Wave Propagation on Turbulent Jets

by S. Thwaites and N. H. Fletcher
Department of Physics, University of New England, Armidale N.S.W. 2351, Australia

Summary

Results of experiments involving the propagation of acoustically generated waves on turbulent plane jets are presented. The jets are produced by a perspex reconstruction of an organ pipe flue and their behaviour is shown to conform to the predictions of similarity and self-preservation; i.e. the jet velocity $V \sim x^{-1/2}$ and the jet-width is proportional to x, where x is the distance from the slit. Thus they are fully turbulent and, as a consequence, well behaved and reproducible. The measured wave velocities for frequencies between 200 Hz and 1200 Hz and slit-widths between 0.5 mm and 1.0 mm are compared with similar measurements on laminar jets. However, the stable nature of the turbulent jet allows much higher blowing pressures, up to 1500 Pa, to be used. The wave velocities u for the turbulent jet vary as $x^{-1/2}$ and so the ratio of the wave velocity to the jet centre-velocity is constant along the jet. This ratio is closely equal to 0.5 except for small values of the efflux velocity and slit-width where it rises to 0.6.

Wellenausbreitung über turbulente Strahlen

Zusammenfassung

Es werden die Ergebnisse von Versuchen zur Ausbreitung akustisch angeregter Wellen über turbulente, ebene Strahlen vorgelegt. Die Strahlen wurden erzeugt durch eine aus Perspex bestehende Nachbildung eines Orgelpfeifen-Generators. Es wird gezeigt, daß ihr Verhalten die Bedingungen der Ähnlichkeit und der Selbsterhaltung erfüllt. D.h., die Strahlbreite ist proportional zu x und für die Strahlgeschwindigkeit gilt $V \sim x^{-1/2}$, wobei x die Entfernung vom Schlitz ist. Die Strahlen sind also völlig turbulent und reproduzierbar. Die für Frequenzen zwischen 200 Hz und 1200 Hz und Schlitzbreiten zwischen 0,5 mm und 1 mm gemessenen Wellengeschwindigkeiten werden mit ähnlichen Messungen an laminaren Strahlen verglichen. Die Stabilität des turbulenten Strahls erlaubt jedoch die Verwendung sehr viel höherer Blasdrücke von bis zu 1500 Pa. Die Wellengeschwindigkeiten u für den turbulenten Strahl ändern sich mit $x^{-1/2}$, wodurch das Verhältnis der Wellengeschwindigkeit zur zentralen Strahlgeschwindigkeit über den Strahl konstant ist. Dieses Verhältnis liegt nahe bei 0,5 außer für kleine Werte von Ausströmgeschwindigkeit und Schlitzbreite, für die es sich auf 0,6 erhöht.

Propagations d'ondes sur jets turbulents

Sommaire

On rend compte d'expériences relatives à la propagation d'ondes engendrées par voie acoustique sur des jets turbulents plans. Les jets sont produits par un modèle en plexiglas reproduisant un tuyan d'orgue à anche. Leur comportement a été trouvé conforme aux prédictions des lois de similarité et de conservation c'est-à-dire que la vitesse V du jet est proportionnelle à $x^{-1/2}$ et que la largeur du jet est proportionnelle à x, qui est la distance à la fente de l'anche. Ainsi ces jets sont complètement turbulents et par conséquent bien conformés et reproductibles. Les vitesses des ondes ont été mesurées à des fréquences de 200 à 1200 Hz et pour des largeurs de fente comprises entre 0,5 et 1,0 mm. On les a aussi comparées des résultats similaires obtenus avec des jets laminaires. Cependant la stabilité des jets turbulents a permis d'utiliser des pressions de soufflage beaucoup plus élevées pouvant aller jusqu'à 1500 Pa. Les vitesses u des ondes sur les jets turbulents varient comme $x^{-1/2}$ et de ce fait le rapport des vitesses u aux vitesses V au centre du jet est constant tout le long du jet. Ce rapport est très approximativement égal à 0,5 excepté pour les petites valeurs de la vitesse d'écoulement et de la largeur de la fente, auquels cas il s'élève jusqu'à 0,6.

1. Introduction

The problem of the propagation of waves on jets disturbed by an acoustical field is one which has benefited relatively little from any recent advances in the mathematical analysis of turbulence so that not much more is known now than was first illuminated in the classic works of Rayleigh [1], [2]. Although Rayleigh studied it for its intrinsic interest, the problem is important in the understanding of musical instruments like the organ flue pipe and flute. The difficulty with the mathematically formidable nature of the problem makes experimental investigation the most useful avenue for gaining some understanding of the phenomenon. To this end this paper presents the results of experiments designed to represent more closely the situation in real organ pipes than previous experiments and to overcome some of the difficulties encountered in them.

Coltman [3] has measured the propagation velocities of waves on an acoustically perturbed laminar jet for blowing pressures up to 150 Pa. Fletcher and Thwaites [4] have performed similar measurements for frequencies from 100 Hz to 1000 Hz and blowing pressures from 20 to 100 Pa. Even the highest pressure of 150 Pa, however, is low for a jet in an organ pipe and the reason for not proceeding to higher pressures was given by Fletcher and Thwaites as being the development of unstable, complex behaviour in the jet when the pressure was greater than this. It is evident from the blowing pressures typical of organ pipes, (200 to 2000 Pa), and the acute nozzle geometries that the jet is in all probability turbulent. The common practice of "nicking", where small serrations are made in the flue, would appear to guarantee turbulent flow. In addition, the necessity of having reliably reproducible and well-behaved jet flow in an organ pipe means that it is unlikely that the flow is laminar.

The experiments done up until now on this problem have all used nozzle geometries similar to that of Fletcher and Thwaites [4] and have been careful to produce laminar flow. In the present investigation, since it was the jet in organ pipes that was of interest, it was decided that it would be exactly this kind that was produced and for this reason the design of an organ flue pipe was followed in the construction of the nozzle. The results are presented in two parts where the first one is concerned entirely with the unperturbed jet. This paper is not intended as a study of plane turbulent jets, a subject which has been amply covered by other workers. It is, however, felt that an understanding of the jet being perturbed is essential in a total understanding of the problem. The results of the experiments in which the jet is re-perturbed are presented in the second section.

2. The jet

The experimental arrangement was as shown in Fig. 1. The jet nozzle used was constructed from perspex to resemble a typical organ flue pipe and, with the pipe attached, it sounded over a normal range of blowing pressures. In this way it was hoped that the jet being investigated was very similar to the jets produced in real organ flue pipes. In particular it was hoped that the jet was fully turbulent and would behave simply. The slit was 30 mm long and its width could be varied from zero to 2 mm. The languid was easily removed and replaced by others of different thickness or geometry.

The x and y coordinates are defined as is customary for planar jets with x along the jet and y across the plane of the jet, the origin being at the slit opening. The jet velocity in the x direction was obtained as a function of x and y from pressure measurements using a Pitot-tube whose traversing mechanism was attached to a multi-turn potentiometer. The Pitot-tube was connected to a pressure transducer.

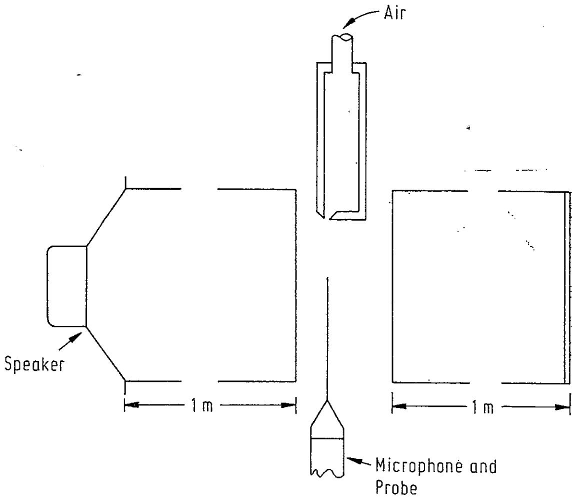

Fig. 1. Schematic diagram of the experimental arrangement. A loudspeaker feeds into a 2 m pipe which is divided in the middle and closed at the far end. The flue directs the jet across the gap and the probe-microphone points into the air-stream.

The particular jets studied were those with slit-widths, $2l$, equal to 0.5, 0.7 and 1 mm. The blowing pressures, P_0, as measured at the nozzle, were 400, 800 and 1500 Pa since these pressures encompass the range from the onset of fully turbulent flow, at a little below 400 Pa, to the highest pressure convenient for the experiments described in the next section. The languid configuration shown in Fig. 1 was used for all the measurements except when stated otherwise. It is a design termed an "inverted languid" by organ builders. The languid thickness, d, was 6 mm.

The pressure along the jet centre plane, $y = 0$, was measured for $x = 0$ to $x = 20$ mm for all the slit-widths and blowing pressures. The jet centre line velocity, $V(x)$, obtained from this was plotted against x. In every case we could write for $V(x)$, when $x \gtrsim 3$ mm,

$$V \cong A\,x^{-1/2}\,. \qquad (1)$$

This result is consistent with measurements of fully turbulent plane jets by a number of experimenters, [5], [6]. The initial portion of the jet when $x \lesssim 3$ mm was characterised by almost no decrease in the velocity, as was also observed in other experiments, and is the region of free mixing layer flow. The region characterised by the $x^{-1/2}$ law is the fully developed jet. The simple form of eq. (1) is to some extent fortuitous and and depends on exact flue geometry. More generally we would expect $(x - x_0)^{1/2}$ $x_0 \sim l$.

Following Townsend [7], the principles of similarity and self-preservation, when applied to plane turbulent flow, lead to the conditions that $V \propto x^{-1/2}$ and $b \propto x$ where b is some measure of the jet half-width. Thus Townsend gives for a plane fully turbulent jet

$$V = V_0 [l/x^{1/2}] f(y/x) \tag{2}$$

where f is some function giving the profile of the jet and V_0 is the velocity of efflux. This same result can be obtained using the speculation of Fletcher and Thwaites [4] that the major effect of turbulence is to increase the effective kinematic viscosity, ν', by an amount depending on the Reynolds number of the flow, so that

$$\nu' = \nu [1 + F(Vb/\nu)] \tag{3}$$

where ν is the kinematic viscosity in the absence of eddy diffusion. If the function F is dominated by its linear term, then substitution of this expression in Bickley's [8] equations for a laminar jet,

$$V = 0.4543 \ldots (J^2/\nu x)^{1/3} \operatorname{sech}^2(y/b) \tag{4}$$

and

$$b = 3.635 \ \ldots (\nu^2/J)^{1/3} x^{2/3}, \tag{5}$$

yields

$$V \propto x^{-1/2} \tag{6}$$

and

$$b \propto x \tag{7}$$

where

$$J = \int_{-\infty}^{\infty} V^2 \, \mathrm{d}y \simeq 2 l V_0^2 . \tag{8}$$

The jet profiles were measured for $2l = 0.5$ mm and $P_0 = 800$ and 1500 Pa at distances varying between 5 and 20 mm from the slit. From the curves of $V(y)$ obtained, the value of y at which V fell to half its value at $y = 0$ was plotted against x and good agreement to a straight line was obtained, as expected, in each case.

Since it was not the intention of these experiments to investigate turbulent jets in detail a sufficient idea of the range of variation of the ratio b/x was obtained from eq. (1) rather than from detailed measurements of every jet. An effective jet half-width, b', was defined using the conservation of momentum flux to be

$$b' = \frac{V_0^2 l}{V^2} . \tag{9}$$

The ratio b'/x was plotted against the Reynold's number $V_0 l/\nu$. Over the range of efflux velocities and slit widths used in all the experiments the ratio varied by a factor of 2. This represents a variation by approximately the same amount in the angle of divergence of the jet. In general this angle increased with increasing Reynolds number but the scatter of the data was sufficient to make it difficult to conclude what the detailed dependence was. The other parameter of the experiment on which b'/x could be reasonably expected to depend is the ratio, d/l, of the languid thickness to the slit width. However, experiments performed for $2l = 0.5$ mm with languids ranging in thickness from 3 mm to 12 mm gave velocities, $V(y)$, and ratio b'/x almost identical to the original values.

As stated before, it is hoped that the jet used in these experiments is typical of jets on organ flue pipes. The important conclusion to be drawn from the results of this section is that the jet closely resembles an ideal, fully turbulent jet and is well behaved.

3. Propagation of acoustically generated perturbation

The experiments described in this section are essentially similar to those described by Coltman [3] and by Fletcher and Thwaites [4] but some important modifications have been made. Fig. 1 gives a schematic picture of the present experiment.

The jet is perturbed at the nozzle by a flow field generated by a 10 cm loudspeaker. It was found that the flow field produced between a pair of speakers facing each other and connected in antiphase as described before was insufficient to perturb the jet enough to make measurements possible. A much larger flow was obtained by placing the 10 cm speaker in one end of a long pipe which was closed at the other end. The pipe used was 2.0 m long and 0.1 m in diameter. The speaker was then driven at those frequencies which give a velocity maximum halfway down the pipe. The pipe was in two segments each 1 m long with a gap at the position of maximum flow and the jet blew, with its plane perpendicular to the pipe axis, into the flow field through this gap.

Measurements of the phase velocity of the disturbances on the jet were performed using a condenser microphone equipped with a very fine probe tube pointing into the air-stream. The zero-crossings of the jet produced pressure pulses in the microphone and by filtering the microphone output at twice the oscillator frequency a signal corresponding to these zero-crossings was obtained. The microphone with the probe tube was traversed along the jet in the x direction from $x = 3$ to $x = 20$ mm and the phase of the filtered signal was plotted against x. These curves were used to obtain the velocity of the wave as a function of x. The perturbation frequencies used to generate the waves were the six frequencies between 200 and 1100 Hz corresponding to the appropriate harmonics of the 2 m pipe.

The phase velocities, u, calculated from the measurements showed a steady decrease with increasing x for all the slitwidths, blowing pressures and frequencies used. In fact for all the results we can write with quite a reasonable degree of accuracy

$$u \propto x^{-1/2} \tag{10}$$

for the fully developed jet. Thus for every jet we have

$$\frac{u}{V} = \text{constant.} \tag{11}$$

This result is interesting in the light of the speculation of Fletcher and Thwaites [4]. They found, both experimentally and on the basis of dimensional considerations, for the phase velocity of waves on a laminar jet

$$u = \text{const} \times \frac{J}{\nu}. \tag{12}$$

They again substituted eq. (3) for the kinematic viscosity, ν, assuming the linear term to be dominant, and obtained for the phase velocity of a wave on a turbulent jet

$$u = \text{const} \times V \tag{13}$$

which is in agreement with eq. (11).

In the present experiments there appeared to be no systematic variation of u/V, with the frequency for any given jet. In fact u/V was close to a constant, independent of frequency, within the accuracy of our measurements ($\pm 10\%$). It was also apparent that u/V changed very little as V_0 and l were changed. However, it was decided to plot u/V against

$$\xi \equiv J/\omega^{1/2}\nu^{3/2},$$

as the simplest possible dimensionless parameter of interest. This is shown in Fig. 2 for all the measurements. As can be seen, u/V varies very little over the whole range except for values of $\xi \lesssim 7 \times 10^5$. For larger values of ξ, u/V attains a nearly constant value corresponding to a phase velocity very close to half the local jet centre-velocity (independently of everything else). The range and accuracy of the experimental data does not allow us to come to any clear conclusion about the functional form of the relation shown in Fig. 2. There may be a genuine limiting value for u/V at large ξ, but the data is equally well fitted by a relation of the form

$$u/V = B\,\xi^{-n} \tag{14}$$

where $n \cong 0.13$, this being the curve drawn in the figure. The current experimental results are sufficiently scattered to obscure a weak frequency dependence of the form implied by eq. (14).

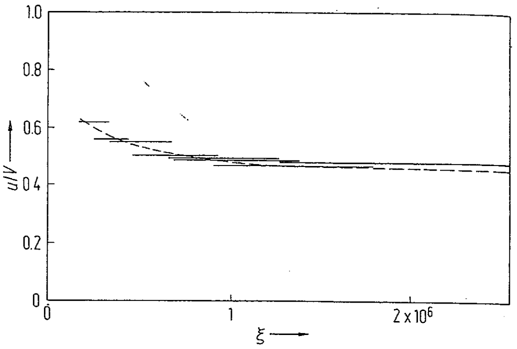

Fig. 2. Normalized wave velocity u/V plotted against the dimensionless parameter $\xi = J/\omega^{1/2}\nu^{3/2}$. Each heavy line represents a set of results having constant $J = 2l\,V_0^2$ and varying frequency. The dashed curve is given by eq. (14).

We know of no other published measurements of the velocities of propagation of waves on turbulent jets with which to compare our results. However even comparison with results for laminar jets [3], [4] yields some interesting consistencies. Coltman [3] studied the wave propagation on a laminar jet having $2l = 1.59$ mm and efflux velocities from 4 to 15 m s^{-1}. The jets travelled up to 7 mm from the slit with almost no spread and thus $V(x)$ was approximately constant. For x greater than a half-wavelength he found the phase velocities to be constant along the jet with the ratio u/V between 0.44 and 0.50. This was independent of frequency. Both these conclusions and the current results are consistent with Rayleigh's [2] equation for the propagation velocity, u, on an inviscid jet. In the limit when $2\pi\, l/\lambda \gg 1$ this becomes

$$u = \frac{V}{2} \tag{15}$$

where λ is the wavelength.

4. Conclusion

The results obtained from these experiments suggest that the jets produced by organ pipe flues closely resemble ideal, fully turbulent jets. They are well behaved and reproducible over the range of slit-widths and blowing pressures present in typical organ pipes. The propagation characteristics of acoustically generated waves on these jets follow simple rules which are, in some aspects, similar to the rules for the propagation of waves on laminar jets. In particular the wave velocity is approximately equal to one half the local jet centre-velocity over most of the range and the propagation is not frequency dispersive.

This paper is part of a programme of research in musical acoustics which is supported by the Australian Research Grants Committee. Suszanne Thwaites is indebted to the University of New England for the award of a Research Solarship.

(Received April 15th, 1979.)

References

[1] Lord Rayleigh, On the instability of jets. Proc. London Math. Soc. **10** [1879], 4 reprinted in Scientific Papers of Lord Rayleigh, Vol. 1. Dover, New York 1964, pp. 361 – 371.

[2] Lord Rayleigh, The theory of sound, Vol.2. Reprinted: Dover, New York 1945, pp. 376–414.

[3] Coltman, J. W., Jet drive mechanisms in edge tones and organ pipes. J. Acoust. Soc. Amer. **60** [1976], 725.

[4] Fletcher, N. H. and Thwaites, S., Wave propagation on an acoustically perturbed jet. Acustica **42** [1979], 323.

[5] Bradbury, L. J. S., The structure of a self-preserving plane jet. J. Fluid Mech. **23** [1965], 31.

[6] Gutmark, E. and Wygnanski, I., The planar turbulent jet. J. Fluid Mech. **73** [1976], 465.

[7] Townsend, A. A., The structure of turbulent shear flow, Cambridge University Press, Cambridge 1956.

[8] Bickley, W. G., The plane jet. Phil. Mag. **28** [1937], 727.