Bell clapper impact dynamics and the voicing of a carillon

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The periodic re-voicing of the bell clappers of the Australian National Carillon in Canberra provided an opportunity for the study of the acoustic effects of this operation. After prolonged playing, the impact of the pear-shaped clapper on a bell produces a significant flat area on both the clapper and the inside surface of the bell. This deformation significantly decreases the duration of the impact event and has the effect of increasing the relative amplitude of higher modes in the bell sound, making it "brighter" or even "clangy." This effect is studied by comparing the spectral envelope of the sounds of several bells before and after voicing. Theoretical analysis shows that the clapper actually strikes the bell and remains in contact with the bell surface until it is ejected by a displacement pulse that has traveled around the complete circumference of the bell. The contact time, typically about 1 ms, is therefore much longer than the effective impact time, which is only a few tenths of a millisecond. Both the impact time and the contact time are reduced by the presence of a flat on the clapper. [DOI: 10.1121/1.1448517]

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I. INTRODUCTION

The National Carillon in Canberra was presented to the people of Australia by Britain to mark the Golden Jubilee of the establishment of Canberra as Australia's national capital in 1913. The carillon was opened by the Queen in 1970, a year that also marked the two hundredth anniversary of James Cook's famous voyage in which the east coast of Australia was mapped for the first time. The carillon itself was designed and built by John Taylor and Company of Loughborough in England, and is housed in an elegant white tower on tiny Aspen Island in Canberra's central Lake Burley Griffin. It has a total of 53 bells and its compass is four and a half octaves from G_2 to C_7. The pitch of the carillon is set a semitone lower than standard pitch at about $A_4 = 415$ Hz.

The carillon is played from a keyboard and pedalboard at a lower level in the tower (where there is also an identical practice clavier using undercut metal slats and resonator tubes). An auxiliary set of hammers or clappers on a small number of bells is used, under automatic control, to sound out the hours and quarter hours with Westminster chimes.

Experience has shown that the sound of the bells changes progressively as they are used. Initially the sound improves "as the clappers get to know the bells," as one authority picturesquely puts it. After a few years of regular use, however, the sound becomes "brighter" with greater development of higher partials. While this high development of upper partials is perhaps desirable in isolated bells, their inharmonicity can make the sound discordant in a carillon where bells are played in harmony.

Examination of the bells shows that the change of tone is associated with the development of elliptical flat areas on both bell and clapper in the impact region. As usage demands, therefore, a re-voicing operation is undertaken in which the clapper surface is ground or filed to restore its curved shape at the impact site. This re-voicing makes the sound "mellow" once again. No attempt is made to reshape the impact site on the bell itself, since this might affect the tuning of its overtones. One of us (W. T. McG.) is responsible for the maintenance (including re-voicing) of the Australian National Carillon, and this afforded the opportunity to study several mechanical and musical aspects of the operation. Conveniently for our study, re-voicing had not been carried out for several years, as more urgent mechanical matters required prior attention. Consequently, the clappers exhibited a level of damage larger than normally acceptable, making the results of re-voicing much more clear. No work at all was required on the bells themselves, so this study is complementary to the recent work of Boutillon and David,[1] whose study concentrated upon the reconditioning of the bells in a carillon.

Rather surprisingly, in this particular carillon, the position on the inside of the bell against which the clapper strikes is not initially smooth. Indeed the bell-tuning operation has apparently been carried out on a lathe with a quite broad cutting tool, with the result that the inside surface bears regular circumferential grooves each 1–2 mm wide and almost 1 mm deep, separated from their neighbors by flat areas also about 1–2 mm wide. After the 30 years of playing that this

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instrument has received, the impact site on the bell wall has been almost flattened over an elliptical area somewhat larger than the flat developed on the clapper, this exaggeration being due to small variations in the position of the impact site because of mechanical slack in the mechanism.

The mechanism itself consists of a pear-shaped clapper of cast iron, mounted on a hinged rod. The original mechanism was modified some years ago, and the weight of each clapper is now balanced by a spiral spring. The clapper is drawn into impact with the bell by a flexible steel cable typically about 30 cm long that connects it to the rods, levers, and cables leading up from the clavier.

It is not our purpose here to survey what is known of the acoustics of bells or of carillon design. The interested reader will find appropriate material in a book by Fletcher and Rossing[2] and in a collection of papers edited by Rossing,[3] included among which is an extensive monograph by Bigelow.[4]

II. MEASUREMENTS

To examine the effects of re-voicing the clappers, three bells were selected for study: Nos. 9 (E_3), 29 (C_5), and 47 ($F^\sharp_6$), effectively spanning the compass of the instrument. The results for the midrange bell (C_5) will be reported in some detail and then comparable results for the other bells quoted.

Bell 29 is about 43 cm in diameter and 36 cm in height to its shoulder and is cast from bronze. Its mass is about 59 kg. The spherical part of its pear-shaped cast-iron clapper has a diameter of about 95 mm. The estimated mass of the clapper is about 5 kg. The impact damage flat on the clapper was approximately elliptical, with dimensions 17 mm×14 mm, while that on the inside of the bell was slightly larger at 20 mm×14 mm. The grooves on the bell surface had been plastically deformed so that the surface was essentially flat, though traces of the groove pattern did remain.

The re-voicing itself was carried out by hand and eye, using a powered angle grinder and then a file for finishing. The resultant clapper surface was of approximately uniform spherical curvature but had some file marks across it of approximate depth 0.1 mm.

Since the plastic washers used in the bell mount isolated it electrically from the clapper, it was possible to examine the contact time τ simply by using the impact as a switch in a simple electrical circuit consisting of a 9 V cell and a 1000 Ω resistor in series and displaying the voltage across the resistor on a storage oscilloscope. When a simple impact was achieved—a point to be considered again later—its duration was about 0.6 ms. The same experimental arrangement was used to measure contact time after re-voicing, the typical duration then being about 1.0 ms, though becoming as long as 1.3 ms for particularly gentle strikes.

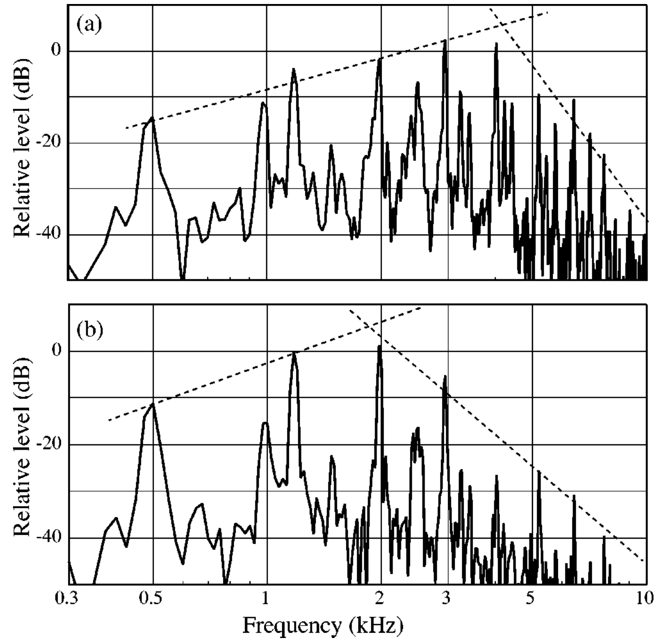

FIG. 1. (a) Sound spectrum of bell 29 before re-voicing; (b) sound spectrum of the same bell after re-voicing.

An interesting feature of this experiment, which will later be seen to be significant, is that, while impacts widely separated in time so that the bell has ceased vibrating were quite clean and reproducible, impacts in rather quick succession with only a few seconds separation displayed jitter in the form of repeated very brief contacts. This jitter can be attributed to the effects of residual vibration of the bell.

The radiated sound signal was recorded using a microphone and tape recorder located in the bell chamber about 4 m from the bell. The microphone position was the same for recordings before and after re-voicing so that comparison was possible irrespective of resonances or other effects within the bell chamber. Recorded signals from a number of bell strokes before and after re-voicing were later subject to frequency analysis using a fast Fourier transform technique, the sample being selected to commence 40 ms after the initiating impulse.

A typical pair of spectra for bell 29 is shown in Fig. 1. The spectral envelope can be adequately described in terms of the two straight lines shown, and the frequency f^* at the intersection of these lines gives a measure of the ‘‘brightness’’ of the sound. This frequency is about 4 kHz before re-voicing and about 2 kHz afterwards. The distribution of

TABLE I. Experimental results.

Bell	Bell diam (cm)	Bell mass (kg)	Clapper diam (mm)	Clapper flat diam (mm)	τ before (ms)	τ after (ms)	f^* before	f^* after
9(E_3)	120	950	200	18	1.5	3.5	900 Hz	600 Hz
29(C_5)	43	59	95	15	0.6	1.2	4 kHz	1.8 kHz
47($F^\sharp_6$)	20	7.7	56	7	0.6	0.8	>4 kHz[a]	>4 kHz[a]

[a]Uncertain because of wind noise and other problems.

modes within the spectral envelope is determined by the design and tuning of the bells[2,4] and need not be considered here though it is, of course, the primary concern of the designer and bell founder.

Similar analyses were carried out for the other two bells, and the complete results are summarized in Table I. The clapper flat diameter quoted is the geometric mean of the two diameters of the contact ellipse. It can be seen that the behavior scales quite consistently for all three bells.

III. THEORY

Two aspects of the impact behavior require theoretical investigation. The first is the impact itself, including contact time, behavior of the contact force, and the way in which these quantities depend upon the size of the damage flat and upon parameters such as impact speed. The second is the vibrational excitation and sound output to be expected from this type of impact.

The whole subject of the impact of one body upon another is of great practical importance, and has been the subject of much theoretical and experimental investigation, dating back to the classic work of Hertz in the nineteenth century. An excellent exposition with extensive references has been given by Goldsmith.[5]

A rigorous theory of impact in the present case necessarily involves the complex elastic deformation of the bell and clapper during the impact and also the vibration and bodily motion of the bell during that time. Such an analysis would be extremely complicated and is in any case unnecessary for the present problem. The analysis to be presented here therefore includes only enough detail to establish the effects of the major physical parameters and to give a reasonable approximation to the numerical values involved.

A. Hertzian impact theory

Most reasonably simple treatments of impact are based upon Hertz's assumption that the impact takes place sufficiently slowly that a static treatment of the elastic distortions is adequate. His theory covers the case of impact of spheres on spheres and of spheres on plane objects, but could reasonably be extended to the more complex geometry of a sphere with a contact flat. Following the treatment in Chapter 4 of Goldsmith,[5] we find that, for a sphere of mass m and radius R impacting on a very massive plate at a speed V, the Hertz theory gives a contact time

$$\tau_H \approx 4.5 \left[\frac{(\delta_1 + \delta_2)^2 m^2}{RV} \right]^{1/5}, \tag{1}$$

where

$$\delta_1 = \frac{1-\mu_1^2}{\pi E_1}; \quad \delta_2 = \frac{1-\mu_2^2}{\pi E_2} \tag{2}$$

and μ_1, μ_2 are the Poisson's ratios and E_1, E_2 the Young's moduli of the sphere and plate, respectively. During the impact, the force follows a curve like

$$F \approx F_{\max} \sin(\pi t / \tau_H), \tag{3}$$

where

$$F_{\max} \approx 0.44 \frac{m^{3/5} R^{1/2} V^{6/5}}{(\delta_1 + \delta_2)}. \tag{4}$$

Equations (1)–(4), which apply only for an undamaged clapper, serve as a check for the extended but less accurate theory to be developed in Sec. III B.

These quasistatic equations can apply only in the limit of very slow impacts which allow any elastic vibrations generated to dissipate completely in a negligible fraction of the impact time. For more rapid impacts it is necessary to consider the effects of wave generation and reflection from the boundaries of the impacting objects. This is difficult enough in the case of a small sphere impacting on a clamped rod[5] and becomes impossibly complicated for more complex geometries. For very rapid impacts, of the type exemplified by impact of a small sphere on a large object such as an extended plate, the situation is somewhat simplified, since the deformation of the sphere can be taken to be quasistatic, and the generation of elastic waves in the plate can be approximated by a simple wave impedance, provided there is not time during the impact for any reflections from the plate boundaries to return to the impact site. The latter assumption is a reasonable approximation for the case of clapper impact on a bell, and will be the basis of the theory developed in Sec. III B.

B. An extended impact theory

It is necessary now to formulate a simple theory from which predictions can be made of the impact behavior of a sphere with an initial flat upon the surface of a bell. The geometry assumed during the impact is as shown in Fig. 2. The coordinate of the center of the clapper, assumed spherical for simplicity, relative to a fixed plane is y_1, and that of the surface of the bell, assumed plane, is y_2, the coordinate of this surface before the impact being y_0. During the im-

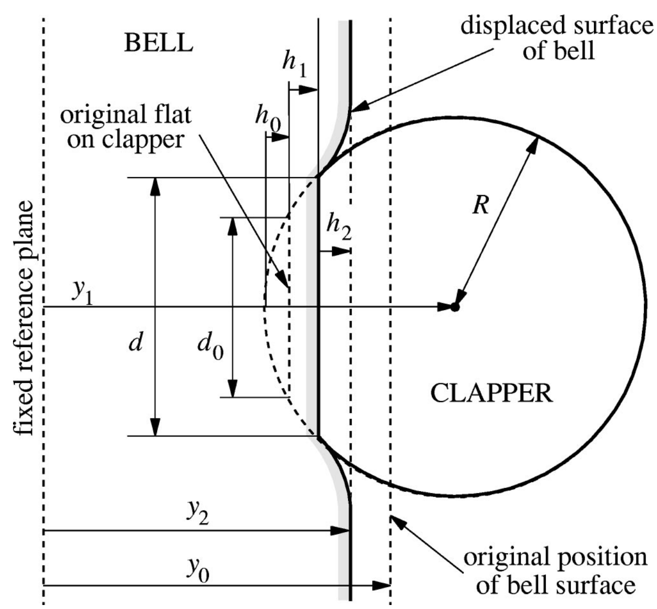

FIG. 2. Assumed geometry of the bell and clapper during impact. The bell is displaced from its initial position y_0 because of elastic wave generation, while the clapper deforms the surface elastically and is itself deformed, the diameter of the contact flat increasing from its initial value d_0 to d.

pact, the clapper penetrates the bell surface by a distance h_2 and the depth of the flat below the extension of the spherical surface increases from h_0 to h_0+h_1. At the same time the diameter of the contact flat increases from its initial magnitude d_0 to a new time-dependent magnitude d. It is assumed that all distortions are elastic rather than plastic, at least during the course of a single impact event.

The first approximation to be made is to assume that the bell is large enough and the impact short enough that there are no significant reflections during the contact time. Since the bell has axial symmetry, this assumption is equivalent to the proviso that there is not enough time during the impact for a wave to travel round the circumference of the bell and return to the impact point before the clapper rebounds, a matter that will be discussed later. The large-scale behavior of the bell during impact can now be approximated by

$$F=-Z\frac{dy_2}{dt}, \tag{5}$$

where F is the force imposed by the clapper impact, y_2 is the coordinate of the bell surface as shown in Fig. 2, and Z is the characteristic impedance for flexural waves on the bell. The total mass of the bell is sufficiently much larger than that of the clapper that it is reasonable to ignore its bodily displacement. Exact evaluation of Z is difficult, because of the complex geometry of the bell, but an approximation can be made by assuming it to behave like an infinite flat plate. An analysis has been given by Skudrzyk,[6] who shows that, for the case of a plate or iron or soft steel, the characteristic impedance is

$$Z_\infty \approx 100 b^2 \text{ kg s}^{-1}, \tag{6}$$

where b is the thickness of the plate in millimeters. Because the elastic parameters of bell bronze and cast iron are very similar, we expect this result to be a good approximation for the bell material. [For other materials, Z_∞ is proportional to $(E\rho)^{1/2}$, where E is the Young's modulus and ρ the material density.] Since, however, the clapper strikes the bell close to a free edge, the effective impedance Z is expected to be about $0.5Z_\infty$. The thickness of the bell wall is not uniform, but is in the range 20–30 mm for bell 29. Adopting this approximation as a guide, therefore, we expect that Z is in the range $2-5\times 10^4$ kg s^{-1} for this bell. In some of the calculations to follow, a value of 3×10^4 kg s^{-1} will be assumed.

Referring now to Fig. 2, if d_0 is the diameter of the impact damage flat, then this can be related to the parameter h_0 by

$$h_0 \approx \frac{d_0^2}{8R}, \tag{7}$$

where R is the radius of the clapper ball, or more precisely its radius of curvature, and the approximation is good provided $h_0 \ll R$. Similarly, the parameter h_1, which is the extent of the compression, is related to the diameter d of the contact flat during the impact and to the initial geometric parameter h_0 by

$$h_1 \approx \frac{d^2}{8R} - h_0, \tag{8}$$

provided $h_1+h_0 \ll R$, as is the case in practice. Calculation of the elastic strain in the clapper is very complicated, but it is adequate for our present purposes to assume that it is distributed over a volume of diameter d and depth d/K_1, where K_1 is rather greater than unity. The total force F on the impact surface then approximately satisfies

$$\frac{dF}{dh_1} = \frac{K_1}{d}\left(\frac{\pi d^2}{4}\right)\frac{E_1}{1-\mu_1^2} = \frac{K_1 d}{4\delta_1}, \tag{9}$$

where E_1 and μ_1 are the Young's modulus and Poisson's ratio, respectively, of the clapper material and δ_1 is defined by Eq. (2). The uncertainty implicit in this result is encapsulated in the factor K_1. Its magnitude will be derived later by comparison with the more accurate Hertz theory for a case where they overlap. Substituting Eq. (8) into Eq. (9) and integrating gives

$$F = K_1 \frac{(2R)^{1/2}}{3\delta_1}[(h_1+h_0)^{3/2} - h_0^{3/2}]. \tag{10}$$

The elastic deformation of the bell surface presents a similarly difficult problem but, to the same degree of approximation, and by the same arguments used to treat the clapper, we can write

$$F = K_2 \frac{(2R)^{1/2}}{3\delta_2}[(h_2+h_0)^{3/2} - h_0^{3/2}], \tag{11}$$

where K_2 is another constant of order unity. It is to be expected that $K_1 \approx K_2$. From Eqs. (10) and (11),

$$(h_2+h_0)^{3/2} = \frac{K_1\delta_2}{K_2\delta_1}(h_1+h_0)^{3/2} + \left(1-\frac{K_1\delta_2}{K_2\delta_1}\right)h_0^{3/2}. \tag{12}$$

There is also a further geometrical connection between the coordinates, namely

$$h_0+h_1+h_2 = y_2 - y_1 + R. \tag{13}$$

Note that, in all these equations, h_0, h_1, and h_2 have been defined so as to be always positive, and F must also be positive since there is no adhesion at the contact.

Finally, the motion of the center of mass of the clapper obeys

$$\frac{d^2y_1}{dt^2} = \frac{F}{m}, \tag{14}$$

where the mass m of the clapper is rather greater than the spherical mass because of its pear shape and attached mechanism. Equations (5)–(14) now define the motion of the whole system.

C. Numerical solution

For a numerical solution, Eqs. (14) and (5) are written in the first-order form

$$\frac{dy_1}{dt} = z, \tag{15}$$

$$\frac{dz}{dt} = \frac{F}{m}, \tag{16}$$

$$\frac{dy_2}{dt} = -\frac{F}{Z} \tag{17}$$

with boundary conditions $y_2 = y_0$, $y_1 = y_0 + R - h_0$, $dy_1/dt = -V$, $dy_2/dt = 0$, and $h_1 = h_2 = 0$, where y_0 is an arbitrary constant and h_0 is defined by the diameter d of the initial flat by Eq. (7). It is necessary to write the force F exclusively in terms of the coordinates y_1 and y_2, and to do this h_2 must be eliminated from Eqs. (12) and (13) and the resulting equation solved numerically for h_1. The three equations (15)–(17) then serve as the basis for the computation.

In the carillon under study, the bells are made of cast bronze for which $E_2 = 1.05 \times 10^{11}$ Pa and $\mu_2 = 0.36$, while the clappers are cast iron with $E_1 = 1.2 \times 10^{11}$ and $\mu_1 = 0.27$.[7] These values are sufficiently similar that it is reasonable to assume equality, so that $\delta_1 = \delta_2$. In addition, since exact values of the constants K_1 and K_2 in Eqs. (10) and (11) are unknown, it makes sense to assume that $K_1 = K_2 = K$. These simplifying assumptions then lead to the result $h_1 = h_2$ so that, from Eq. (13),

$$h_1 + h_0 = h_2 + h_0 = \frac{R + y_2 - y_1}{2} \tag{18}$$

and

$$\delta_1 = \delta_2 = 2.4 \times 10^{-12}\ \mathrm{Pa}^{-1}. \tag{19}$$

These assumptions lead to a great simplification in the subsequent calculations.

The one major formal uncertainty in the extended theory is encapsulated in the value of the elastic parameter K, when all that is known is that it is of order unity. The accompanying uncertainty can, however, be resolved by comparing the computed result for contact time in the case $d_0 = 0$ and $Z = \infty$ with the known accurate results of the Hertzian treatment for this case as given by Eq. (1). This comparison can be made by assuming that $Z = \infty$ in the simple model and adjusting K to obtain agreement, and leads to the result $K \approx 5$. This value is then used in subsequent evaluations.

The theory can now be used to investigate the effect of impact flat diameter upon impact time. Because of uncertainty in the effective value of Z for the bell, and to illustrate the importance of this parameter, a range of values is studied. Once again, the calculations all refer to bell 29.

Figure 3 shows the calculated behavior of the force between the clapper and the bell for an impact velocity of 1 $\mathrm{m\,s^{-1}}$ and three assumed values of the impedance parameter Z. For the case $Z = \infty$ the force curve is symmetrical about its midpoint, in agreement with the Hertzian result (3), while for a finite value of Z, as in reality, the force curve has a longer tail. This asymmetry increases as the impedance Z assumes lower values. It is encouraging to note that these same features are seen in the shapes of curves calculated for the impact of a sphere on a plate of infinite size, as shown in Fig. 71 of Goldsmith.[5] Because of the asymmetry of the curves it is helpful to define the impact time τ^* to be the time from initial contact to the maximum in the force curve. For a Hertzian impact ($Z = \infty$) clearly $\tau^* = \tau_H/2$, while τ^* is less than half the contact time for finite values of Z.

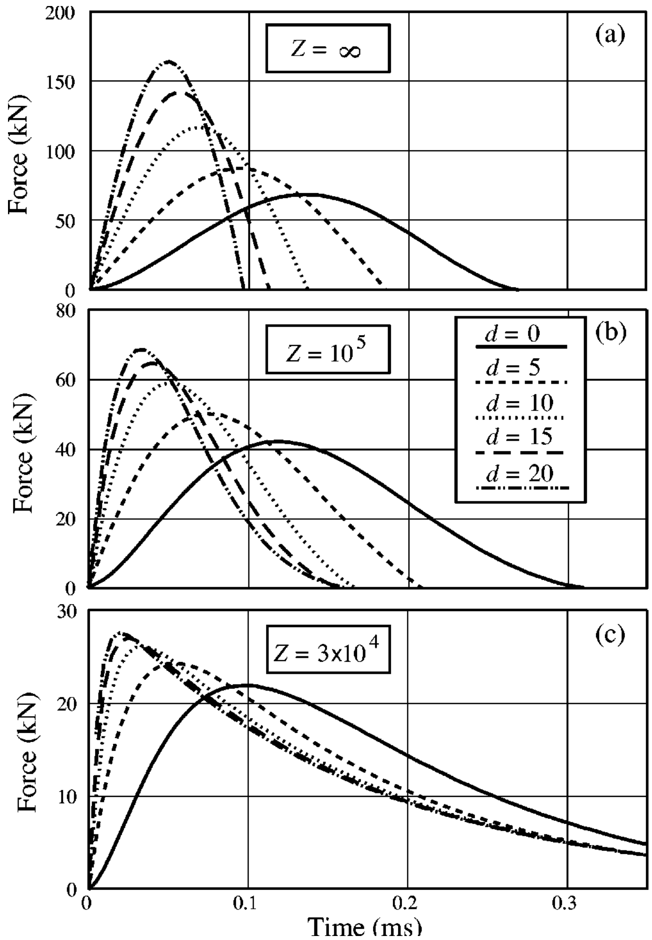

FIG. 3. Calculated time evolution of force F between the clapper and the bell for an impact velocity of 1 $\mathrm{m\,s^{-1}}$ and three different impedance values Z. The diameter of the impact damage flat is shown in millimeters as a parameter. In the case $Z = 3 \times 10^4\ \mathrm{kg\,s^{-1}}$, the clapper remains in contact with the bell surface.

Figure 4 shows the calculated impact time τ^*, as derived from the curves of Fig. 3, as a function of impact flat

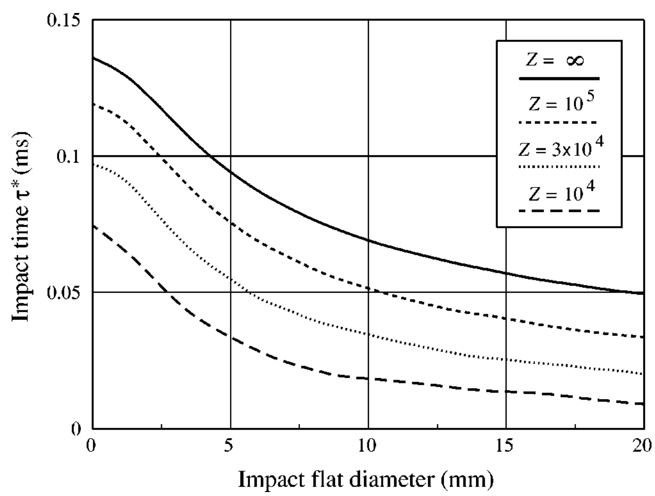

FIG. 4. Calculated impact time τ^* for a spherical clapper of mass 5 kg on a bell surface with effective impedance Z, as a function of the diameter of the damage flat on the clapper surface.

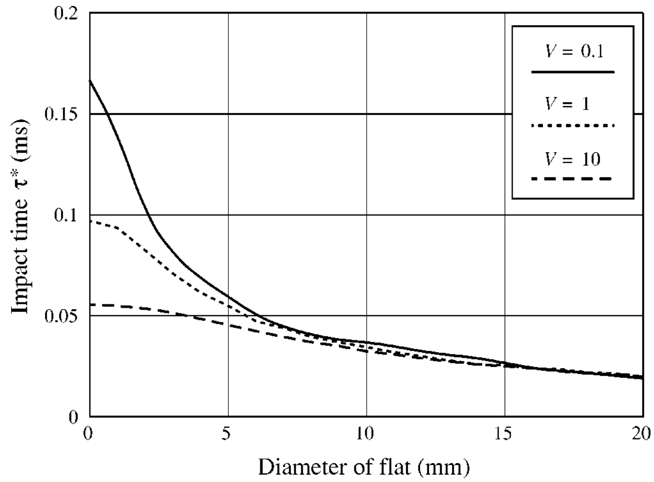

FIG. 5. Calculated variation of the impact time τ^* as a function of impact damage flat diameter for a range of impact speeds, shown in meters per second as a parameter. It is assumed that $Z=3\times10^4$ kg s^{-1}, at which impedance value the clapper comes to rest against the bell after impact.

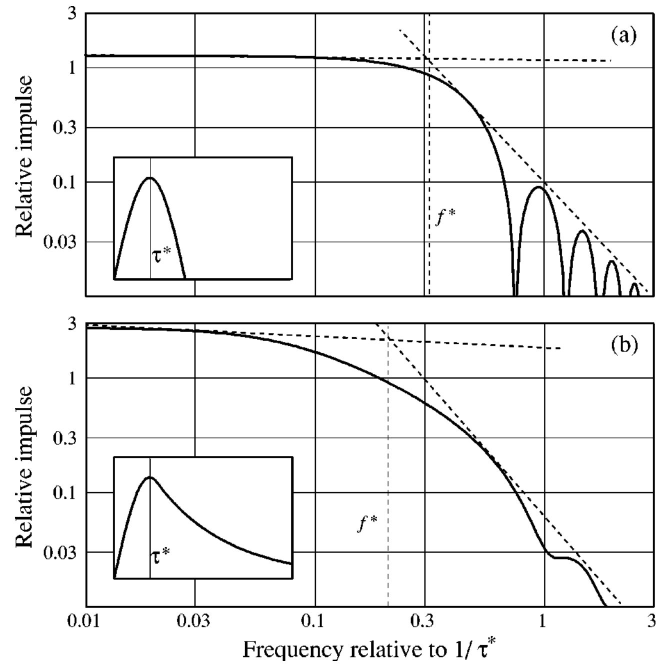

FIG. 6. Fourier transform of the impact force $F(t)$ for (a) a very high bell impedance Z, giving a Hertzian impact as in Fig. 3(a); (b) a low bell impedance giving an asymmetric impact and infinite contact time as in Fig. 3(c). The time variation of the force is indicated in the inset in each case.

diameter for three values of the impedance Z, one of which is the infinite impedance implicitly assumed in the Hertz model. The behavior for $Z=10^5$ kg s^{-1} is not very different from that for $Z=\infty$, though impact times are a little shorter and total contact times a little longer. For Z values smaller than about 5×10^4 kg s^{-1}, however, and specifically for the value 3×10^4 kg s^{-1} considered appropriate for this bell, the clapper actually comes to rest against the bell surface after the impact. The importance of this prediction will be examined in Sec. III D.

Finally, Fig. 5 shows the effect of clapper impact velocity on impact time τ^* for a range of values of impact flat diameter. As already discussed, it is assumed that $Z=3\times10^4$ kg s^{-1} for this bell, so that the clapper remains in contact with the bell after impact and the contact time τ is infinite. For the undamaged clapper, there is a significant shortening of impact time as the impact velocity is increased, but once a flat of moderate diameter develops τ^* becomes independent of impact velocity. This feature is another reason for the clapper voicing program.

D. Other bells

As mentioned earlier, similar measurements and revoicing procedures were carried out on two other bells, one much larger and one much smaller than bell 29. The measurements on all three bells are shown in Table I. Here τ is the total contact time from electrical measurements and f^* is the spectral turnover frequency as shown in Fig. 6. The results for the larger bell, No. 9, are broadly consistent with those analyzed in detail, bell 29. Those for bell 47 are less decisive, since the flat was smaller and the spectral recordings were obscured by wind noise and other problems. It would be of interest to undertake a complete study of spectral balance across the whole carillon, but this was not attempted.

IV. CONTACT TIME DISCREPANCY

There is one aspect in which the theory appears to be in substantial disagreement with experiment, and that is the actual magnitude of the contact time during impact. The experimental contact time for bell 29 ranges from 0.6 ms for the damaged clapper to 1.0–1.2 ms when it has been revoiced. If the bell impedance Z is 10^5 kg s^{-1} or higher, so that the clapper rebounds after contact, then the calculated contact times are about 0.15 and 0.3 ms, respectively—about one-quarter of the measured times. If the bell impedance is substantially lower, as analysis indicates, then the calculated contact times become infinite. While it might be possible to find a precise value of Z giving approximate agreement with experiment for contact times, this balance would be unrealistically sensitive.

One might be tempted to question the reliability of the electrical contact measurements. Other investigators, however, have used almost identical techniques to measure contact times and have obtained results that agree in general magnitude with our own. Jones[8] long ago made measurements on a large bell in place in a chime and found a contact time of about 0.8 ms. More recently Grützmacher *et al.*[9] measured contact times in the range 0.6–1.5 ms in a laboratory setup, in approximate agreement with our results, and also found a decrease in contact time with increasing impact velocity.

An explanation of this discrepancy is actually simple, and is suggested by the observed instability of the contact measurements when the bell is already vibrating. Suppose that the impedance Z that the bell presents to the clapper is less than about 5×10^4 kg s^{-1}, as is appropriate from the plate analogy discussed previously. Then after impact the clapper will remain in contact with the bell surface until it is dislodged by some mechanical impulse from the bell. From Fig. 3, the clapper impact has a sharp rise time τ^* followed by a slow decay, and thus launches a transverse pulse of this shape in both directions around the sound-bow of the bell. These pulses return to the clapper position after making a complete circuit of the bell, during which they are distorted by dispersion effects, and dislodge the clapper from contact with the surface as they return to the impact site. The clapper then moves away from the bell surface and is further accelerated by the return spring of the action. If the clapper has an impact flat, then the impact time τ^* is shorter, and the generated pulse wave-packet is higher in frequency. The phase velocity of flexural waves, however, is proportional to the square root of the frequency, and the group velocity is twice the phase velocity, so that this higher-frequency pulse travels around the bell more quickly and dislodges the clapper after a smaller delay.

These statements about phase and group velocities follow from the equation for bending wave propagation on a plate, which is

$$\frac{\partial^2 z}{\partial t^2} = -S\nabla^4 z, \tag{20}$$

where z is the normal displacement and S is the plate bending stiffness divided by its mass per unit area. If the wave has the form $z=A\sin(\omega t-kx)$, then Eq. (20) shows that $\omega=S^{1/2}k^2$ and, since the phase velocity is $c=\omega/k$, this gives $c=S^{1/4}\omega^{1/2}$. The group velocity c', however, is given by $c'=\partial\omega/\partial k$ which is just $2c$.

To confirm this explanation of long contact times, it is necessary to introduce quantitative considerations. From the analysis of Skudrzyk,[6] the phase velocity of flexural waves of frequency f on an infinite plate is

$$c=\left(\frac{\pi Z f}{4M}\right)^{1/2}, \tag{21}$$

where M is the plate mass per unit area and Z is the characteristic impedance given by Eq. (6). For bell 29, the wall thickness is about 20 mm so that $Z=3\times10^4\ \mathrm{kg\,s^{-1}}$. At 2 kHz, which is the frequency of maximum excitation as judged from the sound spectrum for the voiced clapper, Eq. (21) gives a phase velocity of about 540 $\mathrm{m\,s^{-1}}$ and a group velocity of 1080 $\mathrm{m\,s^{-1}}$. This gives a pulse transit time of about 1.2 ms, in fairly good agreement with the experimental contact time of 1.0–1.2 ms for the voiced clapper. For the unvoiced clapper, the excitation maximum is at about 4 kHz, giving a pulse return time of about 0.8 ms, which is comparable with, though a little longer than, the experimental value of 0.6 ms.

It is possible, however, to estimate these propagation velocities quite independently of the validity of the infinite-plate impedance approximation. In the fundamental "hum" mode, the bell vibrates with two nodal diameters,[2] and the mode can be considered to be a superposition of two counterpropagating waves, each with two wavelengths around the circumference of the sound-bow. Since the bell circumference is about 140 cm at the strike position, this gives a phase velocity of about 350 $\mathrm{m\,s^{-1}}$ at this frequency, and hence a group velocity of about 1400 $\mathrm{m\,s^{-1}}$ at 2 kHz, in moderately good agreement with the above-given estimate. The contact times predicted from this estimate are about 0.9 ms for the voiced clapper and 0.6 ms for the unvoiced clapper, again in fairly good agreement with experiment.

V. SOUND GENERATION

Sound generation by the bell is a combination of the excitation event and subsequent sound radiation. Since the bell itself is not modified during the voicing process, its mode frequencies, impedances, and radiation efficiencies remain unaltered, and it is necessary only to consider the excitation event. The above-developed theory predicts an impact force $F(t)$ with a time evolution that depends upon the impedance Z of the bell wall as shown in Fig. 3. The spectral shape of this impact event is given by its Fourier transform, which is written most conveniently, to within a constant factor, as

$$F(\omega)=\left[\left(\int_0^\infty F(t)\cos\omega t\,dt\right)^2+\left(\int_0^\infty F(t)\sin\omega t\,dt\right)^2\right]^{1/2}. \tag{22}$$

The shape of this function, plotted in the same way as the experimental results of Fig. 1, is shown in Fig. 6 for two different cases: one with large Z, giving a nearly Hertzian impact, and one with small Z, giving a lingering contact. In both cases the envelope of the spectrum consists of two straight lines intersecting at a frequency close to $0.2/\tau^*$ where τ^* is the impact time as previously defined and indicated in the insets. When allowance is made for a radiation efficiency that increases smoothly with increasing frequency, there is close qualitative similarity between the spectral envelope of Fig. 6 and that of the bell sound in Fig. 1, for both unvoiced and voiced clappers.

Comparison with experiment. In making comparisons between theoretical predictions and experimental measurements, it must be borne in mind that this was a field experiment rather than a laboratory experiment. This meant that several important parameters had to be estimated rather than measured in a controlled way. With this proviso, the predictions of the theory are in acceptably good agreement with experimental results.

The turnover frequencies f^* predicted for the bell 29 on the basis of the theory set out above and the impact times given in Fig. 4, assuming that $Z=3\times10^4\ \mathrm{kg\,s^{-1}}$, are about 8 kHz for the unvoiced clapper and 2 kHz for the voiced clapper, compared with the measured values of 4 and 1.8 kHz, respectively. While far from exact, this agreement should be considered as satisfactory in a field experiment rather than a laboratory experiment, particularly since the bell impedance parameter Z, the clapper mass m, and the impact velocity V have all been estimated rather than measured. All of these parameters affect the impact time τ^* significantly, as indicated in Figs. 4 and 5. Choice of a rather larger value for Z would improve the agreement.

The measured behavior of the other bells investigated scales in just about the expected manner, as indicated in Table I, though the experimental data for bell 47 is inconclusive. For bell 9, the linear size scaling is by a factor 2.8, so that, by Eq. (6), the scaled value of Z should be about 2.4×10^5 kg s^{-1}. The scaling of the clapper diameter indicates a mass m of about 46 kg. Inserting these values in the theory, together with the impact flat diameter from Table I, gives τ^* values for the damaged and undamaged clapper of 0.1 and 0.23 ms, respectively. The resulting predicted turnover frequencies are 2000 and 870 Hz, respectively, compared with the measured values of 900 and 600 Hz. This agreement is similar to that for bell 29 and would again be improved by choice of a larger value for Z.

It is not appropriate in such a field experiment to seek to refine the agreement between theory and measurement by varying the parameters. Bearing in mind the many approximations involved, the agreement is fairly satisfactory.

VI. CONCLUSIONS

This study of the dynamics of bell clapper impact has elucidated many features of the problem that were presumably solved by the designers and builders of historic bells and carillons on the basis of experience founded upon trial and error. It is somewhat surprising that the bell clapper actually comes to rest against the bell surface until it is pushed away by a returning vibration pulse. On reflection, however, such a design results in the transfer of maximum energy from the clapper to the bell, and thus the achievement of the loudest possible sound for a given impact velocity. The parameter that principally determines this behavior, assuming the bell design to be fixed, is the clapper mass—too light a clapper will bounce, as well as having rather little energy to transfer.

Bell founders have long recognized that the sound of a carillon changes rather rapidly over the first year or so of its use, and then becomes more stable. This change is associated with the development of impact flats on the clappers, and initially proceeds quite rapidly until the clappers, and the inside surface of the bell, become more resistant to plastic deformation through work hardening. A voicing process that restores the shape of the clappers brings the sound closer to its original state, and the work-hardened clappers are now more resistant to plastic deformation. The work hardening itself, which has almost no effect on the elastic moduli, has no direct influence on impact dynamics or sound. Generally nothing is done about the deformation of the bell surface, though some carillons do allow a 180° rotation of the bells to the other equivalent impact position, a routine that ultimately halves the rate of bell deformation.

It is certain that a carillon sounds different to the musical ear and to the carillonneur after it has been re-voiced. It is interesting to determine what are the qualities of a re-voiced carillon that make it an improvement on its state before the operation.

The effects of re-voicing are two: (1) the impact time is lengthened so that the turnover frequency is reduced, giving a more ''mellow'' sound; and (2) the impact time becomes more dependent upon the impact velocity, so that ''soft'' notes have relatively little harmonic development and ''loud'' notes are both louder and ''brighter,'' as in most other musical instruments. These considerations are those that principally motivate the carillon curator to undertake the operation. In this, a carillon is rather different from an isolated bell or from a peal of bells, since the music it is called upon to produce involves harmonies, rather than simply melodic patterns.

Experience with analysis of the re-voicing of this particular carillon shows that details of the clapper impact are important. The analysis also points up the importance of matching each clapper to its bell in order to give a well-balanced tone quality and loudness across the compass. It is these things, as well as the design of the bells themselves, that distinguish a really fine carillon.

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