Oscillating reed valves—An experimental study

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The results of experiments on the threshold behavior and large-amplitude oscillation of ''outward-swinging door'' vibrating flap valves in an air environment are reported and compared with the predictions of a simple nonlinear theory that parametrizes aerodynamic effects by means of a simple damping coefficient together with a contraction coefficient for the flow. The agreement is acceptably good for the threshold blowing pressure for valve oscillation, the large-signal vibration amplitude, the pressure jump in the transition from threshold to large-signal behavior, and the variation in vibration frequency, all as functions of reservoir volume. The calculated pressure waveform in the reservoir has the observed phase and magnitude but fails to reproduce finer details. It is concluded that the simple theory provides an adequate account of the behavior of such valves. There are just two parameters in the theory, describing jet contraction and aerodynamic damping, respectively. Since these may depend significantly upon the detailed geometry, valves with different shapes may behave in quantitatively different ways.
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INTRODUCTION

Pressure-controlled vibrating valves are important in many contexts. They provide the sound-generating mechanism for the vocal utterances of birds, humans, and many other animals, and also for musical instruments of the brass and reed–woodwind families.

The geometric and dynamic complexity of many of these systems is great, since the vibrating element has many possible modes of deformation, but it has proved helpful to consider three simplified situations in which the motion of the valve can be described by a single displacement parameter x measuring the valve opening. We can then specify the behavior of the valve by a two-element parameter (σ_1, σ_2) in which $\sigma_1 = +1$ if a steady positive pressure applied to the upstream or inlet port of the valve tends to increase the valve opening x, and $\sigma_1 = -1$ if this pressure tends to close the valve. The second parameter σ_2 is defined similarly for a pressure applied to the downstream or exit port of the valve. For convenience, the configuration of a valve will be described simply by $(+,-)$, for example, rather than giving the complete symbol $(+1,-1)$.

We can recognize three distinct types of valves within this simple classification. Valves of type $(-,+)$, which can be pictured as inward-swinging doors or doors that are blown closed, are found in the reed mechanisms of woodwind instruments such as clarinets or oboes. Valves of type $(+,-)$ are like outward-swinging doors or doors that are blown open, and describe the motion of a trumpet player's lips over at least part of the range of the instrument. Valves of configuration $(+,+)$ are like sliding doors or doors that are blown sideways, and represent another possible motion of a trumpet player's lips. This ambiguity in the case of brass-instrument playing arises because description of the motion of the soft tissue of the lip really requires at least two displacement parameters,[1] and ideally a continuum description.[2] The same is true of human vocal folds and the syringeal membranes of birds, both of which could be approximated by $(+,-)$ or $(+,+)$ but are really more complex than this.[3,4]

The remaining valve type, described by $(-,-)$, does not appear to occur naturally or in any musical instruments, or to have any practical utility. This is perhaps because of its tendency to simply blow closed for any applied pressure.

There have been many studies of the behavior of $(-,+)$ valves in woodwind instruments, particularly clarinets, following early work by Backus,[5] but here the air column of the instrument is a dominant influence. A more general study of the acoustics of valves of both $(-,+)$ and $(+,-)$ types was carried out by Fletcher *et al.*,[6,7] while lip valves in brass instruments have been investigated by Elliott and Bowsher,[8] Yoshikawa,[9] Copley and Strong,[10] and others. The behavior of free reeds, as in harmoniums, has been studied by St. Hilaire *et al.*,[11] and more recently by Johnston,[12] Koopman, Cottingham *et al.*,[13,14] and particularly by Bahnson *et al.*[15] A summary of most of this work has been given by Fletcher and Rossing.[16]

The threshold for self-excited oscillation of the three simple valve classes in the absence of an attached resonator has been examined theoretically in another paper by one of the present authors.[17] In that paper it is shown that, if the acoustic impedance presented to the inlet of the valve is $Z_1 = R_1 + jX_1$ and that presented to the outlet is $Z_2 = R_2 + jX_2$, then a necessary condition for the initiation of self oscillation is that

$$\sigma_1 X_1 - \sigma_2 X_2 < 0. \tag{1}$$

This condition (1) is, however, not sufficient. In addition, there is a condition on the blowing pressure (or, more gen-

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erally, the pressure difference across the valve) that depends in detail upon valve geometry and internal damping.

In the case of valves of configuration $(+,-)$, which are considered in the present paper, the requirement $X_1+X_2<0$ can be achieved by supplying the valve from a reservoir of fixed volume V, in which case $X_1=-\rho c^2/V\omega$ (where ρ is the density of air, c the speed of sound, and ω the frequency of the oscillation), and allowing the valve to exhaust to the open air so that $X_2=0$. It is then found[17] that there is a preferred volume for the supply reservoir at which self-oscillation can be maintained at low blowing pressure; for larger or smaller reservoir volumes a greater threshold blowing pressure is required. The first part of our experimental study investigates these predictions.

The small oscillations of a valve are, however, generally unstable, and tend to grow rapidly to large amplitude. This transition, and the limiting state achieved by the large-amplitude oscillations, form another major part of the study. In contrast to the threshold determination, which can be described by linearizing the equations of flow and motion,[17] a description of this large-amplitude behavior necessarily involves proper treatment of the many nonlinearities involved.

I. BACKGROUND THEORY

The theoretical background that the experiments are designed to check has been given before.[17] We repeat here only its elements, and give details of a few refinements required to bring its assumptions closer to the realities of the experiment. The valve flap is taken to be a flexing plate of length L, width W, and effective mass m, clamped across one end. In the original theory, air was assumed to issue through the valve only across its tip, where the height of the opening is $x(t)$. To bring this treatment closer to the realities of experiment, we must allow for an additional escape of air through the opening of average width $a(x)$ along the sides of the valve, and through clearance gaps of width b between the valve tongue and the base plate. The total exit area of the valve is thus approximately

$$F(x)=W[x^2+b^2]^{1/2}+2L[a(x)^2+b^2]^{1/2}. \tag{2}$$

As already discussed, the valve is assumed to be fed from a reservoir of volume V, which is supplied with a steady volume flow U_0 from a high impedance source, and to exhaust to free air so that the downstream impedance is zero. The volume flow $U(t)$ through the valve is given by Bernoulli's equation, modified by a flow-contraction coefficient C, often written C_c in the literature, which has the value $C=0.61$ for flow through a sharp-edged slit.[18,19] The pressure must also be supplemented by a small term to represent the inertia of the air in the channel of length δ at the tip of the flap. The resulting equation for the pressure $p(t)$ in the reservoir is then

$$p=\frac{\rho U^2}{2C^2F(x)^2}+\frac{\partial}{\partial t}\left[\frac{\rho U\delta}{CF(x)}\right], \tag{3}$$

where ρ is the density of air. While the opening x is a linear variable for small oscillations of the valve tongue, it has a more complex form when the valve tongue enters the aperture of the backing plate or ultimately protrudes through its rear face. This is easily taken into account when the equations are solved numerically for large-amplitude oscillations.

The vibration of the valve flap is that of a simple cantilever, assumed to have natural frequency ω_0 and quality factor Q, so that, in the absence of air flow, its damping factor is $k=\omega_0/2Q$. This damping is provided largely by viscous losses in the surrounding air and, in some of our experiments, by added mechanical damping material near the root of the valve flap. As shown in the Appendix, the equation of motion of the flap, expressed in terms of its tip opening x, is

$$\frac{d^2x}{dt^2}+2k\frac{dx}{dt}+\omega_0^2(x-x_0)=\frac{1.5WLp}{m}, \tag{4}$$

where x_0 is the static opening of the valve flap with no applied pressure and m is the effective moving mass of the valve flap, as derived in the Appendix. For the experimental arrangement used, the valve tongue is initially flat and is raised a distance x_0 above the base plate. It then follows, as also shown in the Appendix, that

$$a(x)\approx 0.6x_0+0.4x. \tag{5}$$

In Eq. (4) we might expect the damping coefficient k to be a constant, determined by the combination of internal losses in the valve flap material and viscous damping in the surrounding air. The value of k should thus be determinable from the quality factor Q of the free oscillation damping of the valve flap by the relation $k=\omega_0/2Q$. Schlieren images of the oscillating valve, to be reported elsewhere, show however that a vortex develops downstream during the closing part of the valve cycle, and it would be surprising if this did not contribute damping to the mechanical oscillation of the valve. To allow for this possibility, we introduce an additional damping proportional to the change in momentum flux caused by the motion of the valve flap. Since the change in flow direction is presumably proportional to $(1/v)dx/dt$, where v is the air jet velocity as determined by Bernoulli's principle, it is a reasonable assumption that the extra damping is proportional to $\rho v x_0\, dx/dt$ per unit vibrating length of edge of the valve flap. We can then write

$$k=\frac{\omega_0}{2Q}+\beta\frac{\rho v(W+0.8L)x_0}{m}, \tag{6}$$

where β is a numerical coefficient which we expect to be approximately unity. In the calculations to follow, it is assumed that $\beta=1$, and the resulting aerodynamic damping is then small enough that its precise value is not important.

A third and final equation relates the pressure in the reservoir to the steady inflow U_0, the outflow $U(t)$ through the valve, and the change in reservoir volume caused by the valve vibration. This leads to

$$\frac{dp}{dt}=\frac{\rho c^2}{V}\left(U_0-U-0.4WL\frac{dx}{dt}\right). \tag{7}$$

The quantity $0.4WLx'$ is approximately the volume displaced by a cantilever strip of width W and length L when its tip is displaced from equilibrium by an amount x'.

A. Small-signal approximation

A simpler form of these three equations, linearized for the case of small oscillations, $x \ll x_0$, by writing U, x, and p in the form $U = U_0 + \tilde{U} \sin \omega t$ etc., formed the basis of the earlier analysis of oscillation threshold,[17] though in the examples computed there the flow-contraction effect, the flow inertia in the valve gap, the contribution of valve displacement to reservoir pressure, the flow through the sides of the valve, and the additional aerodynamic damping, were all omitted. For the particular values of the physical parameters of the valve used in the present experiment, in which both blowing pressure and frequency are rather low, it turns out that the contributions of flow inertia, and to a large extent aerodynamic damping, can indeed be neglected, but side flow and the flow-contraction correction must both be taken into account.

In the present paper, linearized versions of the full equations are used for threshold calculations, and the complete equations are solved numerically to predict the large-amplitude behavior. The linearized threshold equations are derived in the same way as before.[17] Neglecting the air-inertia term, the equation for the threshold pressure is now

$$- \frac{(2\bar{p}\rho)^{1/2} W L X_1 [2\bar{p} C (W + 0.8L) + 0.4\omega_0 W L X_1 C F(\bar{x})]}{2\bar{p}\rho + C^2 F(\bar{x})^2 X_1^2} > \tfrac{4}{3} k \omega_0 m, \tag{8}$$

where $\bar{p}$ is the average reservoir pressure, $\bar{x} = x_0 + 1.5\bar{p} W L / m\omega_0^2$ is the equilibrium opening of the valve under this pressure, $F(x)$ is given by (2), and $X_1 = -\rho c^2 / V\omega$ is the imaginary part of the acoustic impedance of the reservoir cavity. The principal modifications to the corresponding result derived in the original treatment are insertion of numerical factors of order unity, derived from the replacement of $W\bar{x}$ by $F(\bar{x})$ to account for air flow from the sides of the valve and through the clearance leaks and from the inclusion of the flow-contraction coefficient C, and insertion of the second term in the numerator, which is derived from the displacement of air by the valve motion. Note that the threshold, and indeed all the behavior later calculated for the valve, depends upon the profile of the valve tongue, if it is not flat, through the form of $F(x)$.

Another consequence of change in reservoir volume, in the case of a swinging-door $(+,-)$ valve, is an increase in vibration frequency with decreasing reservoir volume, because of the acoustic stiffness of the enclosed air. A simple linear treatment suggests that the valve oscillation frequency ω is given by

$$\omega^2 \approx \omega_0^2 + \frac{0.6\rho c^2 W^2 L^2}{mV}. \tag{9}$$

While the general trend of this result is correct, it ignores the effect of flow through the valve and overestimates the frequency shift. To obtain a reliable estimate of the frequency shift, Eqs. (3), (4), and (7) must be integrated numerically and the oscillation frequency determined directly.

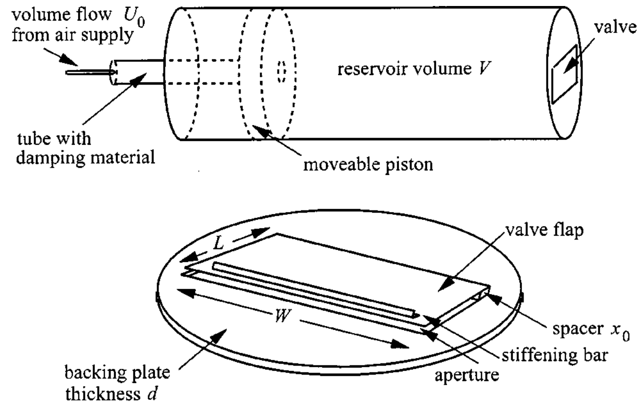

FIG. 1. The experimental setup. The upper part of the figure shows a general view, and the lower part a detailed view of the valve itself.

II. EXPERIMENTS

Figure 1 gives details of the experimental arrangement used. The valve flap was cut from flat brass sheet of either 0.33- or 0.15-mm thickness and was 28 mm in length and 52 mm in width. The relatively large width was chosen in order to give a good approximation to two-dimensional flow through the valve for associated experiments in flow visualization. The valve flap was stiffened by a thin aluminum bar of mass about 0.8 g, glued parallel to the tip to prevent lateral oscillations, which could be observed stroboscopically in the absence of such stiffening. The aperture under the valve was a little larger than the valve flap in both length and width, giving a clearance b of about 0.5 mm on all three sides, so that the valve was ''free'' rather than striking against the plate. A free rather than a striking valve tongue was chosen for study because of its greater simplicity—the exact course of any strike impact has a significant effect on the vibrational behavior of a striking valve flap.

The thickness of the aperture plate was 4 mm, so that the valve flap penetrated to its opposite side during very large-amplitude oscillation. The distance x_0 between the stationary valve flap and the plate could be varied by inserting a thin shim sheet, and was normally fixed at either 0.5 or 1.0 mm. The natural frequency of the valve oscillation was in the range 100 to 300 Hz, corresponding to the lowest natural frequency of the flexing cantilever, the actual frequency depending upon the sheet thickness and the added load, if any. The Q value, which was measured by the free decay behavior, was about 55 for the bare valve and could be reduced to as little as 10 by applying adhesive tape to the flap near its clamped end.

The upstream reservoir was constructed from heavy PVC pipe of inside diameter 101 mm, closed with a solid piston, the position of which could be changed to alter the reservoir volume. Air was supplied to the reservoir through a long, narrow tube terminating in a short coaxial pipe of 20-mm diameter filled with acoustically absorbent wool, as shown in Fig. 1. This arrangement minimized turbulence in the reservoir while adequately defining its volume. The isolation effectiveness of the absorbing material was checked by using, in one experiment, a much narrower tube. The air supply itself was drawn from a high-pressure source through a reducing valve, and therefore constituted a high-impedance constant-flow source, as assumed in the theoretical treatment.

The instrumentation for the measurement of all the important physical parameters included: a rotameter for the steady volume flow U_0, a water manometer for the average reservoir pressure p_0, a pressure transducer for the varying reservoir pressure $p(t)$, an accelerometer for the motion of the valve flap $x(t)$, a condenser microphone for the radiated sound waveform or alternatively for the reservoir pressure, and a fast Fourier transform (FFT) analyzer for the waveform and spectrum of the quantities involved.

A. Oscillation threshold

One of the major predictions of the linearized threshold theory[17] is that valves of configuration $(+,-)$ should be able to begin autonomous oscillation when fed at an adequate pressure from a closed reservoir, which is itself fed from a constant-flow source. The theory predicts that the threshold pressure for oscillation should depend upon the volume of the reservoir, there being in each case an optimal volume, determined by the physical parameters of the valve flap, near which the threshold pressure is lowest.

Most of the physical parameters of the valve were easily measured directly, but its resonance frequency and damping behavior required experimental determination. This was done by flicking the valve in a controlled manner and capturing the decay transient of its oscillation using a microphone and storage oscilloscope. The bare brass valve flap had little intrinsic damping, so that for many of the experiments the mechanical damping was augmented by adding strips of adhesive tape to both sides of the flap close to the clamped edge. In the same way, the resonance frequency of the valve flap could be changed by gluing strip masses near the tip of the valve flap. In all cases an adjustment was required in the calculations to allow for the mass added to the vibrating valve flap.

Figure 2(a) shows the measured pressure threshold for oscillation of the valve, as a function of reservoir volume, for various values of the mechanical Q factor. It is clear that there is indeed a volume-dependent threshold behavior of the type predicted by theory, and that the threshold pressure rises as the valve damping is increased. In Fig. 2(b) the threshold curves calculated from the theory leading to Eq. (8) are shown. There are no adjustable parameters in the theory, and the agreement with experiment is moderately satisfactory, given the approximate nature of the theory, and indeed of the experiment, since the threshold behavior may well depend upon subtleties of air flow.

Figure 3(a) shows similar curves with resonance frequency as the parameter. The resonance frequency was changed by adding mass to the valve tongue near its tip, and the effective mass of the tongue was evaluated from the shift in resonance frequency. The theoretical curves are shown in Fig. 3(b), and again there is broad agreement between theory and experiment. The agreement can be improved considerably by increasing the value of the aerodynamic damping coefficient β, but there is no immediate justification for this.

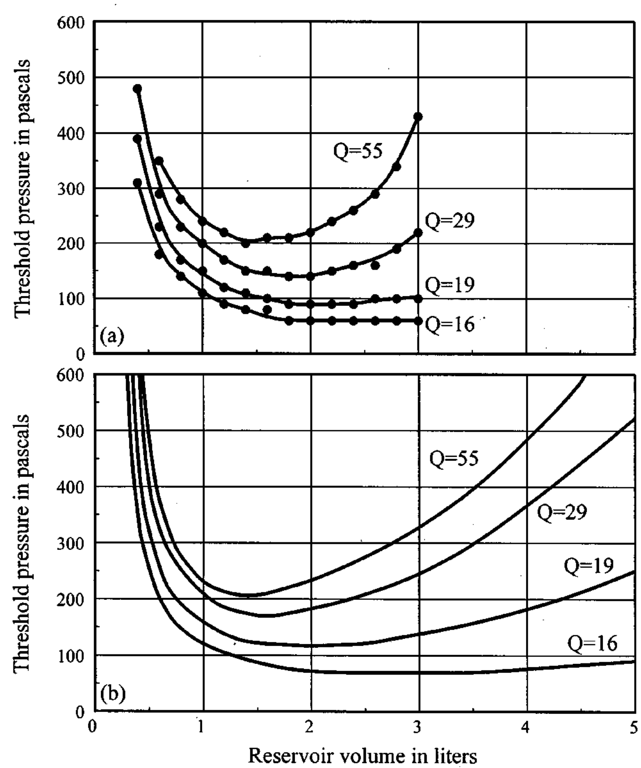

FIG. 2. (a) Measured threshold oscillation pressure of the experimental valve as a function of reservoir volume, with the Q value of the free valve flap as a parameter. For this valve the sheet thickness was 0.15 mm and the free resonance was 105 Hz. (b) Oscillation threshold as calculated from the theory.

B. Large-amplitude behavior

As already noted, the threshold vibration of the system is unstable, and rapidly grows to a limit cycle with a large amplitude and with an increased reservoir pressure. In the course of the experiments, this large-amplitude motion of the valve flap was monitored through a telemicroscope using stroboscopic illumination, and was also measured by twice integrating the output of a subminiature accelerometer attached to the flap near its fixed end. Once oscillation begins, the valve oscillation amplitude increases sharply, as does the reservoir pressure, until a limit cycle is reached in which the valve closes into the aperture plate for a time approaching half of each cycle. For very large oscillation amplitudes (exceeding about 5 mm in the case of our experiments) the valve flap emerges at the back of the aperture plate and allows secondary air flow during this part of the cycle. There is hysteresis in this behavior so that, once oscillation has begun, the air flow into the reservoir, and thus the reservoir pressure, can be reduced without the vibration stopping. The behavior of oscillation amplitude as a function of reservoir pressure is illustrated for the case of a valve with static opening $x_0 = 1$ mm and several different reservoir volumes in Fig. 4(a). In Fig. 4(b) the behavior calculated by integrating Eqs. (3)–(7) numerically is shown, assuming an initial displacement of the valve flap that is analogous to flicking it with a finger. Agreement between theory and experiment is again satisfactory, and the hysteresis can be understood from the fact that an oscillating valve of $(+,-)$ configuration presents a rather high, steady flow impedance because the res-

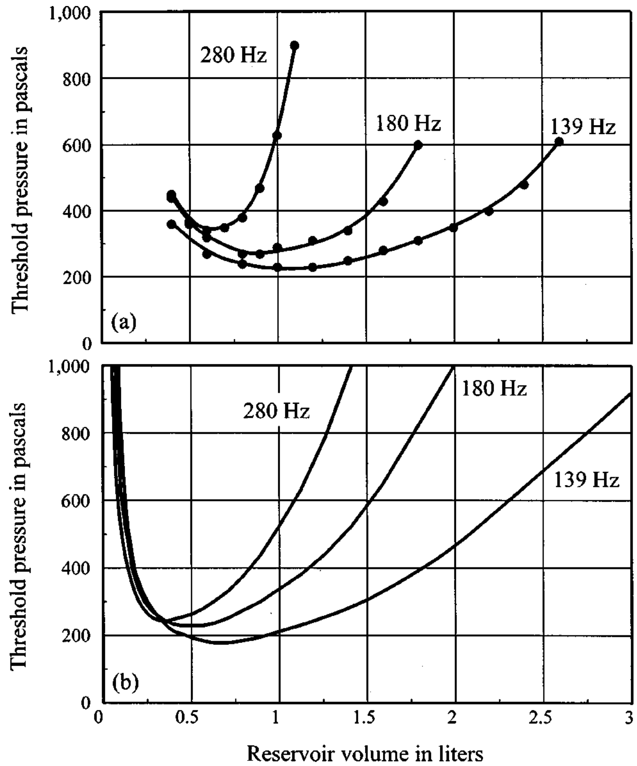

FIG. 3. (a) Measured threshold oscillation pressure of the experimental valve as a function of reservoir volume, with mass-loaded frequency as a parameter. The valve sheet thickness in this case was 0.33 mm and no stiffening bar was required. Measured Q values ranged between 44 and 52. (b) Oscillation threshold as calculated from the theory. The effective mass of the valve flap was scaled as $(\text{frequency})^{-2}$.

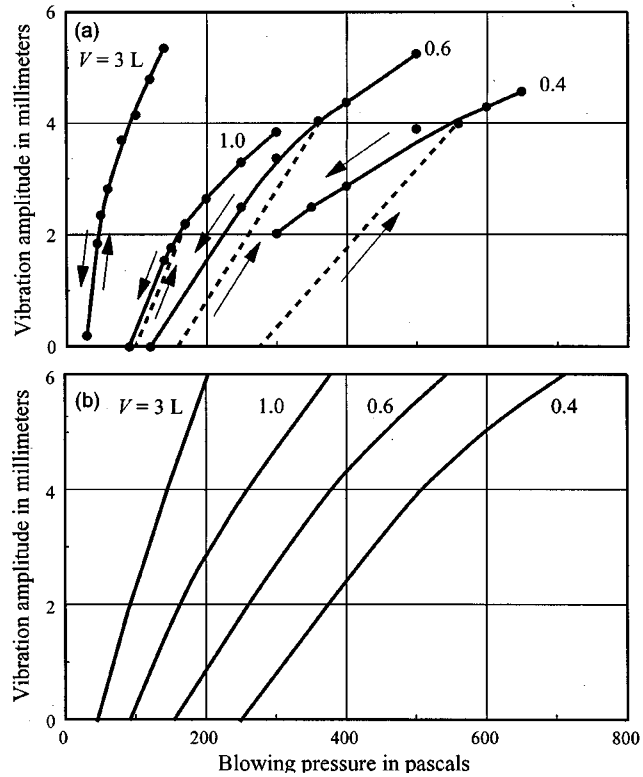

FIG. 4. (a) Measured oscillation amplitude, as a function of mean reservoir pressure, for four different reservoir volumes given in liters as a parameter, for a valve with flap thickness 0.15 mm and free resonance frequency 106 Hz. The arrowed curves show the hysteresis observed. (b) Calculated behavior omitting hysteresis effects.

ervoir pressure generated by its motion is such as to decrease the pressure, and thus the flow, when the valve is open.

The magnitude of the pressure jump when the oscillation increases from threshold to its limit-cycle value is another interesting experimental quantity, which is plotted in Fig. 5. The effect is greater for a small reservoir than for a large one, as might be expected. The pressure jump can be calculated from Eqs. (3)–(7) of the theory, essentially by performing numerical experiments for progressively increasing flow rates until oscillation begins. The calculated jump, shown as a full curve in Fig. 5, is in good agreement with the measurements, which are shown as data points.

It is also interesting to examine the dependence of the oscillation frequency of the valve on reservoir volume. The experimental results are shown as data points in Fig. 6 and the results computed from the large-signal theory as a full curve. The agreement is again acceptably good, the large-signal frequency shift being very much smaller than that predicted from small-signal theory in Eq. (9), which is shown as a broken curve in the figure. The frequency variation is neither predicted nor found to depend significantly upon blowing pressure or valve damping.

Another measurement was of the time-varying pressure within the reservoir, which was measured with either the pressure transducer or a 1/8-in. condenser microphone. The waveform of the pressure oscillation is a distorted sinusoid, as shown in Fig. 7(a), and its phase is very nearly 180° behind that of the valve opening displacement, shown in Fig. 7(b), which confirms that the major contribution to the pressure variation comes from the volume displacement of the moving flap. The motion of the valve flap, as monitored by the twice-integrated output of the accelerometer, is very closely sinusoidal, as is to be expected since it is operating very close to its natural resonance frequency. Integration of Eqs. (3)–(7) gives the form of the reservoir pressure variation shown in Fig. 7(c). The agreement with experiment is good in relation to amplitude and phase, and moderately good in relation to waveform, though clearly some of the details are not captured by the theory. The variation of the

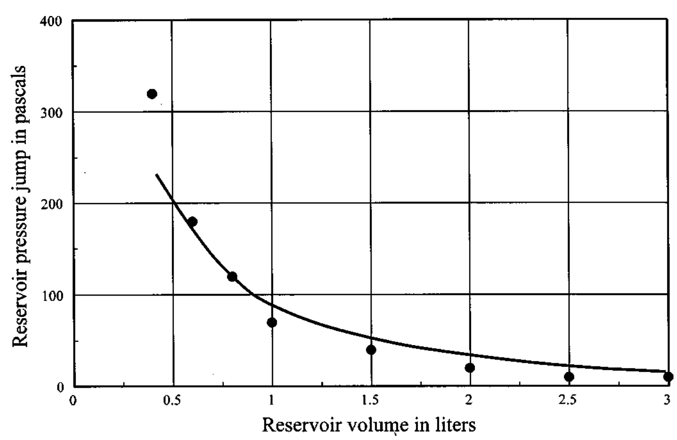

FIG. 5. Measured pressure increase as a threshold oscillation is allowed to grow to its steady state, as a function of reservoir volume (data points) for the valve of Fig. 4. The full curve shows the calculated pressure jump.

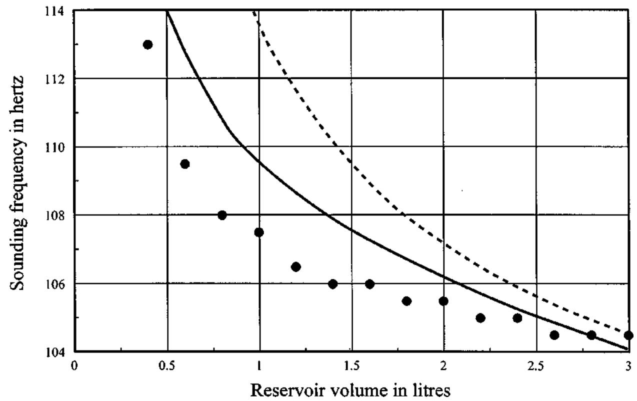

FIG. 6. Measured frequency of oscillation of the valve of Fig. 4 as a function of reservoir volume (data points). The full curve shows the theoretical prediction from numerical integration of the equations, while the broken curve shows the prediction of the simple equation (9). The vibration frequency is insensitive to blowing pressure over a large range.

pressure waveform when the valve is nearly fully open may perhaps be associated with the development of a downstream vortex in the flow, as observed in Schlieren photographs.

The radiated sound spectrum can, in principle, be derived from a knowledge of the total flow waveform $U(t) + 0.4WL\, dx/dt$, which includes both flow through the valve aperture and displacement flow. This is not reported upon here, however, since details of the spectral envelope depend upon rather fine aspects of the valve construction and reservoir geometry.

C. Striking valves

In this study a nonstriking valve was used because of difficulty in specifying precisely the nature of the striking contact. A brief subsidiary study of striking valves was, however, undertaken. To make the behavior as reproducible as possible, the valve flap was made a little larger than the plate aperture in all dimensions; the flap was clamped to the plate without an intervening spacer, and the valve flap was bent slightly to give a tip aperture of about 1 mm. When the valve closed, it did so around its whole periphery.

The experiments followed very much the same lines as those reported in detail above. The main difference observed was that the limit-cycle valve oscillation was of a much smaller amplitude, indeed less that than 2 mm, than for a free valve, presumably because the valve flap loses a great deal of energy during the impact. Stroboscopic study of the valve oscillation revealed complex behavior and the presence of higher cantilever modes, the amplitude of which depended upon the precise geometry of the striking contact. This fact is well-known to the voicers of organ reed pipes, since reed curvature must be properly adjusted to achieve satisfactory sounding behavior, although in that case the reed configuration is $(-,+)$. Much the same thing happens for striking $(+,-)$ valves and, because of this complication, the matter was not pursued further here.

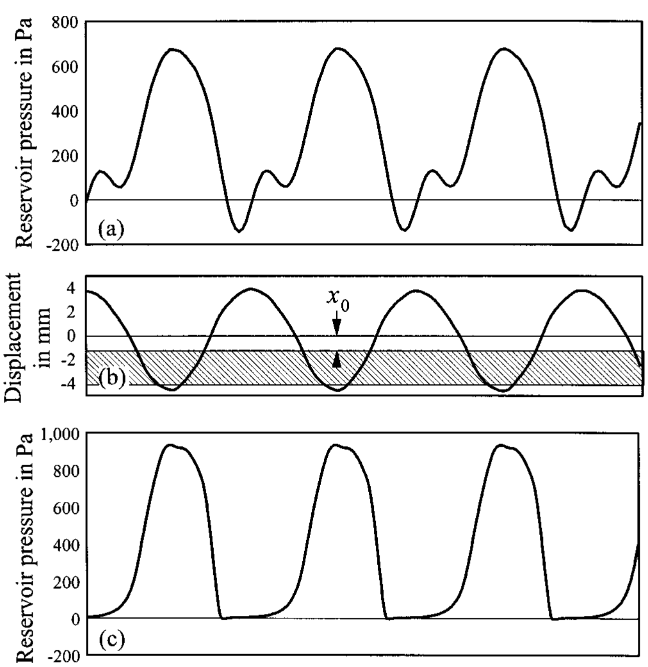

FIG. 7. (a) Measured waveform of the reservoir pressure for reservoir volume 0.8 L and an average reservoir pressure of 350 Pa. (b) Measured displacement of the valve flap (as an indication of relative phase). The shaded region shows the portion of the cycle during which the tip of the flap is within the base-plate aperture. (c) Calculated reservoir pressure waveform.

III. CONCLUSIONS

This experimental study of an oscillating-flap valve of the outward-swinging door $(+,-)$ type shows that its behavior is well accounted for by a simple theory, without invoking any adjustable parameters or aerodynamic complications. The agreement between theory and experiment is not exact, but is as good as could reasonably be expected, bearing in mind the subtleties of reed adjustment that are involved in the sounding behavior of reed-driven musical instruments. A reasonable explanation of this sensitivity to flow conditions is the fact that the contraction coefficient C can actually vary between 0.5 and 1.0 depending upon the precise geometry of the flow aperture.[19] There is also the possibility that the aerodynamic damping coefficient β might be similarly affected.

In the course of the experiments certain interesting aerodynamic phenomena were in fact observed, particularly the formation of a downstream vortex during the closing phase of the oscillation of the valve. The behavior of this vortex will be reported on elsewhere, for its own intrinsic interest, since the present study suggests that it does not have a major effect on the behavior of the valve.

Finally, we note that only the $(+,-)$ outward-swinging door type of valve has been studied here. The fact that the simple theory,[17] with the refinements introduced above, gives a good account of the behavior of valves of this type, however, leads us to expect that it should be able to give a similarly accurate description of valves of $(+,+)$ and $(-,+)$ configuration, which were also included in its formulation.

ACKNOWLEDGMENT

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APPENDIX

Let $\xi(s,t)$ be the form of the displacement of the valve flap, with s measuring distance from its clamped root. The equation of motion of the flap then has the form

$$[\rho_v Wh + m'\,\delta(s-s')]\frac{\partial^2\xi}{\partial t^2} + R\frac{\partial\xi}{\partial t} + K\frac{\partial^4\xi}{\partial s^4} = Wp(t), \tag{A1}$$

where ρ_v is the material density, W the width, h the thickness and K the bending stiffness of the valve flap, and R is its damping coefficient. It is also allowed that there is a further mass m', representing the stiffening bar, fixed to the valve at a distance s' from its root.

Since we are concerned with oscillation near the fundamental cantilever mode of the flap at frequency ω_0, we can write $\xi(s,t) = [x(t) - x_0]\psi(s)$ where $\psi(s)$ is the form of the mode function,[20] normalized so that $\psi(L) = 1$. Multiplying both sides of (A1) by $\psi(s)$ and integrating over the flap length L then gives

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega_0^2(x - x_0) = \frac{\gamma WLp}{m}, \tag{A2}$$

where k is a new damping coefficient given by (6), m is the effective mass of the valve flap as given by

$$m = \rho_v WLh + \frac{m'L\psi(s')^2}{\int_0^L \psi(s)^2 ds} \approx \rho_v WLh + 4m', \tag{A3}$$

and

$$\gamma = \frac{\int_0^L \psi(s)\,ds}{\int_0^L \psi(s)^2 ds} \approx 1.5. \tag{A4}$$

The approximate result in (A3) derives from the form of $\psi(s)$ and from the fact that $s' \approx L$ for the stiffening bar. Since $s' \ll L$ for the damping material added near the clamped edge of the flap, the contribution of its mass to m is negligibly small.

From a knowledge of the form of $\psi(s)$, we can also evaluate the area of the vertical component of the side opening beneath the valve when its tip opening is x. For the case of a planar valve flap, as in the experiment,

$$a(x) = x_0 + (x - x_0)\int_0^L \psi(s)\,ds \approx 0.6x_0 + 0.4x. \tag{A5}$$

If the flap is curved, as in many free-reed musical instruments, then the form of $a(x)$ may differ considerably from this. The form (A5) is, of course, valid only if the flap does not enter the aperture block. If this happens, then a more complicated expression must be used.

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