Stopped-pipe wind instruments: Acoustics of the panpipes

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Stopped-pipe jet-excited musical instruments are known in many cultures, those best-known today being the panpipes or syrinx of Eastern Europe and of the Peruvian Andes. Although the playing style differs, in each case the instrument consists of a set of graduated bamboo pipes excited by blowing across the open tops. Details of the excitation aerodynamics warrant examination, particularly as the higher notes contain amplitudes of the even harmonics approaching those of the odd harmonics expected from a stopped pipe. Analysis shows that the jet offset is controlled by the fluid dynamics of the jet, and is such that appreciable even-harmonic excitation is generated. The theory is largely confirmed by measurements on a player.
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I. INTRODUCTION

Musical instruments consisting of sets of simple stopped pipes over which the player blows an air jet from the lips to produce the sound have an immensely long history. According to an excellent summary by Marcuse,[1] an example made from seven or eight bird bones has been found in a Neolithic cemetery in the Ukraine and dated to between 2300 and 2000 B.C. Similar instruments made from a wide variety of materials were used in classical Greek culture, from which the names "panpipes" and "syrinx" are derived,[1] and also in Africa, Asia, and the Americas. The instruments most familiar today are those with pipes made from bamboo and played by musicians from Eastern Europe and from the Peruvian Andes in South America. A typical example of a good-quality instrument from Eastern Europe is shown in Fig. 1. The playing styles in the two places are very different, the best-known recorded performances from Europe being those of the quasiclassical repertoire by Gheorghe Zamfir, while the Andean musicians adopt a percussive style of playing in popular and folk music. The acoustics of both of these playing styles will be examined in the present paper.

II. PHYSICAL PARAMETERS

As shown in Fig. 1, the panpipes studied are very carefully constructed from bamboo, the base of each pipe being terminated at a septum of the stem. The pipes are glued into a slightly curved inlaid wooden holder. Each pipe is lightly beveled at the top inner edge to present a smooth surface to the player's lips, and the larger pipes have been slightly beveled on the inside.

The instrument is able to sound a diatonic scale based on F♯4 (349 Hz). Intonation is subjectively good (it depends upon the player to some extent) except for the shortest pipe, which sounds C7 instead of the expected B♮6. It is not known whether or not this is intentional.

The pipe diameters decrease steadily with increasing pitch, but the slight irregularities of cross section make accurate measurements a little difficult. Figure 2 shows a plot of measured pipe diameter d as a function of pipe length L, both in millimeters. A power-law regression has been fitted with the result

$$d = 1.32L^{0.43}. \quad (1)$$

The purpose of this scaling is twofold. In the first place it shortens the length of the air jet from the player's lips to the outer edge of the pipe, thus facilitating the playing of high notes, as will be discussed in the following. Second, but relatedly, it tends to equalize the loudness over the compass of the instrument.

It is interesting to compare this scaling law with those commonly adopted for organ flue pipes.[2] Such empirical laws are often expressed as the musical interval over which the pipe diameter doubles, or by giving the diameter ratio for pipes differing by an octave, the latter being typically in the range 1.6–1.7. Expressed as in Eq. (1), this gives the approximate rule $d \propto L^{0.75}$. The pipes in the panpipes thus decrease in diameter more slowly than do the pipes in a typical organ rank. Because the musical compass of the panpipes is only 2.5 octaves compared with the 5 octaves of an organ rank, there is not a large effect on tonal balance across the rank, and indeed this would matter little in any case since the panpipes are played as a simple melody instrument.

An example of panpipes from the Peruvian Andes showed a scaling very similar to the European instruments, though the pipe diameters were much less regular, perhaps reflecting the much lower price of the particular instrument measured. A Thai vot, in contrast, had its pipes arranged in a circle around a solid core, and all pipes were of approximately the same diameter. In the vot, also, the pipe entries were all cut obliquely and it was much more difficult for an untutored player to produce the sound.

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FIG. 1. A set of panpipes from Eastern Europe. Note the diameter scaling.

III. ACOUSTIC THEORY

There is a large literature on the acoustics of jet-excited resonators,[3] but most of it refers to unstopped pipes such as organ diapason ranks, to the familiar transverse flute, or to the recorder. While much of this investigation is relevant to the case of stopped pipes, there are some very significant differences. The only paper on panpipe acoustics of which I am aware is a recent contribution on a waveguide model for panpipe sound by Czyzewski and Kostek.[4] In what follows, the simplest possible theory will be put forward, based upon the jet-drive mechanism proposed by Coltman,[5] Elder,[6] and Fletcher.[7] It is recognized that vortex generation may also play a significant role in the process, as initially proposed by Howe[8,9] and elaborated by Hirschberg and his collaborators,[10,11] but this theory is much more complex and is effectively concealed in the simple jet theory through the concept of a mixing region just inside the pipe mouth, in which momentum flux is conserved and energy is dissipated.

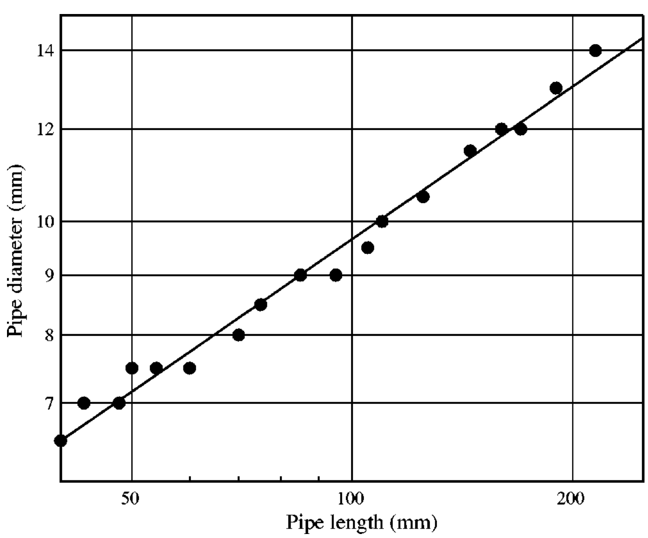

FIG. 2. Pipe scaling of the Eastern European panpipes. The regression line is $d = 1.32L^{0.43}$ with both d and L in millimeters.

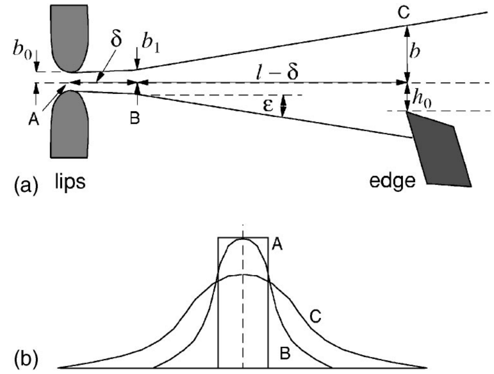

FIG. 3. (a) Development of the jet during propagation across the mouth of the pipe, showing the symbols used in the discussion. In the region between A and B the jet profile changes from ''top-hat'' to Bickley form, and this then broadens linearly by viscous entrainment up to point C. (b) Development of the jet velocity profile, assuming an initial ''top-hat'' shape. Labels A, B, and C correspond in the two parts of the figure.

There is, of course, also considerable energy dissipation by entrainment and vortex formation just outside the mouth of the pipe. While these vortices contribute to sound production in the Howe–Hirschberg model because of their periodic nature, this contribution is small, as in an edge-tone, and is neglected in the mixing-region model.

A very important difference between jet excitation of a closed and an open pipe relates to the steady component of the flow into the pipe. In the case of an open pipe this is of no consequence, because the downstream impedance of the pipe at zero frequency is essentially zero and all the flow can escape. In the case of a stopped pipe, however, the zero-frequency impedance is infinite, so that any steady flow is completely blocked. This has the consequence that the direction of jet flow into a stopped pipe is influenced by the steady balancing component of outflow, and indeed almost completely controlled by this. As a result, the direction of the mean centerline of the jet is essentially fixed by aerodynamic considerations rather than by the player's lip configuration, so that sound production is a relatively stable and simple matter compared, for example, with the difficulty of sounding a comparable end-blown open tube such as the Japanese shakuhachi. The first part of the theoretical development will be devoted to exploring this matter.

IV. AERODYNAMICS

A reasonable approximation to the jet behavior can be deduced by considering the problem in two dimensions. The flow behavior and the symbols used are defined in Fig. 3. When the jet emerges from the player's lips its velocity profile is something between a ''top-hat'' and a parabolic shape, as determined by the effective length of the lip channel. In either case, the central air velocity V_0 is determined simply by the blowing pressure in the mouth and, as in the flute, is typically about 20–40 m/s depending upon the note being played.[12] The flow profile changes shape within a short distance δ after leaving the lips because of viscous drag from the surrounding air, and rapidly assumes the Bickley profile[13]

$$V(z) = V_0 \operatorname{sech}^2(z/b_1), \tag{2}$$

where z is the coordinate transverse to the jet and b_1 is the jet half-width parameter. It is a reasonable approximation to take the jet center-plane velocity V_0 to remain constant during this quite rapid transition, since viscous effects must diffuse in from the edges of the jet, and this assumption will simplify subsequent calculations.

If $2b_0$ is the width of the player's lip aperture and b_1 the jet half-width parameter after profile adjustment, as shown in Fig. 3, then conservation of momentum flux demands that the quantity

$$J=\int_{-\infty}^{\infty} V(z)^2 dz \tag{3}$$

should remain constant during this profile transition, and this requires that $b_1/b_0=3/2$ if the initial profile is top-hat, and $b_1/b_0=4/5$ if it is parabolic. Integrating across the flow profile in each case to determine the volume flow U in the jet shows that $U=1.5U_0$ if the initial flow profile is top-hat, and $U=1.2U_0$ if it is parabolic, where U_0 is the initial volume flow from the player's lips. Since half of the extra entrained flow comes from above and half from below the jet, it follows that the flow entrained from within the pipe by this jet profile adjustment is βU_0, where $0.1<\beta<0.25$.

As the jet travels across the top of the pipe, it broadens further because of viscous drag, as shown in Fig. 3, and entrains more air. The analysis of Bickley[13] gives an expression for this broadening, but measurements on organ-pipe jets at the sort of pressures used in blowing the panpipes show that while the Bickley profile (2) is maintained to a very good approximation for a distance of around 15 mm in a typical case. The jet actually slows and broadens more than predicted, the broadening being approximately linear with distance and representing a spreading half-angle ϵ of about 6°.[14]

There is further entrainment of air as the jet widens in its progress over the remaining distance $l-\delta$ across the top of the pipe, the total path length being taken as l. If the value of the half-width parameter b has changed by a factor α, then in order to preserve the constancy of the momentum flux integral J, given by Eq. (3), the central velocity of the jet must change by a factor $\alpha^{-1/2}$. Taking these two variables into account, the total volume flux in the jet will change by a factor $\alpha^{1/2}$. In the present case, the jet length is typically about 5–10 mm and the initial jet half-width about 0.5–1 mm, while the spreading makes the final jet half-width about 1.5–2 mm, making $\alpha\approx 2$. The jet volume flow after it has traversed the top of the tube is thus about 1.4 times its initial flow, and half of this increase, or 0.2 times the original flow, has been drawn from within the pipe. To this must be added fraction 0.1–0.25 of the initial flow discussed previously, making a total entrained flow from within the pipe of about $0.4U_0$. To be more specific about this analysis, let the jet half-width after profile relaxation be b_1 and the half-width at the lip of the pipe b, then $\alpha=b/b_1$ and the net entrained flow from within the pipe is

$$U'=\frac{U-U_0}{2}=\left(\frac{\alpha^{1/2}-1}{2}+\alpha^{1/2}\beta\right)U_0=\gamma U_0, \tag{4}$$

which is about $0.4U_0$. It is not helpful to attempt to refine these figures, since they are based on the approximation of two-dimensional flow, and in any case vary considerably from player to player and from note to note.

To balance this entrained flow with the total flow injected from the jet, the jet centerplane must be deflected out of the pipe by an amount h_0 so that

$$V\int_{-\infty}^{-h_0} \operatorname{sech}^2(z/b)\,dz=Vb[1-\tanh(h_0/b)]=U', \tag{5}$$

where $V=\alpha^{-1/2}V_0$ is the center-plane velocity of the jet at this position and V_0 is the jet velocity as it leaves the lips. From Eq. (4), this requirement reduces to

$$\tanh(h_0/b)=1-2\gamma\alpha^{-1/2}. \tag{6}$$

Inserting the typical values $\alpha\approx 2$ and $\gamma\approx 0.4$ as discussed before, the necessary offset is $h_0\approx 0.5b$. While this value clearly depends in detail upon the player and the note being played, it shows that the jet is deflected so that its midplane passes outside the lip of the pipe by a significant fraction of the jet half-width, and typically by about 0.5 mm for a note near the low end of the range.

V. ACOUSTIC EXCITATION

It is now important to assess the effect of this jet offset on the acoustic excitation of the pipe. This question has been investigated in detail by Fletcher and Douglas[15] in relation to ordinary organ pipe jets, and the same considerations apply for stopped pipes. The actual acoustic output clearly depends upon both the excitation provided by the jet and the resonant reinforcement of the pipe air column which, for stopped pipes, emphasizes the odd harmonics. The essentials of the computation are as follows.

Because of the behavior of sinuous waves propagating on the jet, and the related adjustment of blowing pressure and jet length by the player, as already referred to, the jet deflection responds almost entirely to the amplitude of acoustic flow in the fundamental of the sound. Because, however, the amplitude of the jet deflection is not small compared with the jet width, the jet flow into the pipe contains harmonics of the fundamental, the amplitudes of which are related to the jet center-plane offset and, of course, to the amplitude of the fundamental oscillation. Theory shows, and experiments confirm, that only odd harmonics are generated when the jet offset is zero, but that the amplitudes of the even harmonics in both the flow and the radiated sound become quite large when the jet offset is greater than about $0.5b$ and the deflection amplitude comparable with b. The calculated behavior of harmonics in the injected flow for the case when the deflection amplitude is $2b$ is shown in Fig. 4, and this calculation has been confirmed by experiment[15] for the case of an open pipe.

The radiated acoustic power, however, depends also upon the total acoustic flow excited at the entrance to the pipe by the transverse oscillations of the jet. For a simple jet of velocity V and cross-section S_j, the acoustic volume flow in the pipe is

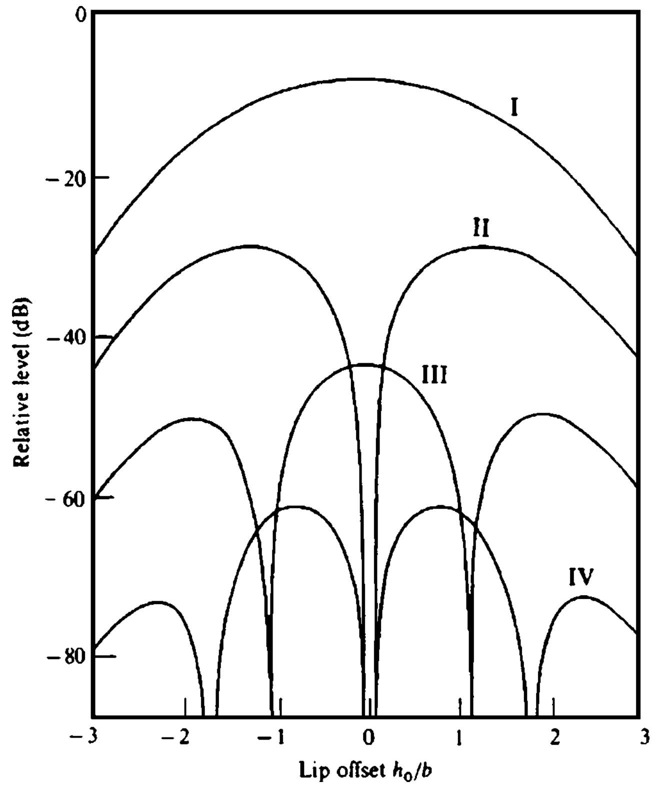

FIG. 4. Jet input flow at harmonics of the fundamental for a Bickley jet profile offset by a fraction h_0/b of the jet width, as calculated by Fletcher and Douglas (Ref. 15). For clarity, each successive curve has been shifted downwards by 10 dB relative to that for the previous harmonic. The calculations were confirmed by experiment on an open pipe.

$$U_p = \frac{(\rho V + j\rho\omega\Delta L)VS_j}{S_p Z_s}, \tag{7}$$

where S_p is the cross-sectional area of the pipe and Z_s is the input impedance of the pipe in series with its end correction.[3,7] This impedance has the approximate value

$$Z_s \approx jZ_0 \cot[(\omega/c - j\sigma)L], \tag{8}$$

where L is the tube length, adjusted for the end correction, $Z_0 = \rho c/\pi r^2$ is the characteristic impedance of the tube, assumed to have radius r, and $\sigma \approx 1.2\times 10^{-5}\omega^{1/2}r^{-1}$ is the wall-loss attenuation. The minimum value of Z_s, corresponding to excitation at an odd harmonic of the tube fundamental, is approximately $\tanh \sigma L \approx \sigma L$, while the maximum value, corresponding to even harmonic excitation, is $\coth \sigma L \approx 1/\sigma L$. The ratio between these two flow amplitude responses is approximately $1{:}(\sigma L)^2$, which is typically about 10^3 or 60 dB for exactly tuned resonances and antiresonances in ideal pipes of these dimensions. The ratio will be reduced, however, for nonideal pipe walls with higher losses, and for resonances, antiresonances, and playing frequencies that do not align exactly with the ideal model.

Another interesting phenomenon occurs in the South American percussive style of playing. Here the jet is launched in brief bursts and at relatively high pressure, so that it becomes turbulent before reaching the lip of the pipe. The turbulence contains components of all frequencies and is therefore able to excite all the modes of the pipe in a fashion that approximately displays the pipe impedance function, as has been shown before in an experimental study of organ pipe noise background by Verge and Hirschberg.[16] In this playing style the fundamental is essentially the only mode that is excited to coherent oscillation.

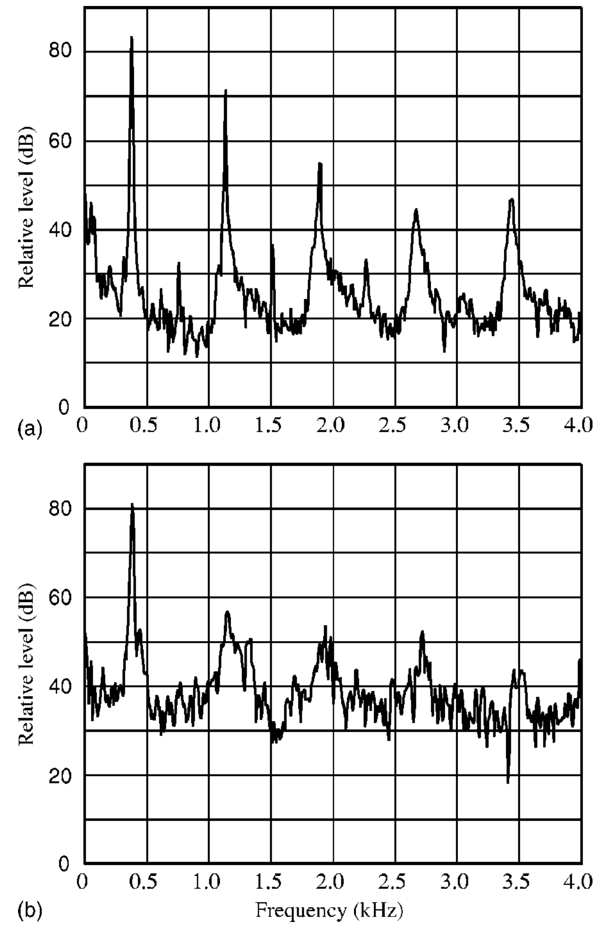

FIG. 5. Radiated sound spectra of the European panpipes for a low note played (a) with normal tone, and (b) percussively.

VI. EXPERIMENTAL STUDY

A brief experimental study demonstrates the generation of even harmonics in normal playing and also the sound spectrum produced in the percussive playing style. While there is clearly a great deal of scope for adjustment of the jet parameters by experienced players, even those with little aptitude can produce acceptable sounds, and the player used for the experiments came into this second category. Figure 5(a) shows the sound spectrum of a low note played in a normal manner. The odd harmonics are very dominant, with the even harmonic levels being 20–50 dB below those of the neighboring odd harmonics, giving the sound a characteristic ''hollow'' quality. Figure 6(a) shows a similar measurement for a mid-range note. The even harmonics are much stronger, though still lying 10–20 dB below the envelope of the odd harmonics. This trend toward equality is continued for higher notes.

There are two reasons for this progression. The first is that the jet is made shorter for high than for low notes, partly because the pipe diameter is smaller and partly as a conscious technique to facilitate phase adjustment in the sound-

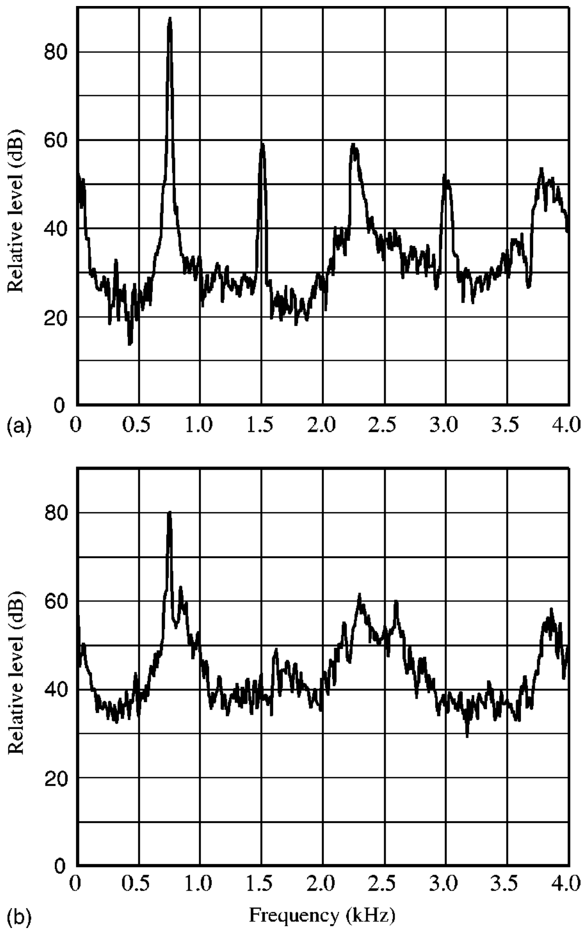

FIG. 6. Radiated sound spectra of the European panpipes for a mid-range note played (a) with normal tone, and (b) percussively.

generation feedback loop. Second, a higher blowing pressure is used for the high notes. Both these changes affect the width of the jet when it impinges upon the edge of the pipe in the same way, high notes having a narrower impact jet than low notes. If b_1 is the half-width of the jet when it relaxes to its steady profile in a distance δ after leaving the player's lips, then the half-width after traveling a total distance l to the opposite edge of the pipe is

$$b = b_1 + \epsilon(l - \delta), \tag{9}$$

where $\epsilon \approx 0.1$, corresponding to the experimental divergence angle of about 6° mentioned previously. Thus b/b_1 is smaller for the short jet lengths than for long. Because of its higher speed, the distance taken for the jet to achieve its steady profile and begin to diverge is greater for fast jets than for slow ones, leaving a smaller value for the divergence length $l - \delta$. From the analysis set out earlier, this means that the fractional offset h_0/b will be greater for high notes than it is for low notes, and consequently there will be greater excitation of even harmonics. On the other hand, it must be conceded that, since the player will generally reduce the height $2b_0$ of the lip opening for high notes,[3,12] this will operate in the opposite direction. Different players may therefore produce a different harmonic balance for high notes.

The spectra produced for the same two notes when played in the percussive style are shown in Figs. 5(b) and 6(b). Only the fundamental is generated in a tonal manner, and there is a noise spectrum with emphasis at odd harmonics of this fundamental, as is to be expected from the resonances (impedance minima, in this case) of the pipe. The broadening of these resonances can be ascribed to the dissipative effect of the jet turbulence. Examination of the "normal playing" spectra of Figs. 5(a) and 6(a) shows that this same noise background is present here, though at a much lower level. One might reasonably surmise that skilled players can reduce this noise still further to produce an excellent pure tonal sound.

VII. CONCLUSIONS

It is hoped that this short paper will lead to a rise in interest in the study of a beautiful and subtle wind instrument. It is not claimed that all problems have been solved by the above-presented analysis, but at least it is a start.

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