Accretion

Let us compute the total available energy.
Considering a proton falling in from infinity, we can write (Longair p. 134)

$$\frac{1}{2} m v^2_{free\text{-}fall} = \frac{G M m}{r}$$

When the matter reaches the surface of the star at r=R,

the kinetic energy of the free-fall (part of it) has to be radiated away as heat.

If the rate at which mass is accreted onto the star is d m/dt,

the rate at which kinetic energy is dissipated at the star surface is ½ dm/dt v^2,

and hence the luminosity of the source is

$$L = \frac{1}{2}\, dm/dt\; v^2_{\text{free-fall}} = \frac{G\; M\;\; dm/dt}{R} \qquad \frac{c^2}{c^2}$$

Accretion efficiency

Efficiency = η = $GM / c^2 R$

$L = \eta\;\; dm/dt\;\; c^2$

LUMINOSITY = $\mathbf{L = \eta \, dm/dt \; c^2}$

$R_{sch} = 2GM/c^2$

Efficiency = $\mathbf{\eta = GM / c^2 R = \frac{1}{2} \; R_{sch} / R}$

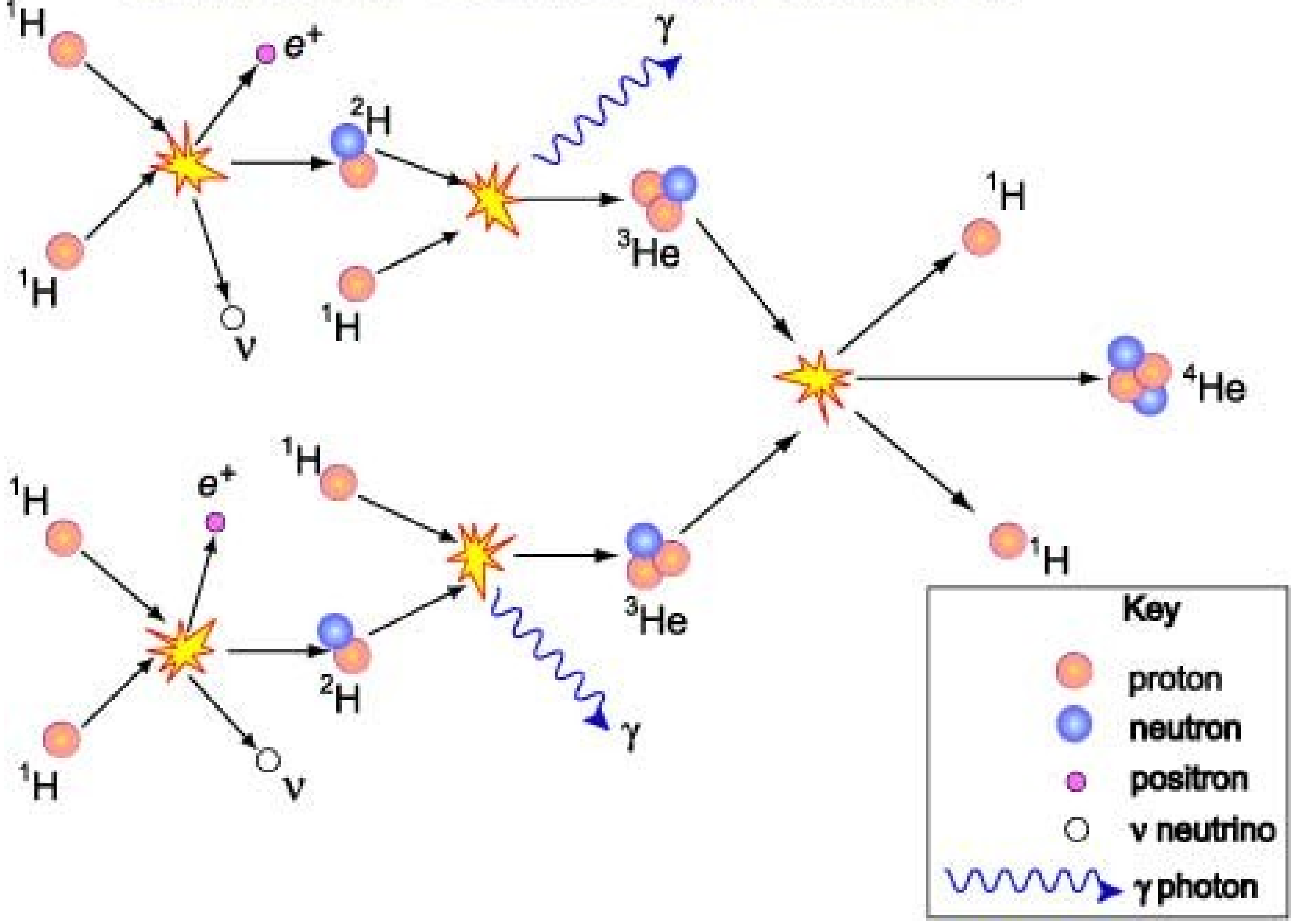
This is a remarkable formula .

It can be seen that written in this form η is the *efficiency* of conversion of the rest mass energy of the accreted matter into heat .

According to the above calculation, the efficiency of energy conversion simply depends upon how compact the star is.

Thus , accretion is a powerful source of energy. This efficiency of energy conversion can be compared with the η of nuclear energy generation.

Main Form of Proton-Proton (pp) Chain in Sun



This efficency of energy conversion can be compared with the η of nuclear energy generation.

Accretion process: Efficiency = $\eta = GM / c^2 R$

Neutron Star – $r_{in} \sim$ 10 km → η= 0.1 ----------------------------> 10%

..of the rest mass energy of the accreted matter into heat) .

Nuclear fusion process:
Efficiency = $\eta = (4\, m_p - m_\alpha) / 4\, m_p$

$$\frac{(4 \times 1.6726\ 10^{-24} - 6.642 \times 10^{-24})}{4 \times 1.6726\ 10^{-24}} = 0.007$$

For nuclear reactions in stars $\eta \sim 0.007$ ----------------------> <1% !!!

Thus , accretion is a powerful source of energy.

Accretion efficiency

Efficiency = η = GM / c^2 R= ½ r_{sch}/ R

$$r_{sch} = 2\ GM/c^2$$

White dwarf M=1 M sol, R=5000 Km → η= 3 x 10^{-4}

Neutron Star – r_{in} ~ 10 km → η= 0.1

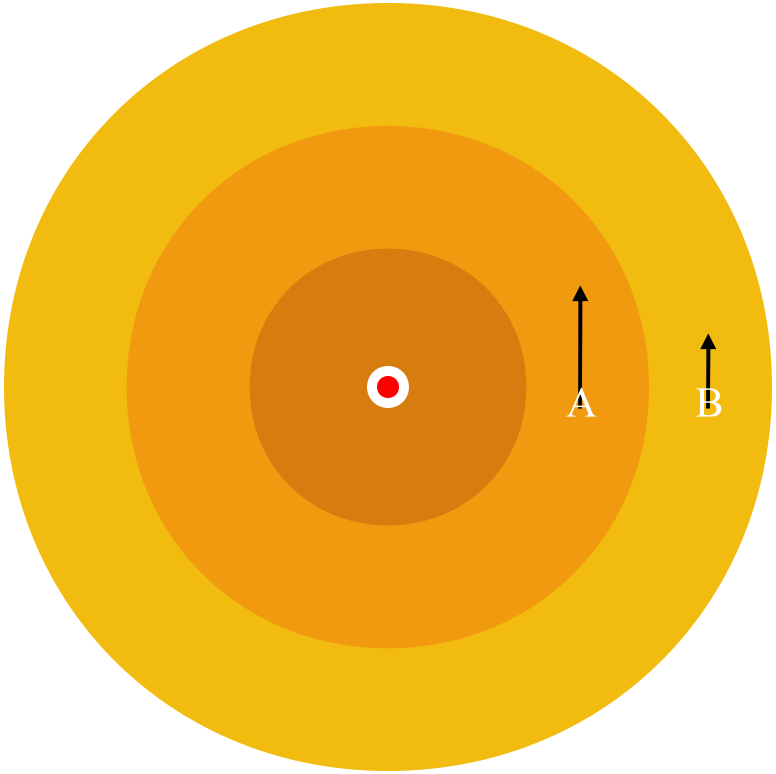
Black Hole - r_{in} = 3rs → η ~ 0.06

But from GR for rotating black holes η = 0.42-------------> >40%

For nuclear reactions in stars η ~ 0.007 ----------------------> <1% !!!

Outward angular momentum transport

Ring A moves faster than ring B. Friction between the two will try to slow down A and speed up B.



Keplerian rotation

So ring A must move inward! Ring B moves outward, unless it, too, has friction (with a ring C, which has friction with D, etc.).

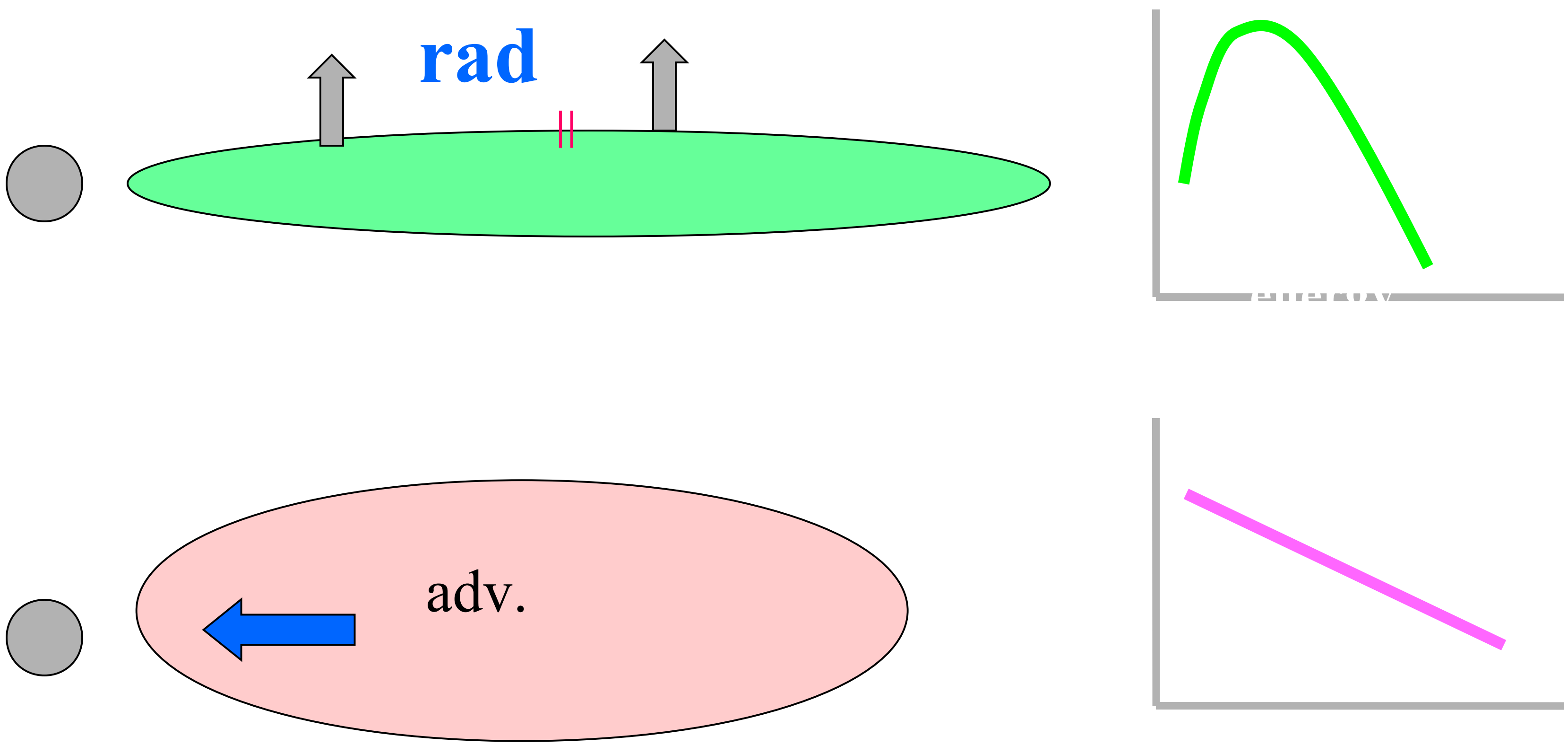
The “standard model”… Viscous accretion disks

- Suppose that there is some kind of “viscosity” in the disk
  - Different annuli of the disk rub against each other and exchange angular momentum
  - Results in most of the matter moving inwards and eventually accreting
  - Angular momentum carried outwards by a small amount of material
- Process producing this “viscosity” might also be dissipative… could turn gravitational potential energy into heat (and eventually radiation)

Standard Accretion Disk Model (Shakura and Sunyaev 1973) : α

MRI (Balbus and Hawley 1991) can generate magnetic turbulence and enhance the efficiency of angular momentum transport

State Transition in Accretion Disks



Eddington Limit

Radiation coming from the disk carries radiation pressure.

Radiation pressure is felt by accreting matter --→

eventually radiation pressure becomes higher than gravitational pull of compact object/star and <u>accretion stops.</u>

Radiation pressure force will be proportional to luminosity (more photons=more radiation pressure)

The limiting luminosity at which an object can accrete is:

$$L_{edd} = \frac{4\,\pi G M m_p}{\sigma_T}$$

σ_T = Thomson cross section

Derived for spherical accretion but approximately correct also for accretion disk

Obtain Ledd by setting Fgrav=Frad

Fgrav (gravitational force per electron) = GM $(m_p+m_e)/r^2 \sim$ GM m_p/r^2

Frad = (Number photons x Thompson cross-section) x p

Energy of typical photon = $h\nu$
The number of photons crossing unit area in unit time at radius r is:
$L/ h\nu\ 4\pi r^2$
<u>Number of collisions</u> per electron per unit time= $L\ \sigma^T/ h\nu\ 4\pi r^2$
Each photon gives a momentum p= $h\nu$ /c to the electron in each collison

$$\text{Frad} = L\sigma_T/ h\nu\ 4\pi r^2 \quad x \quad p = L\sigma_T/4\pi r^2 c$$

(The radiation pressure acts upon the electrons, however protons and electrons coupled by Coulomb interaction)

Obtain Ledd by setting Fgrav=Frad

$$Fgrav = GM\, m_p/r^2$$
$$Frad = L\sigma_T/4\pi r^2 c$$

$$Ledd \;\; \sigma_T/4\pi r^2 c \;\; = \;\; GM\, m_p/r^2$$

$$Ledd \;\; = \;\; 4\pi c\, G\; M\, m_p\; /\sigma_T$$

$$L_{edd} \;\; = \;\; 1.3\; 10^{38}\; M/M_0 \;\; erg/sec$$

X-ray binary luminosities

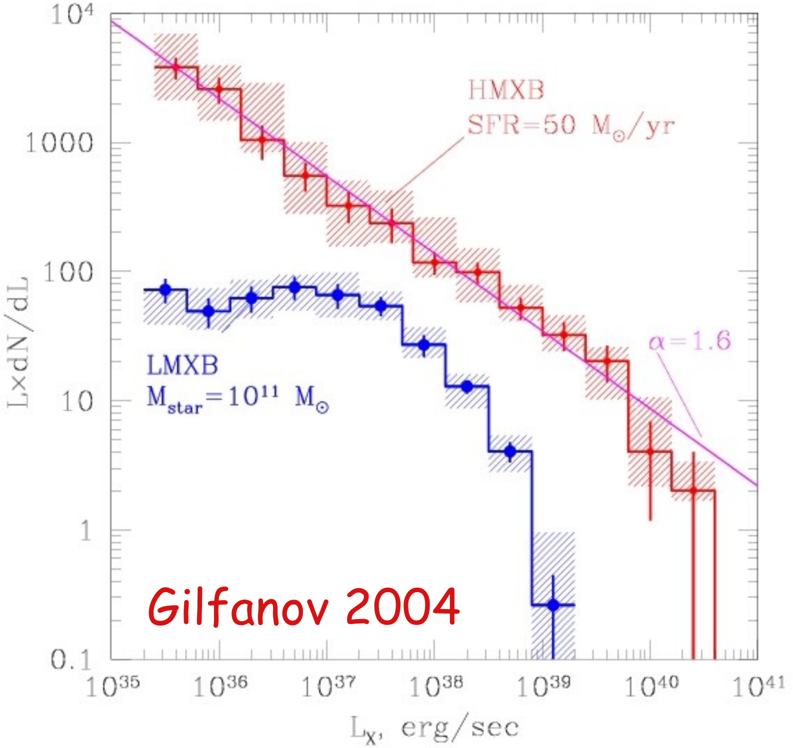
X-ray binaries typically have L_X<<10^{38}erg/s

LMXRBs:

- Flat distribution at faint-end
- max luminosities ~10^{38}-10^{39}erg/s.

HMXRBs:

- Power-law distribution
- Max L_X~ 10^{40}erg/s



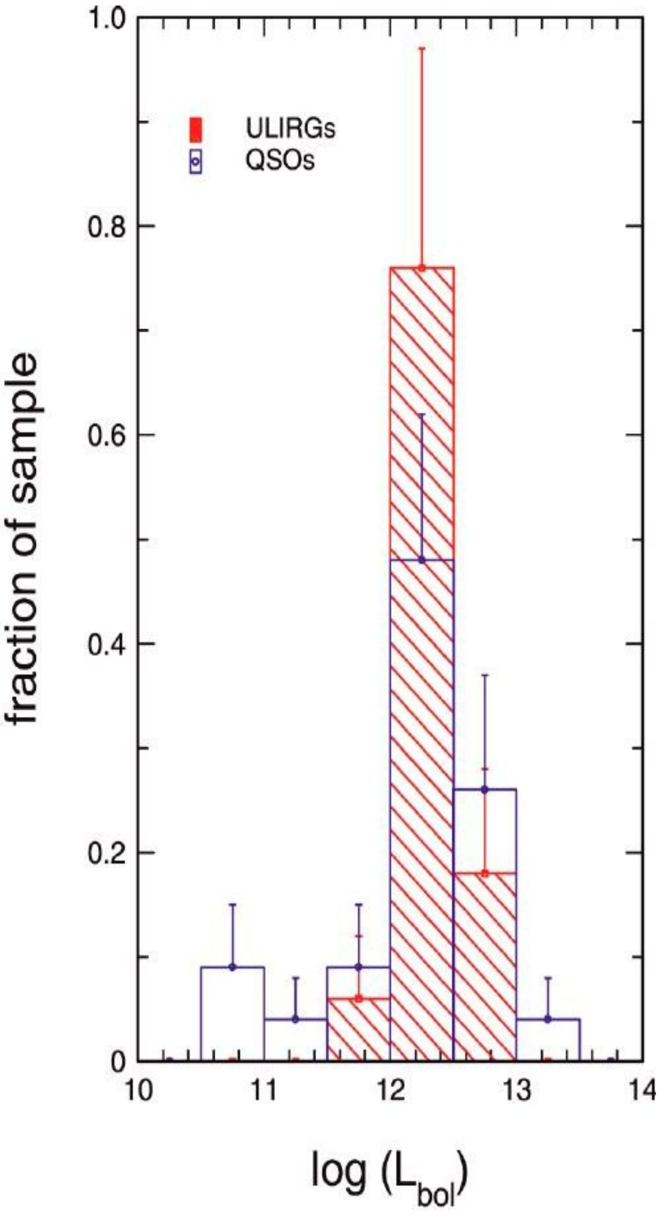
Supermassive BH

$$L_{edd} = 1.3\ 10^{38}\ M/M_0\ \text{erg/sec}$$

$Ledd/L_0 = 10^5\ M/M_0$

for M/M_0 in the range of $10^{6-8} M/M_0$

$Ledd/L0 \quad 10^{11-13}$



3.1 The Nature of the Compact Object

The most reliable method to determine the nature of the compact object is the study of the Doppler shift of absorption lines in the spectrum of its companion. The study of the changing radial velocity during the orbital motion is a technique that has been applied for more than one hundred years to measure the masses of stars in binary-systems. The same method is applied for systems like X-ray binaries, where one component is "invisible". In this case the variations of the radial velocity of the normal companion during its orbit are studied.

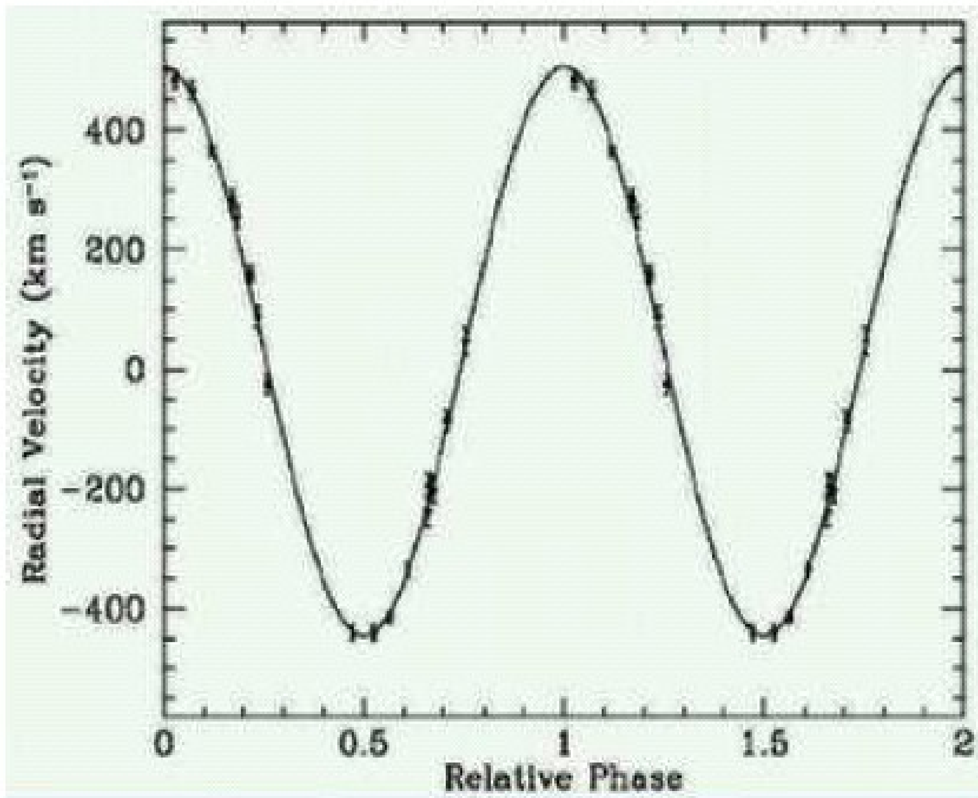
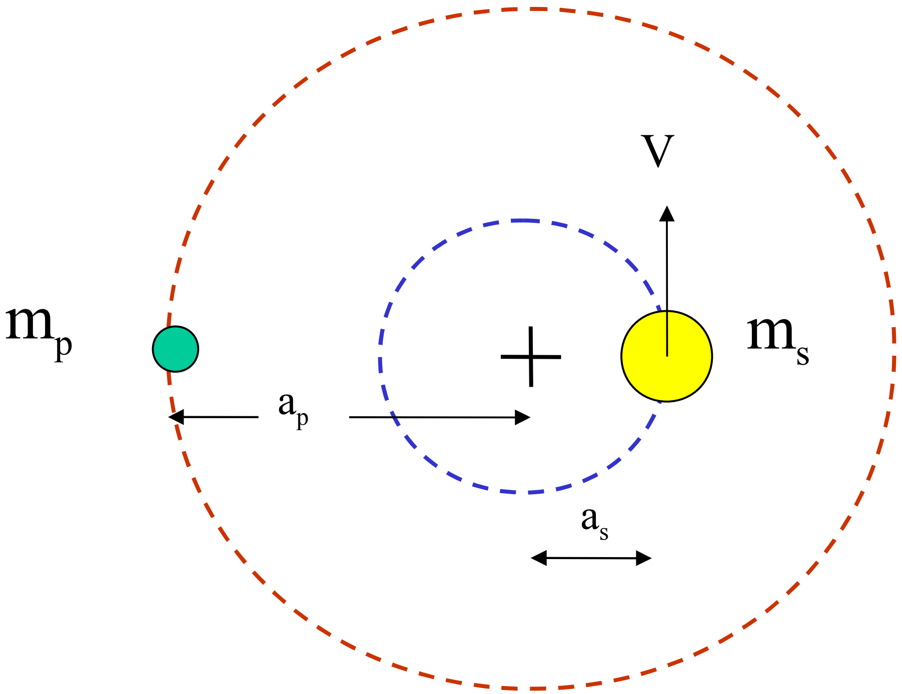


Figure 6: : **Amplitude of the radial velocity variations versus orbital** phase(Filippenko et al. 1999; GRS 1009-45). Using the Doppler shift of spectral



$$P^2 = \frac{4\pi^2 (a_s + a_p)^3}{G(m_s + m_p)}$$

Measuring Masses of Compact Objects

Dynamical study: compact object$_x$ and companion star$_c$

(for binary period, P, and inclination angle, i)

Kepler's 3rd Law: $4\pi^2 (a_x + a_c)^3 = GP^2 (M_x + M_c)$

center of mass: $M_x a_x = M_c a_c$

radial velocity amplitude $K_c = 2\pi a_c \sin i \; P^{-1}$

"Mass Function": $f(M) = P K^3 / 2\pi G = M_x \sin^3(i) / (1 + M_c/M_x)^2 < M_x$

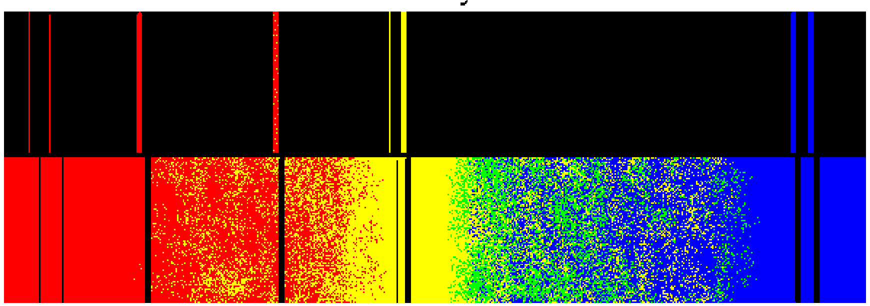
Dynamical Black Hole: $M_x > 3\ M_o$ (maximum for a neutron star)

BH Candidates: no pulsations + no X-ray bursts + properties of BHBs

Doppler shifts

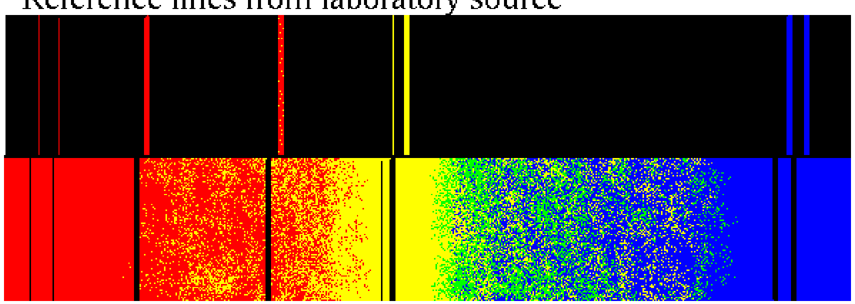
Doppler shifts of the spectral lines yield the radial (i.e. toward the observer) velocity of the star

Reference lines from laboratory source



Absorption lines from star

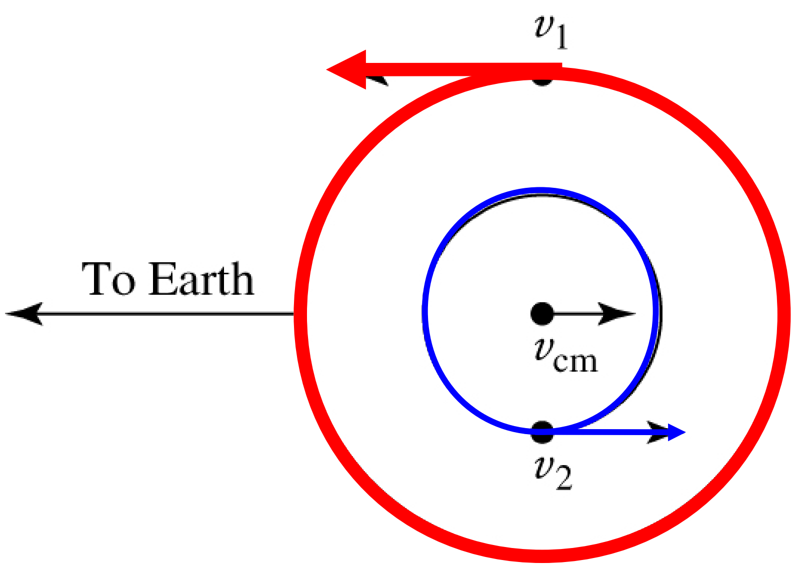
Reference lines from laboratory source

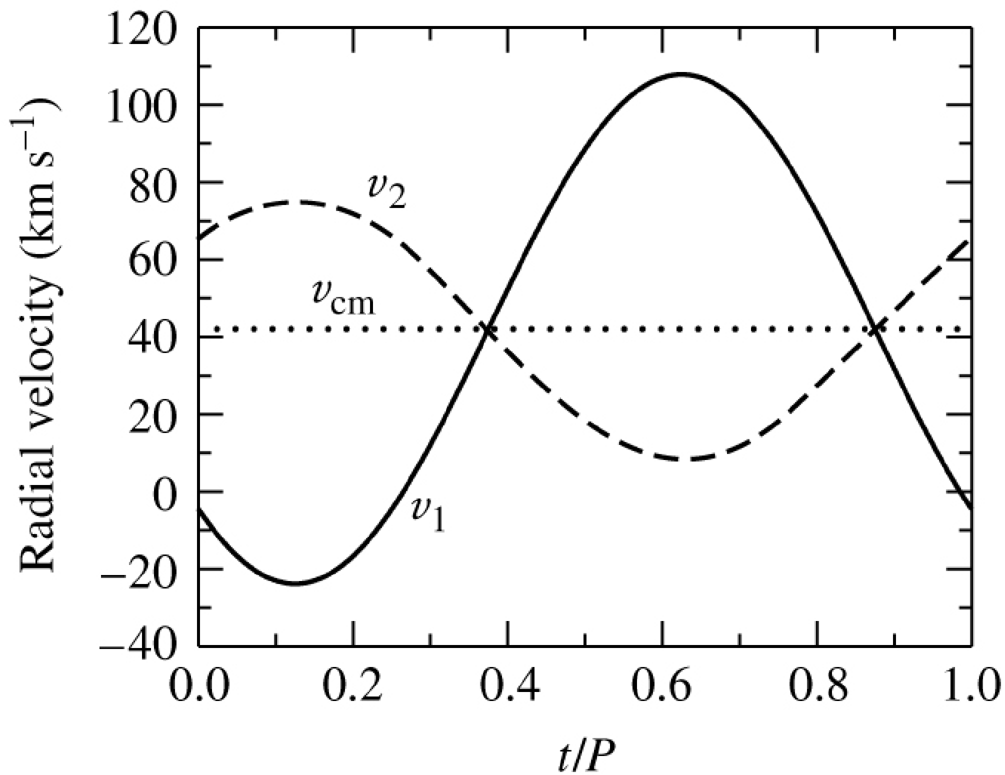


Absorption lines from star

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}}$$

$$\frac{v_r}{c} \approx z \quad \text{if } z << 1$$

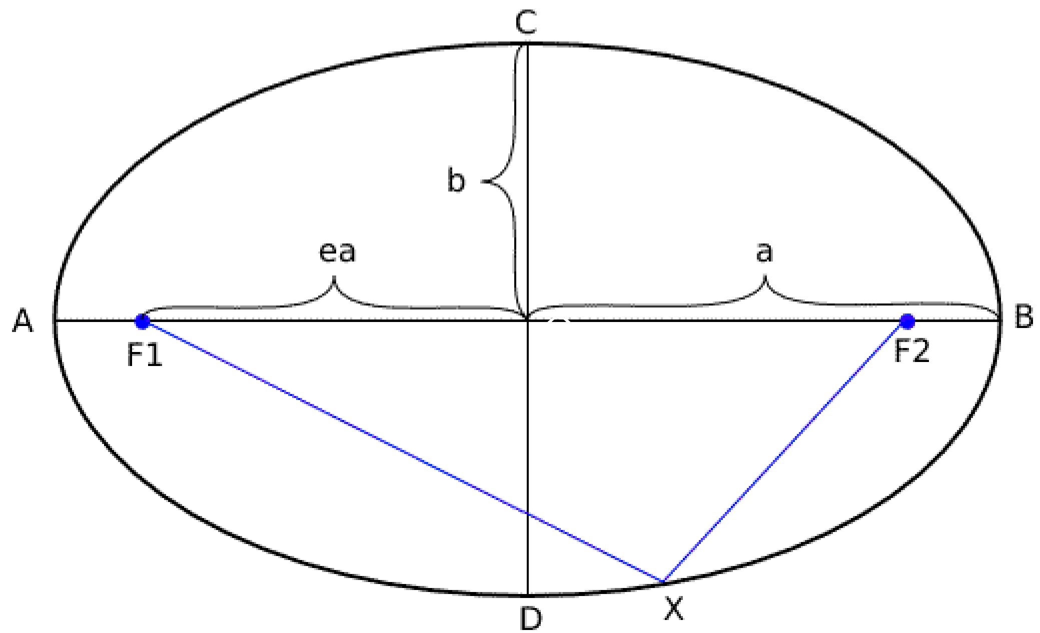




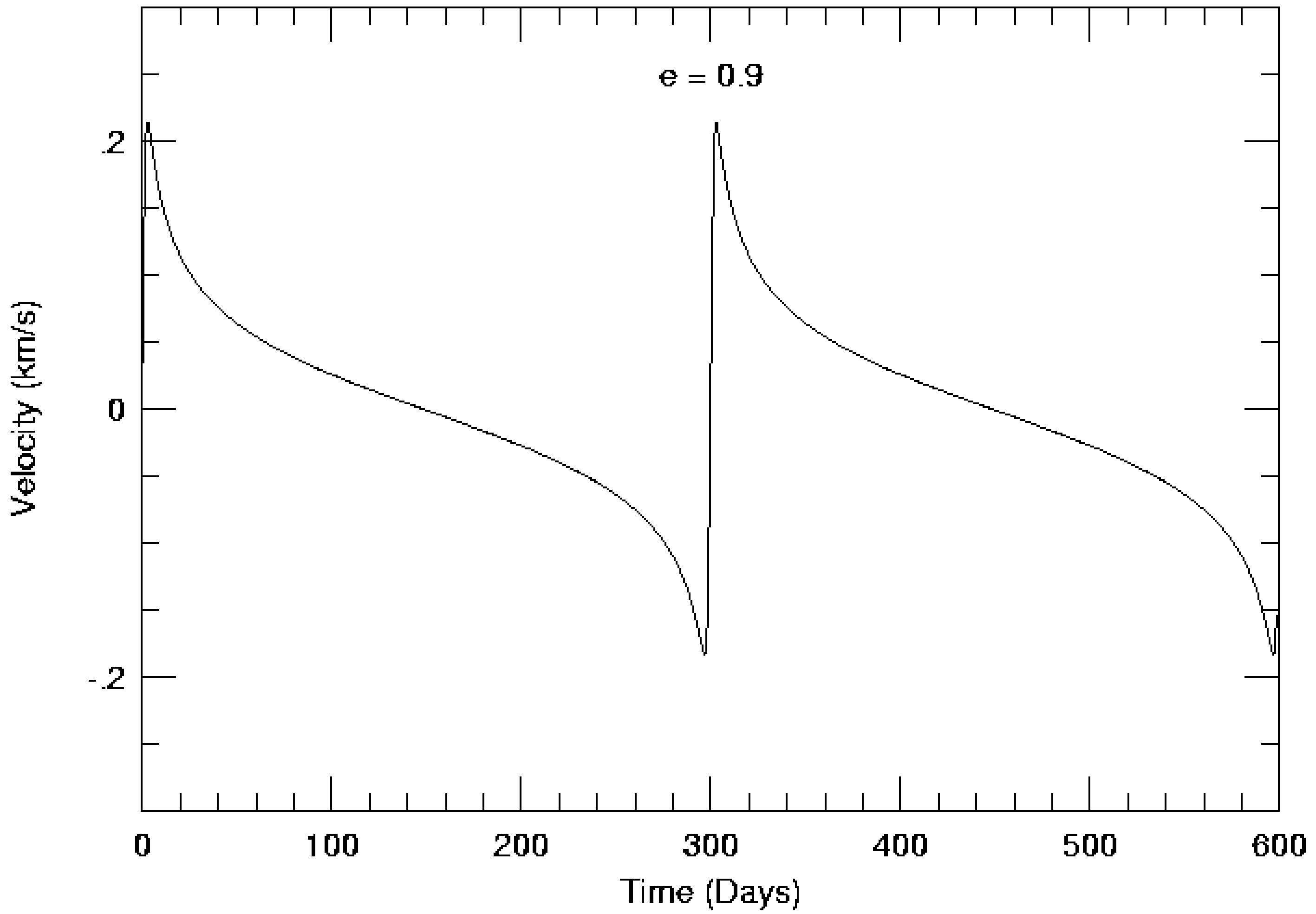
- If the orbit is in the plane of the sky (i=0) we observe *no radial velocity.*
- Otherwise the radial velocities are a sinusoidal function of time. The minimum and maximum velocities (about the centre of mass velocity) are given by

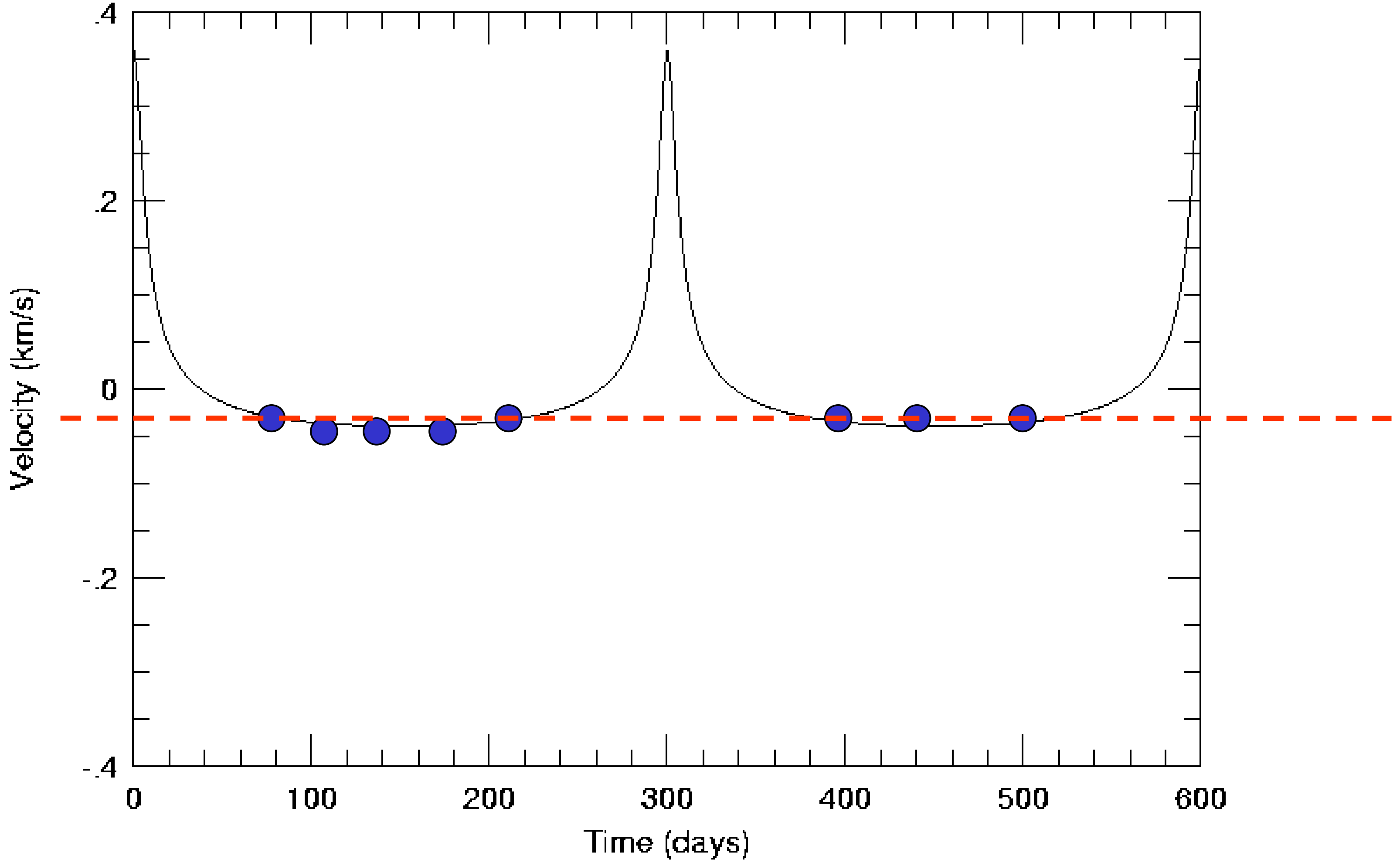
$$v_{1r}^{max} = v_1 \sin i$$
$$v_{2r}^{max} = v_2 \sin i$$

Elliptical Orbits



Radial velocity shape as a function of eccentricity:




.4
.2
0
-.2
-.4
Velocity (km/s)
0
100
200
300
400
500
600
Time (days)

Mass Function

mass M_1 and M_2 with orbital period P

(semi major axis a_1 and a_2 with $a_1 M_1 = a_2 M_2$)

seen under an inclination angle ***I***

radial velocity of component 1 is seen to with amplitude K_1

for a circular orbit

$$K_1 = 2\pi a_1 \sin i / P_b.$$

using Kepler's laws

expressed in observed quantities we can calculated the mass function

$$f(M_2) \equiv \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{4\pi^2}{G} \frac{(a_1 \sin i)^3}{P_b^2} = \frac{K_1^3}{2\pi G} P_b.$$

for known sin I and $M_1 > 0$ this will be a lower limit on the compact star mass;

for a complete solution one needs the light curve as additional data

Best Black Hole X-ray Binaries

Binary	Likely $M_X(M_\odot)$	$f(M) \equiv M_{X,min}(M_\odot)$
4U1543-47	5 ± 2.5	0.22 ± 0.02
GRO J0422+32	10 ± 5	1.21 ± 0.06
GRO J1655-40	7 ± 1	2.73 ± 0.09
SAX J1819.3-2525	10.2 ± 1.5	2.74 ± 0.12
A0620-00	10 ± 5	2.91 ± 0.08
GRS 1124-683	7 ± 3	3.01 ± 0.15
GRS 1009-45	4.2 ± 0.6	3.17 ± 0.12
H1705-250	4.9 ± 1.3	4.86 ± 0.13
GS 2000+250	10 ± 4	4.97 ± 0.10
XTE J1118+480	7 ± 1	6.0 ± 0.3
GS 2023+338	12 ± 2	6.08 ± 0.06
XTE J1550-564	10.5 ± 1	6.86 ± 0.71
XTE J1859+226	10 ± 3	7.4 ± 1.1
GRS 1915+105	14 ± 4	9.5 ± 3.0

Figure 7: : **Black hole candidates.** Compact objects with a mass (M_X) greater than 3 $M_\odot$, upper limit for a stable neutron star. Ramesh Narayan.

http://cgpg.gravity.psu.edu/events/conferences/Gravitation_Decennial/

“It is worth mentioning here that the accumulation of accreted material on the surface of a neutron star triggers thermonuclear bursts. These are called bursts of Type I.

No Type I burst has ever been observed from a compact object where optical observations resulted in a mass above 3 $M_\odot$.
That fact might confirm that in black holes there is no surface where material can accumulate “(Narayan & Heyl 2002).

<u>Observations of Type I bursts give a direct evidence for a neutron star</u>.

Compact Object Mass

Neutron Star Limit: 3 M

$(dP/d\rho)^{0.5} < c$

Rhoades & Ruffini 1974

Chitre & Hartle 1976

Kalogera & Baym 1996

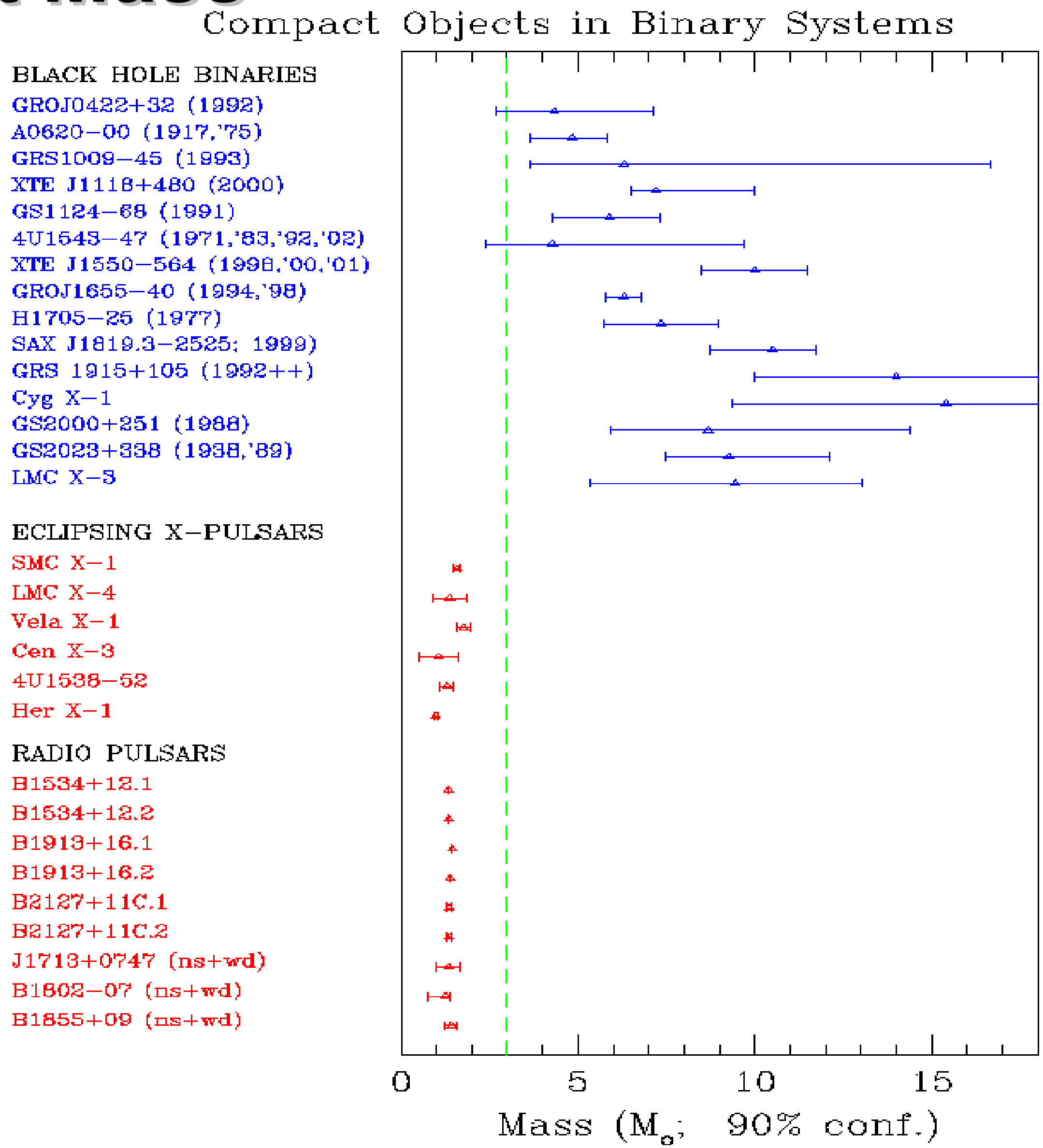
Black Holes (BH)

$M_x = 3\text{-}18\ M_o$

Neutron Stars (NS)

(X-ray & radio pulsars)

$M_x \sim 1.4\ M_o$



Demorest, P. B.; Pennucci, T.; Ransom, S. M.; Roberts, M. S. E.;
Hessels, J. W. T.

Nature, Volume 467, Issue 7319, pp. 1081-1083 (2010).

.... Here we present radio timing observations of the binary millisecond pulsar J1614-2230 ......**We calculate the pulsar mass to be (1.97+/-0.04) Msolar**

Inventory of Black Hole Binaries

BH Binary: Mass from binary analyses

	Dynamical BHBs
Milky Way	18
LMC	2
local group	1 (M33)
---------------------	---------------------
total	**21**

Transients 17

Black Holes in the Milky Way

Jerry Orosz