air jet exciting a closed cavity and an open pipe. In the former case the jet appears to re-attach to the lip so that there is no jet flow into the cavity.



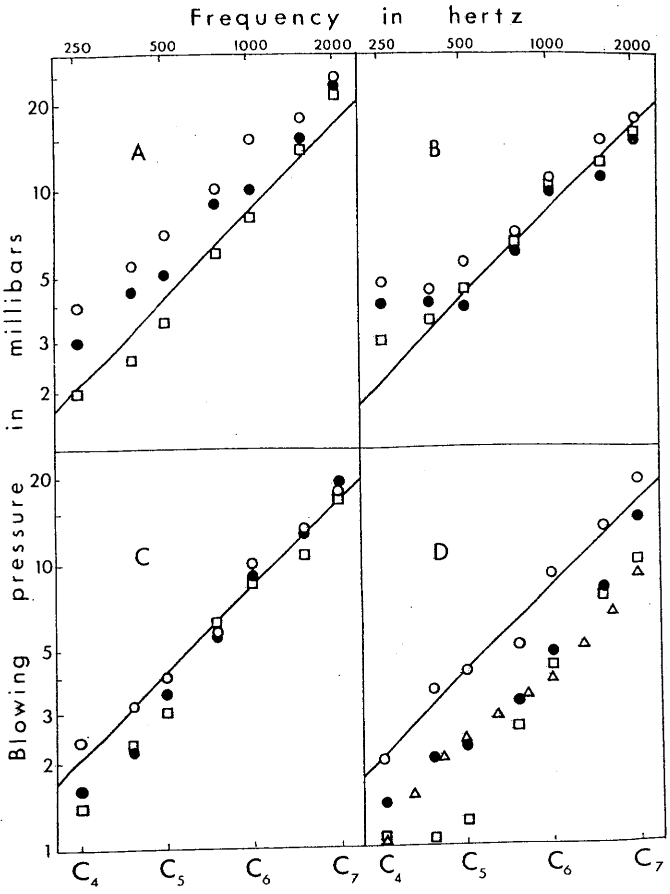
4. Assumed geometry for calculation of the interaction of a jet, with varying deflection, and a pipe resonator. [Fletcher]



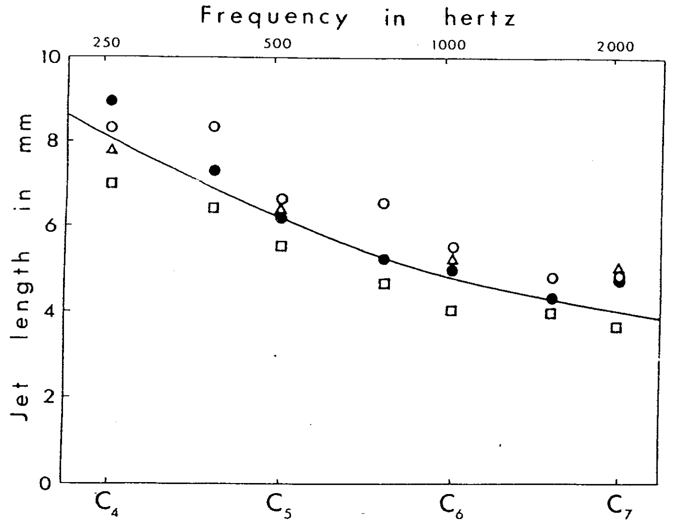
5. Assumed network for interpreting the admittance behavior of an air jet interacting with a flute tube, and the resulting generalized admittance behavior. [Thwaites and Fletcher]



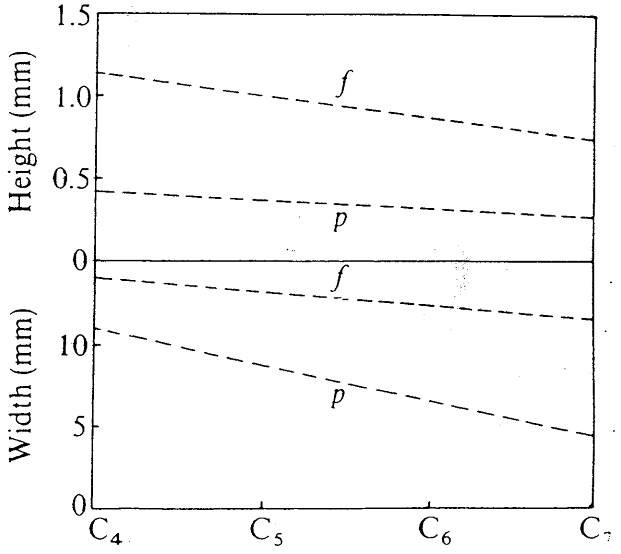
6. Ranges of relative blowing pressure over which the first three modes of an open pipe can be excited by an air jet. The actual blowing pressures depend on the jet length and pipe length. Square symbols indicate values of blowing pressure necessary to achieve reliable single-mode excitation.



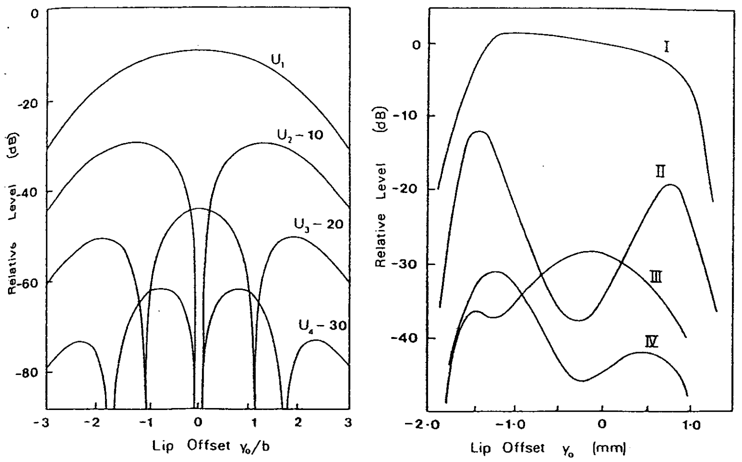
7. Blowing pressure as a function of the pitch of the note being played, as measured for three professional flute players. [Fletcher]



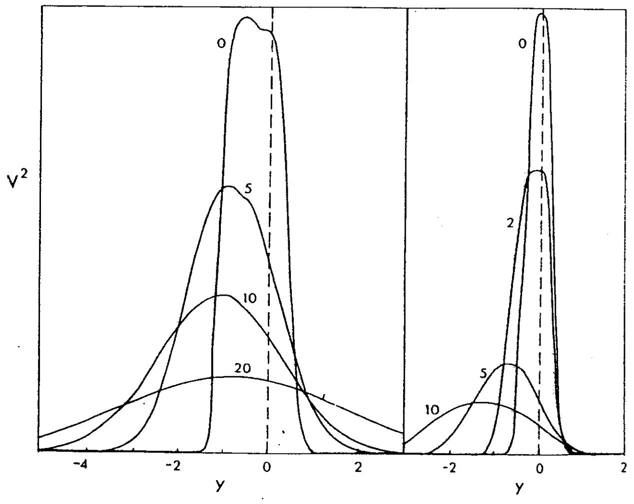
8. Jet length as a function of the pitch of the note being played, as measured for three professional flute players. [Fletcher]



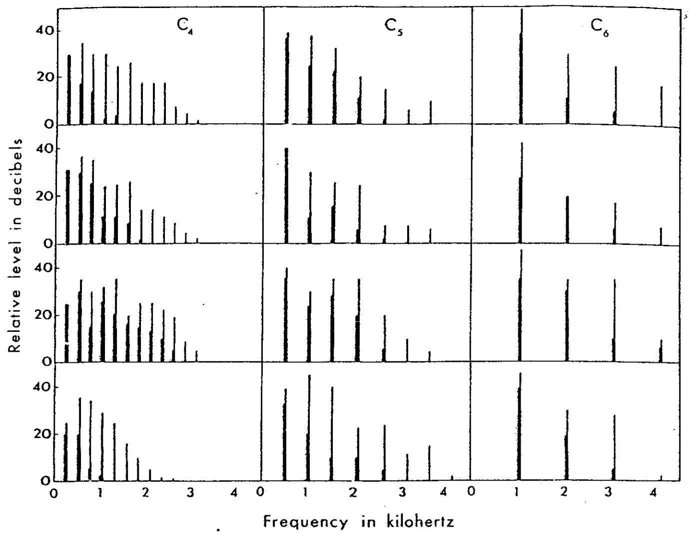
9. Width and height of the lip opening, as functions of the pitch of the note being played and the dynamic level, as measured for three professional flute players. [Fletcher]



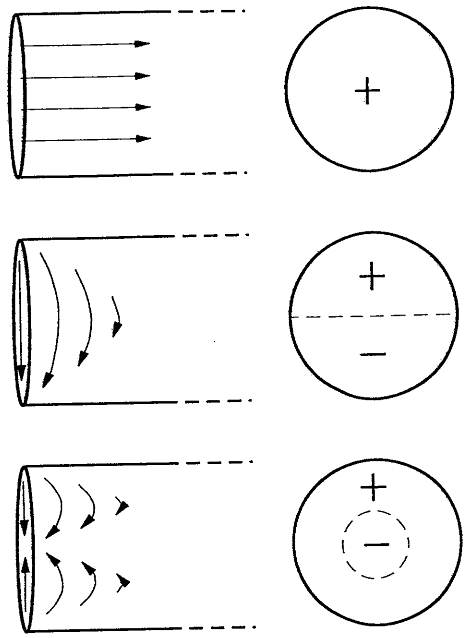
10. Calculated and measured dependence of harmonic levels on jet offset for an organ pipe. The curves in the calculated figure are offset for clarity. [Fletcher and Douglas]



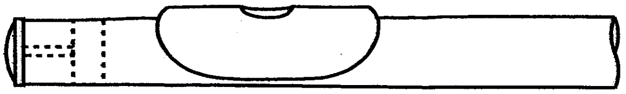
11. Variation of the jet velocity profile (displayed as the square of the velocity) as a function of distance from the flue (given in millimetres) for two jets. In the left-hand figure the flue width is 1 mm, while in the right-hand figure it is 0.25 mm. [Fletcher]



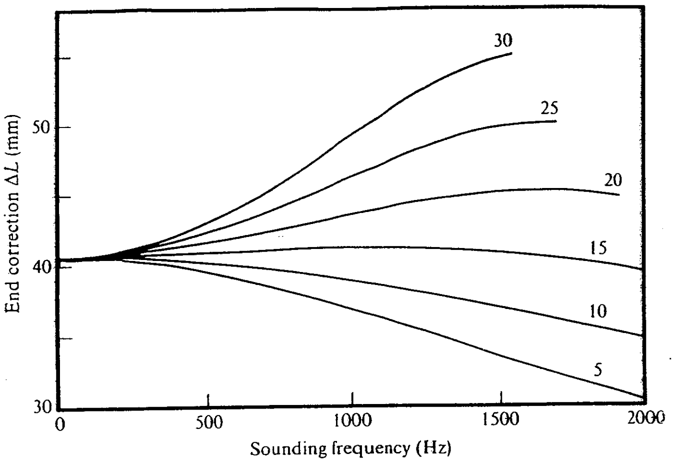
12. Acoustic spectra of three notes as played by four experienced flute players at piano (heavy lines) and forte (fine lines) dynamic levels. [Fletcher]



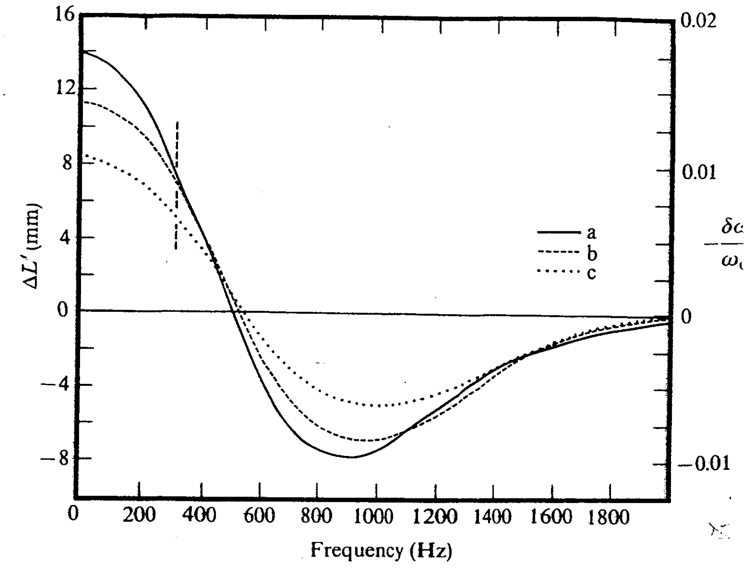
13. The plane wave mode (upper figure) and two evanescent higher modes in a circular pipe.



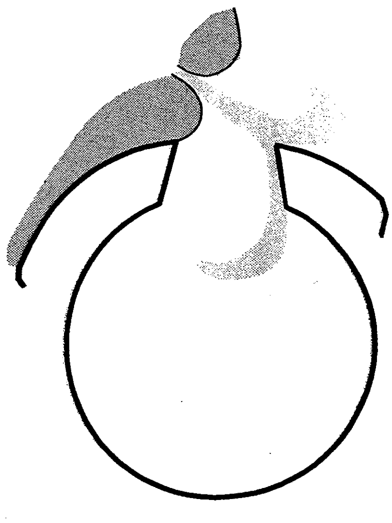
14. The head joint of a metal flute, showing the taper and the cavity between the embouchure hole and the adjustable stopper.



15. Variation with frequency of the end-correction ΔL at a flute embouchure, as affected by the position of the stopper, shown in millimeters distance from the center of the embouchure hole. [Fletcher and Rossing]



16. Variation with frequency of the total end-correction $\Delta L'$ of a flute head joint, as affected by head taper. [Benade and French]



17. Cross-sectional view of the embouchure hole of a metal flute, showing the undercut chimney.